Proceedings of the 38th Conference on Decision & Control
Phoenix, Arizona USA • December 1999

Positive Feedback Stabilization of Compressor Surge

Frank Willems [1], Maurice Heemels [2], Bram de Jager [1] and Anton Stoorvogel [3]
Eindhoven University of Technology
P.O. Box 513, 5600 MB Eindhoven, The Netherlands

Abstract

Stable operation of axial and centrifugal compressors is limited towards low mass flows due to the onset of surge. The stable operating region can be enlarged by active control. In this study, we use a control valve which is nominally closed and only opens to stabilize the system around the desired operating point. Hence, only non-negative control values are allowed which complicates the controller design considerably. A novel positive feedback controller is proposed with clear design parameters to obtain a desirable closed-loop behavior. The technique has successfully been applied to a compression system model. For arbitrarily large control valve capacities, the system can be stabilized in the entire operating region. Simulations show that the surge point mass flow can be reduced up to 15% for the relatively small control valve to be implemented on the actual installation. Using this efficient control strategy, the stabilized operating point is reached with zero control valve mass flow.

1 Introduction

Compressors are widely used for the pressurization of gases. Applications involve air compression for use in aircraft engines and industrial gas turbines, and pressurization and transportation of gas in the process and chemical industries [1]. Towards low mass flows, the stable operating region of axial and centrifugal compressors is bounded due to the occurence of aerodynamic flow instabilities: *rotating stall* and *surge* [4]. These instabilities can lead to the failure of the compressor system because of large mechanical and thermal loads in the blading, and limit its performance and efficiency. Suppressing these phenomena improves life span and performance of the machine. One way to cope with these instabilities is active control [2]. In this approach, the dynamics of the compression system are modified by feeding back perturbations into the flow field. This results in an extension of the stable operating region beyond the "natural" stability boundary.

This study focuses on active control of surge in a laboratory-scale gas turbine installation. Surge is an unsteady, axisymmetric oscillation of the flow through the entire compression system. It is seen in the compressor map as a limit cycle oscillation, see Fig. 1. In this study, a control valve is used for active surge control. Similar to [8], this valve is nominally closed and only opens to stabilize the system in the nominal operating point. As the control valve position can only become positive, *positive feedback stabilization* [5] is applied. The overall efficiency of the compressor system will be improved compared to studies which only accept a nonzero nominal control valve mass flow or pressure drop. The main contribution of this study is the proposal of a new positive feedback structure that guarantees stabilization of the linearized compression system. In contrast with [8], the influence of the control valve dynamics and the control valve constraint can be dealt with in the presented stability analysis. The construction of the feedback has clear design parameters which can be used to obtain a desirable behavior of the closed loop system. The control strategy has successfully been applied to a compression system model.

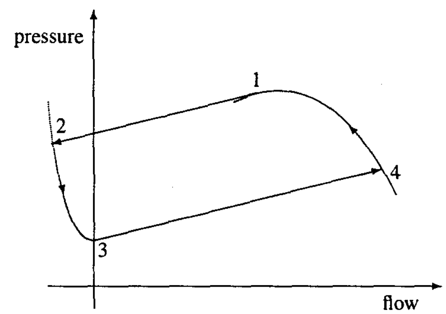

Figure 1: Compressor map with surge cycle.

This paper is organized as follows. First, the positive feedback stabilization problem is considered, which will be applied for the control of the compressor. Section 3 discusses the studied compression system whereas the Greitzer compression system model is described in Section 4. Section 5 deals with the stability of the linearized compression system in the presence of valve saturation and valve dynamics. Simulation results are presented and discussed in Section 6. Finally, conclusions are drawn and directions for future research are given.

2 Positive Feedback Stabilization

Consider a linear system (A, B) given by

$$\dot{x}(t) = Ax(t) + Bu(t),$$

where $u(t) \in \mathbb{R}$ is the *scalar* control input and $x(t) \in \mathbb{R}^n$ the state at time t. The input functions are assumed to belong to the Lebesgue space L_2 of measurable, square integrable

[1]Faculty of Mechanical Engineering, Tel: +31 40 247{2827,2784}, Fax: +31 40 2461418, Email: {frankw,A.G.de.Jager}@wfw.wtb.tue.nl
[2]Faculty of Electrical Engineering, Tel: +31 40 2473587, Fax: +31 40 2434582, Email: w.p.m.h.heemels@tue.nl
[3]Faculty of Mathematics and Computing Science, Tel: +31 40 2472387, Fax: +31 40 2442489, Email: wscoas@win.tue.nl

(*i.e.*, $\int_0^\infty \|u(t)\|^2 dt$ is finite) functions on $\mathbb{R}_+ := [0, \infty)$ taking values in $\mathbb{R}$. Moreover, the controls are only allowed to take *non-negative* values. Hence, the control constraint set is equal to the positive half line $\mathbb{R}_+$. The objective is to construct a non-negative state feedback of the form

$$u(t) = \max(0, Kx(t)) = \begin{cases} 0 & \text{if } Kx(t) \leqslant 0 \\ Kx(t) & \text{if } Kx(t) \geqslant 0 \end{cases}$$

with K being a constant row vector. Switching occurs between the 'controlled mode' (linear state feedback is active) and the 'uncontrolled mode'.

Definition 2.1 (Positive Feedback Stabilizability) (A, B) is said to be *positive feedback stabilizable*, if there exists a K such that all solutions x of

$$\dot{x}(t) = Ax(t) + B\max(0, Kx(t)) \tag{1}$$

are contained in L_2^n.

In the following theorem we describe a solution to this problem in case the matrix A has only one unstable complex conjugate pole pair. For control of the compressor, it suffices to consider this particular situation. Observe that the proof is constructive and allows synthesis of a feedback $\max(0, Kx)$. In the formulation of the theorem, $\sigma(A)$ denotes the set of eigenvalues of A.

Theorem 2.2 *Suppose that (A, B) has a scalar input and A has at most one pair of unstable, complex conjugate eigenvalues. The problem of positive feedback stabilizability is solvable if and only if (A, B) is stabilizable[1] and $\sigma(A) \cap \mathbb{R}_+ = \varnothing$.*

Proof: Since we only need the sufficiency part in this paper, we refer to [5] for the necessity part.

If A has no unstable complex eigenvalues, $\sigma(A) \cap \mathbb{R}_+ = \varnothing$ implies that A is stable and consequently, $K = 0$ results in a stable closed loop system (1). Hence, consider the case where A has one pair of complex conjugate eigenvectors with nonzero imaginary parts. There exist a nonsingular transformation S and a decomposition of the new state variable $\bar{x} = Sx$ in $(x_1^\top, x_2^\top)^\top$ such that the system description becomes

$$\dot{x}_1(t) = A_{11}x_1(t) \qquad\qquad + B_1 u(t) \tag{2a}$$
$$\dot{x}_2(t) = \qquad\qquad A_{22}x_2(t) + B_2 u(t) \tag{2b}$$

with A_{11} anti-stable (*i.e.*, $-A_{11}$ stable), A_{22} stable and (A_{11}, B_1) controllable. The stability of A_{22} implies that for any $u \in L_2$ the corresponding state trajectory $x_2 \in L_2$ (for arbitrary initial state). Hence, if we can construct a feedback of the form $u = \max(0, K_1 x_1)$ (depends only on x_1) that positively stabilizes (2a), the proof is complete.

We concentrate on (2a). Note that $x_1(t) \in \mathbb{R}^2$. Since (A_{11}, B_1) is controllable, the eigenvalues of $A_{11} + B_1 K_1$ can be placed arbitrarily by suitable choice of K_1. Denote the eigenvalues of A_{11} by $\sigma_0 \pm j\omega_0$ with $\omega_0 \neq 0$. We claim that if K_1 is designed such that the eigenvalues of $A_{11} + B_1 K_1$ are contained in[2]

$$\{\lambda = \sigma + j\omega \in \mathbb{C} \mid \sigma < 0 \text{ and } \mid \frac{\omega}{\sigma} \mid < \mid \frac{\omega_0}{\sigma_0} \mid\} \tag{3}$$

then the resulting closed-loop system (2a) with $u = \max(0, K_1 x_1)$ is stable. A solution corresponding to initial state x_0 will be denoted by x_{x_0} (omitting the subscript 1). Consider the following two cases.

1. Eigenvalues of $A_{11} + B_1 K_1$ are real. We claim that the system will eventually remain in the stable controlled mode. Indeed, suppose that $K_1 x_0 < 0$. As long as $K_1 x_{x_0}(t) \leqslant 0$,

$$K_1 x_{x_0}(t) = K_1 e^{A_{11}t} x_0 = c\, e^{\sigma_0 t}\cos(\omega_0 t + \phi) \tag{4}$$

for certain real constants $c \neq 0$ and ϕ. This implies that a sign switch must occur. Denote the time of the first sign switch by t_0 and the corresponding state by $\tilde{x}_0$, respectively. For a positive time interval the system evolves according to the dynamics of the controlled mode $\dot{x}(t) = (A_{11} + B_1 K_1)x(t)$. Observe that $K_1 e^{(A_{11}+B_1K_1)(t-t_0)}\tilde{x}_0$ can have at most one zero, because $A_{11} + B_1 K_1$ has only two real (possibly equal) eigenvalues. Since there is a zero for $t = t_0$, there will be no switch of dynamics beyond t_0 and the system stays in the stable controlled mode, so clearly, $x_{x_0} \in L_2$. Note that the above reasoning also applies, when $K_1 x_0 \geqslant 0$ and $K_1 x_{x_0}(\tau) < 0$ for some $\tau > 0$, by replacing x_0 by $x_{x_0}(\tau)$.

2. Eigenvalues of $A_{11} + B_1 K_1$ are complex, say $\sigma \pm j\omega$. Eventually, the system will switch between the two modes as long as the state $x_{x_0}(t)$ does not become equal to zero. This is most easily seen from (4). For the controlled mode, similar arguments can be used. From this, it can also be seen that the time spent in the controlled mode equals $\frac{\pi}{|\omega|}$ and similarly, in the uncontrolled mode $\frac{\pi}{|\omega_0|}$. The norm of the state decays in one complete cycle of the controlled and uncontrolled mode by $e^{\frac{\pi\sigma}{|\omega|}} \cdot e^{\frac{\pi\sigma_0}{|\omega_0|}}$, which is strictly less than 1 due to the choice of the eigenvalues of $A_{11} + B_1 K_1$ in (3), so $x_{x_0} \in L_2$. □

We would like to extract the following observations from the proof. Under the assumptions of the theorem with one unstable pole pair, the closed-loop system is stable if and only if the eigenvalues of $A_{11} + B_1 K_1$ are taken inside the cone (3). Moreover, the rate of decrease of the state variable can be determined. When the eigenvalues of $A_{11} + B_1 K_1$ are real, the decay is determined by the dominant eigenvalues of $\dot{x} = (A + BK)x$. If the eigenvalues are complex, it can be seen that the duration of one cycle of the controlled ($u = Kx$) and uncontrolled ($u = 0$) phase is $\frac{\pi}{|\omega|} + \frac{\pi}{|\omega_0|}$ in which the norm of the state x_1 decreases by a factor $e^{\frac{\pi\sigma}{|\omega|}} \cdot e^{\frac{\pi\sigma_0}{|\omega_0|}}$. The decay is determined by this factor and the eigenvalues of A_{22} (the stable eigenvalues of A). This elucidates how the eigenvalues $\sigma \pm j\omega$ of $A_{11} + B_1 K_1$ should be chosen to obtain desirable closed-loop behavior. Finally, note that the equilibrium of the closed-loop system is not only stable in the sense of Def. 2.1, but also globally exponentially stable and asymptotically Lyapunov stable.

Extending the proof above to multiple unstable pole pairs seems complicated.

[1] in the ordinary sense, *i.e.*, there exists a matrix K such that $A + BK$ is stable.

[2] In case $\sigma_0 = 0$ it suffices to place the eigenvalues of $A_{11} + B_1 K_1$ in the open left half plane.

3 Compression System

Positive feedback stabilization is applied to suppress surge in a laboratory-scale gas turbine installation. For surge control experiments, the system will be operated in the configuration shown in Fig. 2. The single-stage centrifugal compressor is part of a turbocharger (BBC VTR 160L) and it is driven by the turbine. To independently vary the compressor mass flow and the power delivered by the turbine, externally supplied compressed air is used. It flows via the vessel into the combustion chamber where natural gas is added and burned. The hot exhaust gasses expand over the turbine and deliver the power to drive the compressor. The compressor discharges via the compressor blow-off valve and the instrumented blow-off duct into the atmosphere. The relatively slow blow-off valve represents the pressure requirement of the system, *e.g*, downstream processes. For active surge control the system will be equipped with a relatively fast control valve, as shown in Fig. 2. This valve has to stabilize the compression system around its nominal operating point. Further details about the gas turbine installation can be found in [6].

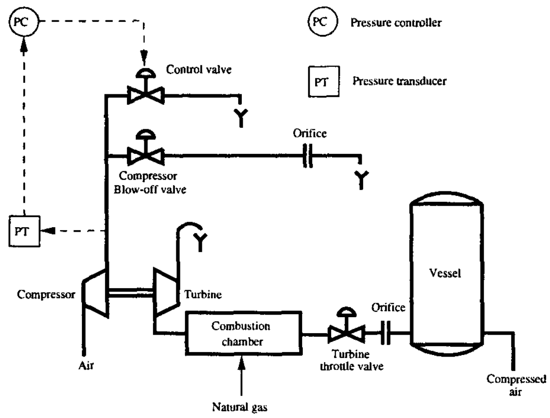

Figure 2: Scheme of the studied gas turbine installation.

4 Greitzer Compression System Model

The behavior of the uncontrolled compression system during surge can reasonably be described by the Greitzer compression system model, as reported in [6]. This lumped parameter model is modified to account for the effect of the control valve, as shown in Fig. 3. The flow in the ducts is assumed to be incompressible and one-dimensional. Compressibility effects are associated with the isentropic compression of the gas in the plenum while inertia effects are lumped on the acceleration of the gas in the compressor duct. Moreover, temperature effects and the influence of rotor speed variations on system behavior are neglected. The original model is described concisely in [3].

Assuming quasi-steady compressor behavior leads to the following set of dimensionless equations:

$$\frac{d\hat{\Phi}_c}{d\tilde{t}} = \beta[\Psi_c(\Phi_{c0} + \hat{\Phi}_c) - \psi_0 - \hat{\psi}] \qquad (5)$$

$$\frac{d\hat{\psi}}{d\tilde{t}} = \frac{1}{\beta}[\Phi_{c0} + \hat{\Phi}_c - \Phi_t(\hat{\psi}) - \Phi_b(\hat{u}_b, \hat{\psi})] \qquad (6)$$

where Φ_c is the dimensionless compressor mass flow and ψ is the dimensionless plenum pressure rise defined as:

$$\Phi_c = \frac{\dot{m}_c}{\rho_a A_c U_t} \quad \text{and} \quad \psi = \frac{\Delta p_p}{\frac{1}{2}\rho_a U_t^2},$$

with compressor mass flow $\dot{m}_c$ and plenum pressure rise $\Delta p_p = p_p - p_a$ and the dimensionless time $\tilde{t} = t \cdot \omega_H$ is obtained using the Helmholtz frequency $\omega_H = a\sqrt{\frac{A_c}{V_p L_c}}$. The subscript 0 indicates quantities in the nominal operating point $\psi_0 = \Psi_c(\Phi_{c0})$ whereas ^ indicates deviations from the nominal operating point. The meaning and values of the parameters used in the Greitzer model can be found in Table 1.

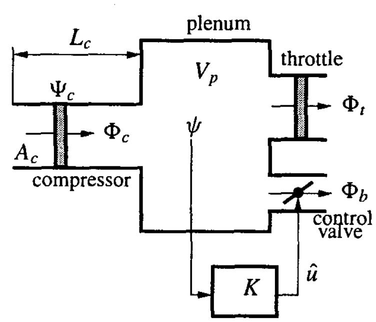

Figure 3: Compression system model.

Table 1: Parameters used in the Greitzer model.

Symbol	*Value/Meaning*
Speed of sound a [m/s]	340
Air density ρ_a [kg/m^3]	1.20
Compressor flow through area A_c [m^2]	$9.56 \cdot 10^{-3}$
Compressor duct length L_c [m]	1.00
Plenum volume V_p [m^3]	$3.75 \cdot 10^{-2}$
Helmholtz frequency f_H [Hz]	27.34
Throttle parameter c_t [-]	0.2994
Rotor radius r_t [m]	0.09
Rotational speed N [$10^3 \cdot$ rpm]	18 - 25
Rotor tip speed $U_t = \frac{2\pi r_t}{60} N$ [$10^2 \cdot$ m/s]	1.70 - 2.35
Greitzer stability parameter $\beta = \frac{U_t}{2\omega_H L_c}$	0.49 - 0.69
Slope of compr. charact. m_c	$\frac{d\Psi_c}{d\Phi_c}\vert_{\Phi_{c0}}$
Slope of equiv. throttle charact. $\frac{1}{m_{te}}$	$\frac{\partial(\Phi_t + \Phi_b)}{\partial\psi}\vert_{(\psi_0, u_{b0})}$
Slope of control valve charact. V	$\frac{\partial\Phi_b}{\partial u_b}\vert_{(\psi_0, u_{b0})}$

To describe the behavior of the compressor, throttle, and control valve, quasi-steady characteristics are used. The measured compressor characteristic $\Psi_c(\Phi_c)$ is approximated by a commonly used cubic polynomial in Φ_c:

$$\Psi_c(\Phi_c) = \Psi_c(0) + \mathrm{H}\left[1 + \frac{3}{2}\left(\frac{\Phi_c}{\mathrm{F}} - 1\right) - \frac{1}{2}\left(\frac{\Phi_c}{\mathrm{F}} - 1\right)^3\right]$$

where the peak of the compressor characteristic corresponds to $\Phi_c = 2F$ and the valley point is laid at $\Phi_c = 0$. Further details about the applied approximation can be found in [6].

For subsonic flow conditions, the dimensionless throttle and control valve characteristics are given, respectively, by:

$$\Phi_t = c_t u_{t0}\sqrt{\psi_0 + \hat{\psi}} \text{ and } \Phi_b = c_b(u_{b0} + \hat{u}_b)\sqrt{\psi_0 + \hat{\psi}}$$

with the dimensionless nominal throttle position u_{t0} and the dimensionless nominal control valve position u_{b0} (for positive feedback stabilization: $u_{b0} = 0$). The throttle and control valve parameter c_t and c_b are a measure for the capacity of the fully opened valve, whereas c_t is estimated from available steady-state measurements.

For the moment, we assume that the control valve dynamics can be neglected. This implies that a linearization of the model around (Φ_{c0}, ψ_0) results in the following second order system:

$$\begin{bmatrix} \dot{\hat{\Phi}}_c \\ \dot{\hat{\psi}} \end{bmatrix} = \underbrace{\begin{bmatrix} \beta m_c & -\beta \\ \frac{1}{\beta} & -\frac{1}{\beta m_{te}} \end{bmatrix}}_{=:A} \begin{bmatrix} \hat{\Phi}_c \\ \hat{\psi} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ -\frac{V}{\beta} \end{bmatrix}}_{=:B} \hat{u}_b \quad (7)$$

5 Active Control

Greitzer [4] shows that the *uncontrolled* compression system (7) is stable if and only if:

$$m_c < m_{te} \text{ and } m_c < \frac{1}{\beta^2 m_{te}}$$

Roughly speaking, this corresponds with operating points on the compressor characteristic where $\Phi_{c0} > 2F$. Active control can enlarge the range of operation points for which the system is stable [2].

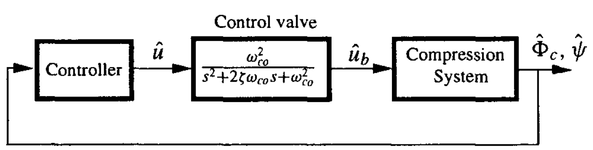

Figure 4: Block scheme of the compression system model

5.1 State feedback

Using active control we have to deal with the limitations of the actual dimensionless valve position $\hat{u}_b$, which is constrained to take values between 0 (closed) and 1 (fully open). In [7], it is shown that the linearized system (7) is *positive feedback stabilizable* for $0 \leqslant \Phi_{c0} \leqslant 2F$. Note that for sufficiently small values of $x = [\hat{\Phi}_c, \hat{\psi}]^T$ the feedback satisfies the upper bound on $\hat{u}_b$. Hence, local stability is always guaranteed. To obtain a large domain of attraction, the amplitude of the control input has to be made as small as possible. Application of the *Kalman-Jakubovič-Popov* (KJP) equality learns that for LQ-control the least control energy (L_2-norm) is needed to stabilize the system if the closed-loop poles approach the mirror images of the "unstable" open-loop poles. Nevertheless, in that case the upper constraint on the valve position may still be violated. Another solution to overcome the problems caused by the upper constraint is to increase the capacity c_b of the control valve.

5.2 Static output feedback

A drawback of the controller derived above is that accurate state measurements of $[\hat{\Phi}_c, \hat{\psi}]^T$ are required. In the installation, only reliable measurements of the plenum pressure are available. Therefore, it is interesting to know what can be achieved with static output feedback $\hat{u}_b = \max(0, K\hat{\psi})$. As K is simply a scalar, standard root locus techniques can be used to decide if a feedback K exists that places the eigenvalues of $A + BKC$ (with $C = [0\ 1]$ and A, B as in (7)) in the cone (3). It can be verified that such a feedback exists for operating points with $N = 25,000$ [rpm] and $\Phi_{c0} \geqslant 1.7F$. This corresponds to a 15% extension of the stable operating region.

5.3 Extended model with valve dynamics

In the previous cases, the dynamics of the valve are neglected ($\hat{u}_b = \hat{u}$). Here, the control valve is supposed to behave as a linear second order system [8] (see Fig. 4):

$$\frac{d^2\hat{u}_b}{d\tilde{t}^2} + 2\zeta\omega_{co}\frac{d\hat{u}_b}{d\tilde{t}} + \omega_{co}^2\hat{u}_b = \omega_{co}^2\hat{u} \quad (8)$$

where $\omega_{co} = \frac{2\pi f_{co}}{\omega_H}$. Linearization of (5), (6), and (8) around $(\Phi_{c0}, \psi_0, \dot{u}_{b0}, u_{b0})$ results in the following dimensionless state equations for the complete system ($\dot{u}_{b0} = u_{b0} = 0$).

$$\begin{bmatrix} \dot{\hat{\Phi}}_c \\ \dot{\hat{\psi}} \\ \ddot{\hat{u}}_b \\ \dot{\hat{u}}_b \end{bmatrix} = \begin{bmatrix} \beta m_c & -\beta & 0 & 0 \\ \frac{1}{\beta} & -\frac{1}{\beta m_{te}} & 0 & -\frac{V}{\beta} \\ 0 & 0 & -2\zeta\omega_{co} & -\omega_{co}^2 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{\Phi}_c \\ \hat{\psi} \\ \dot{\hat{u}}_b \\ \hat{u}_b \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \omega_{co}^2 \\ 0 \end{bmatrix} \hat{u} \quad (9)$$

Then, we can only influence $\hat{u}$ instead of controlling $\hat{u}_b$ directly. This means that we do not have *control* constraints, but *state* constraints. One possibility to tackle this more difficult problem with state constraints is to implement an inverse model of the control valve dynamics. In case of *exact cancellation* of the control valve dynamics, the system (9) reduces to the second order system (7) and the controllers proposed in the Sections 5.1 and 5.2 can be used. However, two problems obstruct the real implementation of this method: (i) the cancellation is never exact (although simulations show that the controller is robust for mismatches), and (ii) implementation of the exact inverse valve dynamics requires differentiation of measured signals which is not reliable due to non-smoothness and noise. Therefore, the following realization is applied:

$$\hat{u} = \frac{\omega_1^2}{\omega_{co}'^2}\left(\frac{s^2 + 2\zeta'\omega_{co}'s + \omega_{co}'^2}{s^2 + 2\zeta_1\omega_1 s + \omega_1^2}\right)\hat{u}^* \quad (10)$$

with $\hat{u}^*$ one of the controllers proposed in the Sections 5.1 or 5.2, and ω'_{co} and ζ' approximations of ω_{co} and ζ. If the approximation of the inverse model is accurate enough for the relevant frequencies of the system, one expects that the closed-loop behavior remains stable. This will be validated by simulations.

Note that $\hat{u}_b$ is not necessarily non-negative, although $\hat{u}^*$ is. However, if the transfer function from $\hat{u}^*$ to $\hat{u}_b$ is given by:

$$H_2(s) = \frac{\omega_2^2}{s^2 + 2\zeta_2\omega_2 s + \omega_2^2} \tag{11}$$

and $\zeta_2 > 1$ (overdamped system), then $\hat{u}_b$ is non-negative as well. This follows from the non-negativity of the impulse response corresponding to (11) and of $\hat{u}^*$.

5.4 State feedback for extended model

For (9), it is possible to construct a *state feedback*:

$$\hat{u} = \max(0, K[\hat{\Phi}_c \; \hat{\psi} \; \dot{\hat{u}}_b \; \hat{u}_b]^\top)$$

that guarantees the stability of the system. Indeed, for $0 \leqslant \Phi_{c0} \leqslant 2\text{F}$ **and** stable control valve dynamics, the conditions of Theorem 2.2 are satisfied. If the control valve dynamics are overdamped ($\zeta > 1$), the non-negativity of $\hat{u}(t)$ implies that $\hat{u}_b(t)$ is non-negative for all t, but the upper constraint on $\hat{u}_b$ can still be violated. For $\zeta \leqslant 1$, an alternative approach is to implement a realization as in (10) with $\zeta_1 > 1$. If the control valve dynamics are exactly cancelled ($\omega'_{co} = \omega_{co}$ and $\zeta' = \zeta$), a situation arises where the control valve dynamics are replaced by (11) with $\omega_2 = \omega_1$ and $\zeta_2 = \zeta_1 > 1$. The main disadvantage of this state feedback is the need for measurements of the states of the control valve and compression system. Current research is concerned with the existence of stabilizing static output feedback controllers or the use of state observers as applied in, *e.g.*, [1].

6 Simulation Results

Simulations are done with the nonlinear compression model in MATLAB/SIMULINK. In all simulations, the *uncontrolled* model is initially disturbed from its nominal operating point for $N = 25,000$ [rpm]. This results in a limit cycle oscillation for $\Phi_{c0} < 2\text{F}$. Then, after 0.25 [s] the controller is switched on and the response is observed. Preliminary, the state $[\hat{\Phi}_c, \hat{\psi}]^\top$ is assumed to be available from measurements. The control valve that will be implemented on the installation has a capacity of 7% of the throttle capacity.

Table 2: Pole placement in case of state feedback ($N = 25,000$ [rpm], $\Phi_{c0} = 1.7\text{F}$, and $\lambda_{open\ loop} = 0.4057 \pm j0.8886$).

Case	*Closed-loop poles*
I	$\lambda_1 = -1$; $\lambda_2 = -0.1$
II	$\lambda_{KJP} = -0.4067 \pm j0.8886$

First, the state feedback controller discussed in Section 5.1 is examined. In the simulation model, saturation of $\hat{u}_b$ is included, but the control valve dynamics are omitted initially ($\hat{u}_b = \hat{u}$). Fig. 5 shows the results for the two cases listed in Table 2. The upper left-hand figure shows the system's response in the compressor map. In this map, the compressor characteristic is plotted for reference. Furthermore, the time traces of the compressor mass flow and plenum pressure perturbations and the actual control valve position are shown. It is seen that in Case I the system is stabilized after two surge cycles. The nominal operating point is finally reached with zero control valve mass flow. Obviously, the domain of attraction of the nominal operating point includes the surge cycle of the uncontrolled system. In Case II, the energy of $\hat{u}_b$ is minimized in order to keep $\hat{u}_b$ as small as possible. This results in a slower response than in Case I since the system operates frequently in the uncontrolled mode. During this period, the state has time to grow. Additional simulations show that if the closed-loop poles

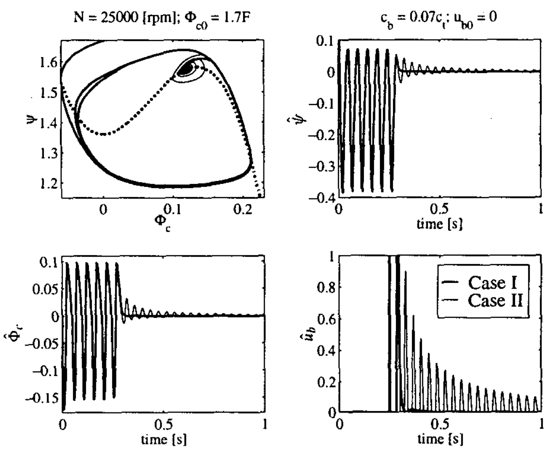

Figure 5: Influence of pole placement for state feedback.

are shifted further to $-\infty$ the upper constraint is limiting. Although the linearized system is stable, for $\Phi_{c0} < 1.7\text{F}$ the applied control system can only reduce the amplitude of the perturbations and the nominal operating point is not reached. By sufficiently increasing c_b, the domain of attraction is enlarged such that surge is stabilized in the entire region $0 \leqslant \Phi_{c0} \leqslant 2\text{F}$. For instance, stabilization in $\Phi_{c0} = 1.0\text{F}$ from the surge cycle is possible with zero control valve mass flow for $c_b = 0.23c_t$.

Reliable, transient mass flow measurements are (currently) not available in the installation. As a result, the stabilization of surge is studied using a static output feedback controller (Section 5.2) based on plenum pressure measurements. For $\Phi_{c0} = 1.7\text{F}$, the results are shown in Fig. 6. The results for $K = 27.0$ and 28.8 correspond with the closed-loop poles $\lambda_{1,2} = -0.1110 \pm j0.2260$ and $\lambda_{1,2} = -0.0051$ and -0.2858, respectively. As shown in [7], for $N = 25,000$ [rpm] stabilization from surge is limited to $1.7\text{F} \leqslant \Phi_{c0} \leqslant 2\text{F}$ using static output feedback. In this case, increasing c_b has no effect.

To study the effect of the control valve dynamics on system behavior, the simulation model is extended with a linear second order valve model (8). The bandwidth of the control valve is $f_{co} = 60$ [Hz]. In this valve model, the control valve position $\hat{u}_b$ is constrained during integration and the control valve velocity is reset to zero if the posi-

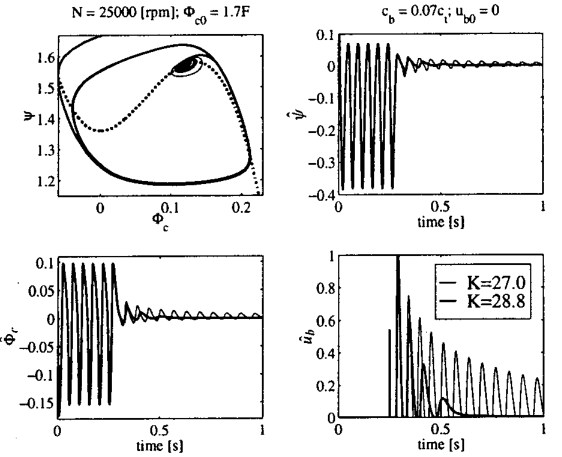

Figure 6: Results for static output feedback.

tion is constrained. The state feedback controller designed for system (7) is applied to control the nonlinear compression system *with* control valve dynamics. As the control valve dynamics can make the closed-loop system unstable, these dynamics are compensated using (10) with $\omega_{co}' = \omega_{co}$, $\zeta = \zeta' = 1.1$, $\omega_1 = \frac{2\pi 10^3}{\omega_H}$ and $\zeta_1 = 1.1$. Note that this guarantees local stability according to Section 5.3. The results are shown in Fig. 7. It is seen that in Case I the system is

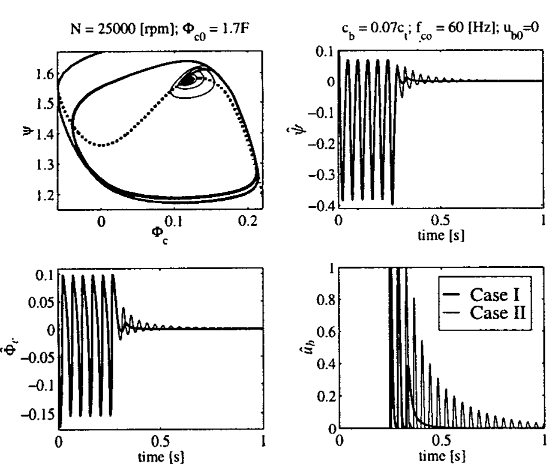

Figure 7: Results for state feedback with compensation of control valve dynamics.

stabilized in the desired operating point, but the response time is slower than in Fig. 5. This is caused by the remaining dynamics after compensation. In Case II, on the other hand, the response is faster for this system with valve dynamics. Furthermore, the robustness of the nonlinear system for non-exact cancellation is studied for Case I. For the range $0.8\omega_{co} \leqslant \omega_{co}' \leqslant 1.2\omega_{co}$, the surge cycle is still stabilized.

7 Conclusions and Future Research

Active surge control has been examined for a compression system with a centrifugal compressor. For this compression system, a positive stabilizing feedback controller has been proposed. The stability of the controlled linearized system *without* control valve dynamics can be guaranteed in the *entire* operating region $0 \leqslant \Phi_{c0} \leqslant 2\text{F}$ by the theory developed in Section 2. Although the actual control valve position is bounded between 0 and 1, it is shown that fully developed surge can be stabilized for sufficiently large control valve capacities c_b. For the relatively small control valve to be implemented on the installation, the surge cycle oscillations can be stabilized for $\Phi_{c0} \geqslant 1.7\text{F}$. This corresponds with a reduction of 15% in surge point mass flow. Similar results are obtained using a static output feedback controller based on plenum pressure measurements. If persistent disturbances and measurement noise are absent, the desired operating point is reached with zero control valve mass flow. The static output feedback strategy is planned to be implemented on the installation in the near future. Then, the simulation results can be validated.

To prove stability of the linearized system *with* control valve dynamics, state feedback can be applied. A drawback of this state feedback is that measurements of the instantaneous mass flow and valve position and velocity are required. Current research focuses on the existence of stabilizing output feedback controllers or the use of state observers as applied in [1]. Furthermore, the nominal operating point is supposed to be known *a priori*. However, pressure requirements in, *e.g.*, jet engines or gas compressor stations, are generally not known. For these systems, techniques have to be developed to determine the desired equilibrium points or controllers have to be applied that do not use ψ_0, *e.g.*, based on $\dot{\psi}$. As the linearized system (7) varies for different nominal operating points, we will also search for controllers that stabilize surge in a large range of Φ_{co} such that gain scheduling is not required or use LPV based controllers.

References

[1] O.O. Badmus, S. Chowdhury, and C.N. Nett. Nonlinear control of surge in axial compression systems. *Automatica*, 32(1):59–70, January 1996.

[2] A.H. Epstein, J.E. Ffowcs Williams, and E.M. Greitzer. Active suppression of aerodynamic instabilities in turbomachines. *J. Propulsion*, 5(2):204–211, March 1989.

[3] E.M. Greitzer. Surge and rotating stall in axial flow compressors. Part I: Theoretical compression system model. *ASME J. Engineering for Power*, 98(2):191–198, April 1976.

[4] E.M. Greitzer. The stability of pumping systems - The 1980 Freeman scholar lecture. *ASME J. Fluids Dynamics*, 103(2):193–242, June 1981.

[5] W.P.M.H. Heemels and A.A. Stoorvogel. Positive stabilizability of a linear continuous-time system. Technical Report 98I-01, Section Measurement and Control, Faculty of Electrical Engineering, Eindhoven University of Technology, 1998.

[6] Corine Meuleman, Frank Willems, Rick De Lange, and Bram De Jager. Surge in a low-speed radial compressor. ASME paper no. 98-GT-426, June 1998.

[7] Frank Willems. Valve selection for compressor surge control. Technical Report WFW 98.042, Fac. of Mechanical Engineering, Eindhoven University of Technology, November 1998.

[8] Frank Willems and Bram De Jager. Active compressor surge control using a one-sided controlled bleed/recycle valve. In *Proc. of the 37th IEEE Conference on Decision and Control*, pages 2546–2551, Tampa, FL, December 1998.