PERGAMON

Automatica 38 (2002) 311–318

automatica

www.elsevier.com/locate/automatica

Brief Paper

Positive feedback stabilization of centrifugal compressor surge☆

Frank Willems[a,c], W.P.M.H. Heemels[b,*], Bram de Jager[c], Anton A. Stoorvogel[d,e]

[a] *Currently at TNO Automotive, Powertrain Design and Development, P.O. Box 6033, 2600 JA Delft, The Netherlands*
[b] *Department of Electrical Engineering, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands*
[c] *Department of Mechanical Engineering, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands*
[d] *Department of Mathematics and Computer Science, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands*
[e] *Department of Information Technology and Systems, Delft University of Technology, Delft, The Netherlands*

Received 13 November 2000; revised 15 June 2001; received in final form 7 August 2001

Abstract

Stable operation of axial and centrifugal compressors is limited towards low mass flows due to the occurrence of surge. The stable operating region can be enlarged by active control. In this study, we use a control valve which is fully closed in the desired operating point and only opens to stabilize the system around this point. As a result, only nonnegative control values are allowed, which complicates the controller design considerably. A novel positive feedback controller is proposed which is based on the pole placement technique. This controller has been successfully applied to a laboratory-scale gas turbine installation. Initial experiments show that the surge point mass flow can be reduced by at least 7%. Using this efficient control strategy, stable operation in the desired operating point is maintained with small average control valve mass flow. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Positive feedback; Compressor; Piecewise linear controller; Active surge control; Feedback stabilization; Control valves

1. Introduction

Control problems involving a positivity constraint on the input variables are studied extensively in the literature (see Brammer, 1972; Farina & Benvenutti, 1997; Heemels, Van Eijndhoven, & Stoorvogel, 1998; Heymann & Stern, 1975; Pachter, 1980; Sissaoui, Collins, & Harley, 1988; Smirnov, 1996; Zaslavsky, 1990 and the references therein). The continuing interest in these control problems is well explained and motivated by many applications in which the attainable values of the control function are inherently constrained in the sense that the direction of its influence cannot be changed. One might think of electrical networks with diode elements, mechanical systems with one-way valves (as in the compressor example studied here), furnace/boiler temperature control without cooling, ecological (soil fertilization) and medical systems (drug infusion), and so on.

Many feedback variants of the stabilization problem for linear time-invariant dynamical systems with positive controls are of interest. In Zaslavsky (1990) the existence of a piecewise continuous state feedback has been shown that renders the origin locally stable (under the condition of "positive controllability"). A more general result has been proven in Smirnov (1996). Smirnov shows that the conditions for "open-loop positive stabilizability" are necessary and sufficient for the existence of a stabilizing Lipschitz-continuous state feedback. In contrast with these results we will a priori impose a simple and easily implementable structure of the feedback. To be specific, the feedback consist of the maximum of a linear state feedback and zero. In principle, the resulting closed-loop system falls within the realm of "switched systems" (Liberzon & Morse, 1999) and has to be studied

☆ This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Xiaohua Xia under the direction of Editor Mituhiko Araki.

* Corresponding author. Tel.: +31-40-2473587; fax: +31-40-2434582.

E-mail addresses: willems@wt.tno.nl (F. Willems), w.p.m.h.heemels@tue.nl (W.P.M.H. Heemels), a.g.de.jager@wfw.wtb.tue.nl (B. de Jager), a.a.stoorvogel@tue.nl (A. A. Stoorvogel).

0005-1098/02/$ - see front matter
PII: S0005-1098(01)00202-3

with techniques from this area. To be more precise, the closed-loop system can be considered as a piecewise linear system (see, e.g., Sontag, 1981), a "max system" (Branicky, 1994) or a linear complementarity system (see, e.g., Heemels, Schumacher, & Weiland, 2000; van der Schaft & Schumacher, 1996). The general problem is widely open and even the simplest case of interest cannot be tackled by common stabilization techniques (see Section 2). However, under certain additional requirements (satisfied by the compressor system that we would like to stabilize) we can actually show that the conditions for "open-loop positive stabilizability" are necessary and sufficient to guarantee the existence of a stabilizing feedback of this particular form. In addition to the simple structure of the feedback controller, another advantage with respect to Smirnov (1996) and Zaslavsky (1990) is that we can base the synthesis on classical control techniques.

An interesting application of the presented theory is the stabilization of surge in a compressor system. Towards low mass flows, the stable operating region of axial and centrifugal compressors is bounded due to the occurrence of surge (Greitzer, 1981). This aerodynamic flow instability can lead to severe damage of the machine due to large mechanical and thermal loads in the blading, and restricts its performance and efficiency. Suppressing this phenomenon improves life span and performance of the machine. One way to cope with this instability is active control (Epstein, Ffowcs Williams, & Greitzer, 1989). In this approach, the dynamics of the compression system are modified by feeding back perturbations into the flow field. This results in an extension of the stable operating region beyond the "natural" stability boundary. A comprehensive overview of the characteristics of surge is given in, e.g., Gravdahl and Egeland (1998) or Greitzer (1981).

To control surge, we use a pressure sensor in combination with a control valve. The valve is closed in the desired equilibrium point and, to stabilize the system in this operating point, it can only be opened. As a result, the control valve position can only become positive. The equilibrium point is reached with zero control valve mass flow if persistent disturbances and measurement noise are absent. This control strategy improves the overall efficiency of the compression system compared to studies that only accept a nonzero nominal control valve mass flow or pressure drop (Botros, Campbell, & Mah, 1991; Gravdahl & Egeland, 1998). In addition, feedback is based on the easily measurable plenum pressure and smaller control valve capacities can be used compared to cases in which the desired equilibrium point is associated with nonzero average control valve mass flow (Willems, 2000). A discussion of various experimental active surge control systems can be found in Willems and de Jager (1999).

The main contribution of this study is twofold. First, we propose a novel positive feedback structure that guarantees stabilization of a linear system. The construction of the feedback allows tuning of the closed-loop behavior. Second, this theory is applied to study the stabilization of surge in a linearized compression system with a constraint on the control valve position. This new and efficient surge control strategy is shown to be successful on an experimental set-up.

2. Positive feedback stabilization

Consider a linear system (A,B) given by

$$\dot{x}(t)=Ax(t)+Bu(t),$$

where $u(t)\in\mathbb{R}$ is the *scalar* control input and $x(t)\in\mathbb{R}^n$ the state at time t. The input functions are assumed to belong to the Lebesgue space L_2 of measurable, square integrable (i.e., $\int_0^\infty \|u(t)\|^2\,\mathrm{d}t$ is finite) functions on $\mathbb{R}_+:=[0,\infty)$ taking values in $\mathbb{R}$. Moreover, the controls are only allowed to take *nonnegative* values, i.e., $u(t)\in\mathbb{R}_+$. The objective is to construct a nonnegative state feedback of the simple form

$$u(t)=\max(0,Lx(t))=\begin{cases}0 & \text{if } Lx(t)\leqslant 0,\\ Lx(t) & \text{if } Lx(t)\geqslant 0.\end{cases} \quad (1)$$

Definition 2.1 (*Positive feedback stabilizability*). (A,B) is said to be *positive feedback stabilizable* (with controllers of form (1)), if there exists a row vector L such that all solution trajectories of

$$\dot{x}(t)=Ax(t)+B\max(0,Lx(t)) \quad (2)$$

are contained in L_2^n.

Note that $x\in L_2^n$ and $\dot{x}\in L_2^n$ implies that $\lim_{t\to\infty}x(t)=0$. The closed-loop system switches between the "controlled mode" ($\dot{x}=(A+BL)x$) and the "uncontrolled mode" ($\dot{x}=Ax$) on the basis of the switching plane $Lx=0$. As mentioned in the introduction, the closed-loop system belongs to the classes of "switched", "piecewise linear", "linear complementarity" and "max-systems". In the simplest case of A being unstable (and Lx scalar-valued) many of the common techniques for stability analysis based upon constructing a common (or sometimes called simultaneous) quadratic Lyapunov function (Boyd & Yang, 1989; Narendra & Balakrishnan, 1994), the circle criterion and Popov criterion (see, e.g., Khalil, 1992), the piecewise quadratic Lyapunov functions as proposed in Johansson and Rantzer (1988) and the analysis in Branicky (1994, 1998) and Liberzon and Morse (1999) do not apply (Heemels & Stoorvogel, 1998).

In the following theorem, we describe a solution to the problem above in case the matrix A has only one unstable complex conjugate pole pair. For the control of the compressor, it suffices to consider this particular situation. The proof will be constructive and synthesis of a feedback $\max(0,Lx)$ is possible. In the formulation of the theorem $\sigma(A)$ denotes the set of eigenvalues of A.

Theorem 2.2. *Suppose that* (A,B) *has scalar input and* A *has at most one pair of unstable, complex conjugate eigenvalues. The problem of positive feedback stabilizability is solvable if and only if* (A,B) *is stabilizable*[1] *and* $\sigma(A)\cap\mathbb{R}_+=\emptyset$.

Proof. The proof of the necessity part can be derived from Smirnov (1996) or it can be based on the positive controllability results (Brammer, 1972; Heymann & Stern, 1975) as described in Heemels and Stoorvogel (1998). In this paper, we will actually use the sufficiency part only.

If A has no unstable complex eigenvalues, $\sigma(A)\cap\mathbb{R}_+=\emptyset$ implies that A is stable and consequently, $L=0$ results in a stable closed-loop system (2). Hence, consider the case where A has one pair of complex conjugate eigenvalues with nonzero imaginary parts. There exist a nonsingular transformation S and a decomposition of the new state variable $\bar{x}=Sx$ in $(x_1^\top,x_2^\top)^\top$ such that the system description becomes (use, e.g., the real Jordan decomposition Lütkepohl, 1996, p. 71)

$$\dot{x}_1(t)=A_{11}x_1(t)+B_1u(t), \tag{3a}$$

$$\dot{x}_2(t)=A_{22}x_2(t)+B_2u(t) \tag{3b}$$

with A_{11} anti-stable (i.e., $-A_{11}$ stable), A_{22} stable and (A_{11},B_1) controllable. The stability of A_{22} implies that for any $u\in L_2$ the corresponding state trajectory $x_2\in L_2$ (for arbitrary initial state). Hence, if we can construct a feedback of the form $u=\max(0,L_1x_1)$ (depends only on x_1) that positively stabilizes (3a), the proof is complete.

We concentrate on (3a). Note that $x_1(t)\in\mathbb{R}^2$. Since (A_{11},B_1) is controllable, the eigenvalues of $A_{11}+B_1L_1$ can be placed arbitrarily by suitable choice of L_1. Denote the eigenvalues of A_{11} by $\sigma_0\pm \mathrm{j}\omega_0$ and observe that $\omega_0\neq 0$. We claim that if L_1 is designed such that the eigenvalues of $A_{11}+B_1L_1$ are contained in[2]

$$\left\{\lambda=\sigma+\mathrm{j}\omega\in\mathbb{C}\mid \sigma<0 \text{ and } \left|\frac{\omega}{\sigma}\right|<\left|\frac{\omega_0}{\sigma_0}\right|\right\}, \tag{4}$$

then the resulting closed-loop system (3a) with $u=\max(0,L_1x_1)$ is stable. To show this, denote a solution of (3a) corresponding to initial state x_0 by x_{x_0} (omitting the subscript 1). Consider the following two cases.

Eigenvalues of $A_{11}+B_1L_1$ *are real.* We claim that the system will eventually remain in the stable controlled mode. Indeed, suppose that $L_1x_0<0$. As long as $L_1x_{x_0}(t)\leqslant 0$,

$$L_1x_{x_0}(t)=L_1\mathrm{e}^{A_{11}t}x_0=c\ \mathrm{e}^{\sigma_0 t}\cos(\omega_0 t+\phi) \tag{5}$$

for certain real constants $c\neq 0$ and ϕ. This implies that a sign switch must occur. Denote the time of and state at the first sign switch by t_0 and $\tilde{x}_0$, respectively. For a time interval of positive length the system evolves according to the dynamics of the controlled mode $\dot{x}(t)=(A_{11}+B_1L_1)x(t)$. Observe that $L_1\mathrm{e}^{(A_{11}+B_1L_1)(t-t_0)}\tilde{x}_0$ can have at most one zero, because $A_{11}+B_1L_1$ has only two real (possibly equal) eigenvalues. Since there is a zero for $t=t_0$, there will be no switch of mode dynamics beyond t_0 and the system stays in the stable controlled mode. Hence, it is clear that $x_{x_0}\in L_2$. Note that the above reasoning also applies when $L_1x_0\geqslant 0$ and $L_1x_{x_0}(\tau)<0$ for some $\tau>0$ by replacing x_0 by $x_{x_0}(\tau)$.

Eigenvalues of $A_{11}+B_1L_1$ *are complex*, say $\sigma\pm\mathrm{j}\omega$. Eventually, the system will switch between the two modes as long as the state $x_{x_0}(t)$ does not become equal to zero. This is most easily seen from (5) and the similar expression for the controlled mode. From this, it can even be observed that the time spent in the controlled mode equals $\pi/|\omega|$ and similarly, in the uncontrolled mode $\pi/|\omega_0|$. The norm of the state decays in one complete cycle of the controlled and uncontrolled mode by $\mathrm{e}^{\pi\sigma/|\omega|}\mathrm{e}^{\pi\sigma_0/|\omega_0|}$. Since this expression is strictly less than 1 due to the choice of the eigenvalues of $A_{11}+B_1L_1$ in (4), it holds that $x_{x_0}\in L_2$. □

We would like to extract the following observations from the proof. Under the assumptions of the theorem with one unstable pole pair, the closed-loop system is stable if and only if the eigenvalues of $A_{11}+B_1L_1$ are taken inside the cone (4). Moreover, the rate of decrease of the state variable can be determined. When the eigenvalues of $A_{11}+B_1L_1$ are chosen to be real, the decay is determined by the dominant eigenvalues of $\dot{x}=(A+BL)x$. If the eigenvalues are complex, it can be seen that the duration of one cycle of the controlled ($u=Lx$) and uncontrolled ($u=0$) phase is $\pi/|\omega|+\pi/|\omega_0|$ in which the norm of the state x_1 decreases by a factor $\mathrm{e}^{\pi\sigma/|\omega|}\mathrm{e}^{\pi\sigma_0/|\omega_0|}$. In this case the decay is determined by this factor and the eigenvalues of A_{22} (the stable eigenvalues of A). This elucidates how the eigenvalues $\sigma\pm\mathrm{j}\omega$ of $A_{11}+B_1L_1$ should be chosen to obtain desirable closed-loop behavior. Finally, note that the equilibrium of the closed-loop system is not only stable in the sense of Definition 2.1, but also globally exponentially stable and asymptotically Lyapunov stable (see, e.g., Khalil, 1992 for the exact definitions).

3. Compression system

Positive feedback stabilization is applied to eliminate surge in a laboratory-scale gas turbine installation. This installation consists of a centrifugal compressor, which is mounted on the same rotational axis as the turbine. For studies involving surge, the system is operated in the configuration shown in Fig. 1, with the dashed connecting line removed. The compressor pressurizes the incoming

[1] In the ordinary sense, i.e., there exists a matrix G such that $A+BG$ is stable.

[2] In case $\sigma_0=0$ it suffices to place the eigenvalues of $A_{11}+B_1L_1$ in the open left half plane.

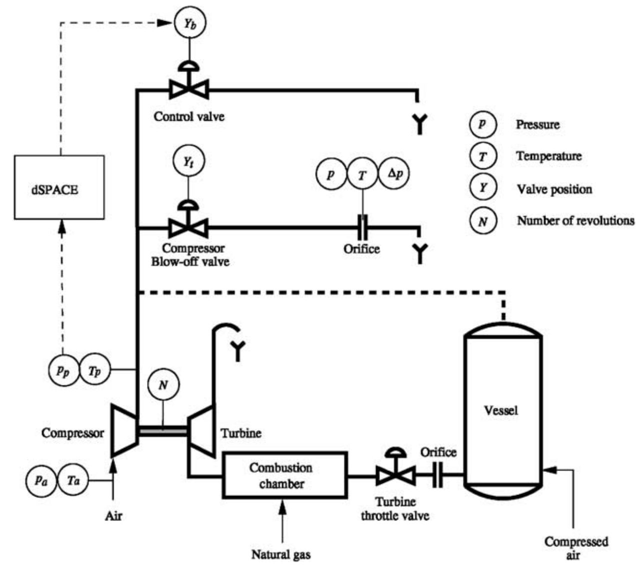

Fig. 1. Scheme of the gas turbine installation.

air, which is discharged via the compressor blow-off valve into the atmosphere. Natural gas is burned in the combustion chamber using externally supplied compressed air. The hot exhaust gases expand over the turbine and deliver the power to drive the compressor. In this configuration, the rotational speed can be varied up to 25,000 rpm due to the limited mass flow rate of the externally supplied compressed air. For higher compressor speeds, the gas pressurized by the compressor can supplement the externally supplied compressed air, by using the dashed connecting line and closing the blow-off valve. In that case, the system is run as a standard gas turbine. This configuration includes a large vessel. Therefore, surge is much more powerful and no extended surge measurements are possible, to prevent damage to the machine.

For active surge control, similar to DiLiberti, Van den Braembursche, Konya, and Rasmunen (1996), a relatively fast control valve is placed in parallel with the blow-off valve, as shown in Fig. 1; the compressor blow-off valve is too slow for surge control in the studied system. This blow-off valve represents the pressure requirement of the system, e.g., downstream processes or losses due to resistance in the piping, whereas the control valve has to stabilize the compression system around its desired operating point. Information about pressure variations during surge are obtained from a high-frequency response pressure transducer p_{p} located at the compressor outlet. Furthermore, transients of the rotational speed N and of the blow-off valve position Y_{t} are observed. Further details about the gas turbine installation and controller implementation can be found in Meuleman, Willems, de Lange, and de Jager (1998) and Willems and de Jager (2000).

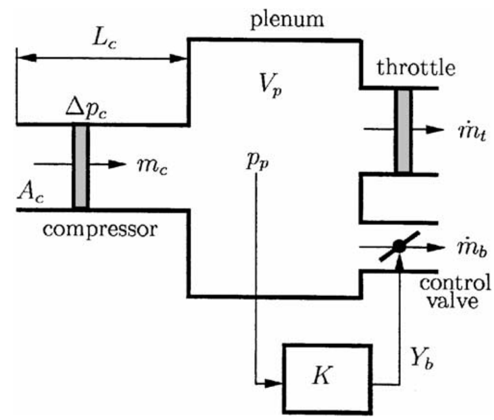

Fig. 2. Compression system model.

4. Greitzer compression system model

The dynamic behavior of the uncontrolled compression system during surge can reasonably be described by the Greitzer compression system model (Greitzer, 1976), as reported in (Meuleman et al., 1994). This model is modified to account for the effect of the control valve, as shown in Fig. 2. The incompressible flow in the compressor, throttle, and control valve duct is described by a one-dimensional momentum equation and the principle of mass conservation is applied to the plenum. Assuming the inertia effects in the throttle and control valve duct to be negligible, leads to the following set of dimensionless equations:

$$\frac{\mathrm{d}\Phi_{\mathrm{c}}}{\mathrm{d}\tilde{t}} = \beta[\Psi_{\mathrm{c}}(\Phi_{\mathrm{c}}) - \psi],$$
$$\frac{\mathrm{d}\psi}{\mathrm{d}\tilde{t}} = \frac{1}{\beta}[\Phi_{\mathrm{c}} - \Phi_{\mathrm{t}}(u_{\mathrm{t}},\psi) - \Phi_{\mathrm{b}}(u_{\mathrm{b}},\psi)], \qquad (6)$$

where the dimensionless time $\tilde{t} = t\omega_{\mathrm{H}}$ is obtained using the Helmholtz frequency $\omega_{\mathrm{H}} = a\sqrt{A_{\mathrm{c}}/V_{\mathrm{p}}L_{\mathrm{c}}}$. The meaning and value of the parameters used in the Greitzer model are listed in Table 1.

The behavior of the compressor and of the valves is described by algebraic relations. For each rotational speed, the measured compressor characteristic is approximated by a frequently applied cubic polynomial in Φ_{c}:

$$\Psi_{\mathrm{c}}(\Phi_{\mathrm{c}}) = \Psi_{\mathrm{c}}(0) + H\left[1 + \frac{3}{2}\left(\frac{\Phi_{\mathrm{c}}}{F} - 1\right) - \frac{1}{2}\left(\frac{\Phi_{\mathrm{c}}}{F} - 1\right)^3\right],$$

where $\Phi_{\mathrm{c}} = 2F$ corresponds to the compressor mass flow at the top of the characteristic. The parameters $\Psi_{\mathrm{c}}(0)$, H, and F are determined from steady-state measurements of the compressor characteristics (Willems, 2000, Chapter 2).

For subsonic flow conditions, the dimensionless throttle and control valve mass flows are described, respectively, by

$$\Phi_{\mathrm{t}} = c_{\mathrm{t}}u_{\mathrm{t}}\sqrt{\psi} \quad \text{and} \quad \Phi_{\mathrm{b}} = c_{\mathrm{b}}u_{\mathrm{b}}\sqrt{\psi},$$

Table 1
Parameters used in the Greitzer model

Parameter	Value/meaning
Speed of sound a (m/s)	340
Ambient air density ρ_a (kg/m^3)	1.20
Compressor duct area A_c (m^2)	7.9×10^{-3}
Compressor duct length L_c (m)	1.8
Plenum volume V_p (m^3)	2.03×10^{-2}
Rotational speed N (10^3 rpm)	18–25
Blade tip speed U (10^2 m/s)	1.70–2.36
Greitzer stability parameter $\beta = U/2\omega_H L_c$	0.30–0.41
Throttle parameter c_t	0.3320
Dimensionless mass flow Φ	$\dot{m}/\rho_a A_c U$
Dimensionless plenum pressure ψ	$2(P_p - P_a)/\rho_a U^2$
Slope of compressor charact. m_c	$\mathrm{d}\Psi_c/\mathrm{d}\Phi_c\|_{\Phi_{c0}}$
Slope of combined valve charact. $\frac{1}{m_{te}}$	$\partial(\Phi_t + \Phi_b)/\partial\psi\|_{(\psi_0, u_{t0}, u_{b0})}$
Slope of control valve charact. V	$\partial\Phi_b/\partial u_b\|_{(\psi_0, u_{b0})}$

where ψ is the dimensionless plenum pressure rise and the throttle and control valve parameter c_t and c_b are a measure for the capacity of the fully opened valve. The dimensionless throttle and control valve position are defined as $u_t = Y_t/Y_{t,max}$ and $u_b = Y_b/Y_{b,max}$, so u_t and u_b vary between 0 (closed valve) and 1 (fully opened valve). It can easily be verified from (6) that the intersection point of the compressor characteristic and the combined valve characteristic is the equilibrium point of the studied compression system:

$$\Phi_{c0} = \Phi_{t0} + \Phi_{b0} \quad \text{and} \quad \psi_0 = \Psi_c(\Phi_{c0}).$$

Linearization of (6) around the desired equilibrium point $(\Phi_{c0}, \psi_0, u_{t0}, u_{b0})$ results in the following second-order system (Willems, 2000):

$$\begin{bmatrix} \dot{\hat{\Phi}}_c \\ \dot{\hat{\psi}} \end{bmatrix} = \underbrace{\begin{bmatrix} \beta m_c & -\beta \\ \dfrac{1}{\beta} & -\dfrac{1}{\beta m_{te}} \end{bmatrix}}_{=:\mathbf{A}} \begin{bmatrix} \hat{\Phi}_c \\ \hat{\psi} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ -\dfrac{V}{\beta} \end{bmatrix}}_{=:\mathbf{B}} \hat{u}_b. \qquad (7)$$

The subscript 0 indicates quantities corresponding to the nominal operating point and ^ expresses deviations from the nominal operating point.

5. Active surge control

Greitzer (1981) shows that the *uncontrolled* compression system (7) is stable if and only if $m_c < m_{te}$ and $m_c < 1/\beta^2 m_{te}$. Roughly speaking, this corresponds with operating points on the compressor characteristic where $\Phi_{c0} > 2F$. Active control can enlarge the range of operation points for which the compression system is stable (Epstein et al., 1989).

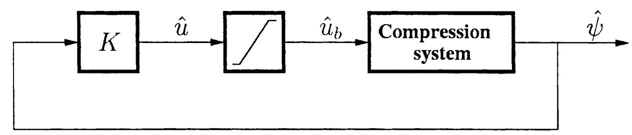

Fig. 3. Block scheme of the linearized compression system model.

5.1. Static output feedback

In the examined installation, reliable transient mass flow measurements are not available. Therefore, it is studied what can be achieved with static output feedback based on plenum pressure measurements (see Fig. 3):

$$\hat{u} = -K\hat{\psi}, \qquad (8)$$

where $\hat{\psi} = \psi - \psi_0$. Then, the system *without* constraints on $\hat{u}$ is stabilizable if and only if the following condition holds (Simon, Valavani, Epstein, & Greitzer, 1993):

$$m_c < \frac{1}{\beta}. \qquad (9)$$

The Greitzer stability parameter β is proportional to the rotational speed N. Consequently, stabilization is more difficult for increasing N, because smaller slopes m_c of the compressor characteristics are allowed.

To avoid wasteful bleed of compressed air, the control valve is closed in the desired equilibrium point ($u_{b0} = 0$). As a result, the dimensionless control valve position $\hat{u}_b$ is constrained between 0 and 1. Up to now the positivity constraint is ignored in the literature on surge control. It is verified from Theorem 2.2 that the linearized system (7) is *positive feedback stabilizable* for $0 \leqslant \Phi_{c0} < 2F$. For sufficiently small values of $\hat{\psi}$, the feedback satisfies the upper bound on $\hat{u}_b$. Hence, *local* stability can be guaranteed. To obtain a large domain of attraction, the control input has to be made small. The optimal K would maximize the domain of attraction, but the nonlinear tools for doing so are currently not available.

Due to the theory developed in Section 2 and the fact that K in (8) is a scalar, standard root locus techniques can be used to decide if a feedback K exists that places the eigenvalues of $A + BKC$ (with $C = [0 \ \ -1]$ and A, B as in (7)) in the cone (4). For $N = 25{,}000$ rpm, results are plotted in Fig. 4. The upper figures show the root-loci for equilibria corresponding to three different Φ_{c0} values and for K varying between -20 and -5. The dash-dotted lines indicate the bounds of the cone (4), which are plotted for reference. From these plots, it is seen that a stabilizing feedback exists for operating points with $N = 25{,}000$ rpm and $\Phi_{c0} \geqslant 1.87F$ using static output feedback. Recall that the uncontrolled linearized system is stable for $\Phi_{c0} > 2F$. As a result, a 6.5% extension of the stable operating region is expected. If $N = 18{,}000$ rpm, the linearized system can be stabilized by (8) for $\Phi_{c0} > 1.65F$, which corresponds with a 17.5% increase. To determine the control gains K, for which both closed-loop poles are located in

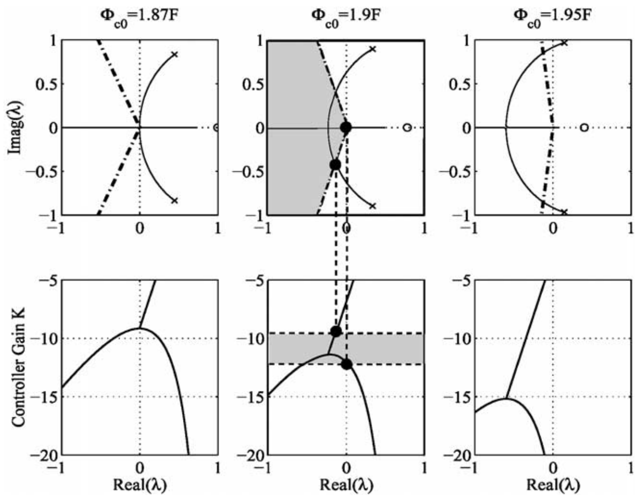

Fig. 4. Root-loci (cone (—), open-loop poles (×), open-loop zero (○); $N = 25,000$ rpm, $c_b = 0.1c_t$, and $u_{b0} = 0$).

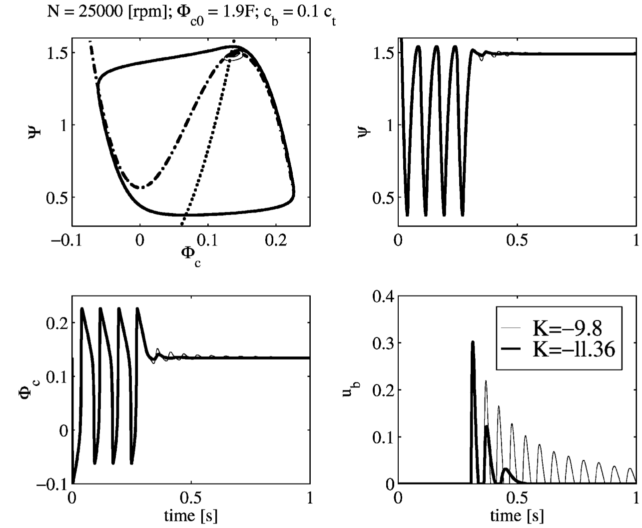

Fig. 5. Simulation results for the nonlinear compression system model.

the cone, in the lower figures the control gain is plotted. From Fig. 4, it is concluded that the lower constraint on $\hat{u}_b$ does not affect the stabilizability of the linearized system; when for all K a closed-loop pole is not contained in the cone, it is also not located in the open complex left-half plane for all K. However, the lower bound on $\hat{u}_b$ reduces the *range* of stabilizing control gains.

5.2. Simulation results

In the simulation, the nonlinear compression model is used including control valve saturation. We consider a case with maximal realizable speed: $N = 25,000$ rpm and $\Phi_{c0} = 1.9F$. According to (9), in this case stabilization is relatively difficult to accomplish using static output feedback based on plenum pressure measurements, because m_c is close to $1/\beta$.

To illustrate the effect of the control gain K on the system's response, two cases are considered. If $K = -9.8$, the closed-loop poles are complex ($\lambda_{1,2} = -0.1446 \pm j0.3832$) and lay just inside the cone (4). The control gain $K = -11.36$ is the smallest $|K|$ for which the closed-loop poles are real ($\lambda_1 = -0.2139$; $\lambda_2 = -0.2280$).

In the simulations, the *uncontrolled* system is initially disturbed from its nominal operating point. For $\Phi_{c0} < 2F$, this results in a limit cycle oscillation associated with surge. After 0.25 s the controller is switched on and the system's response is observed. In Fig. 5, the upper left-hand figure shows the system's response in the compressor map. In this map, the dash-dotted and dotted lines are the compressor and throttle characteristic, respectively, which are plotted for reference. Furthermore, the time traces of the plenum pressure rise ψ, the compressor mass flow Φ_c, and the control valve position u_b are shown. It is seen from Fig. 5 that for $K = -11.36$ the system is stabilized after three cycles and the desired equilibrium point is reached with zero average control valve mass flow ($u_b = 0$). Obviously, the domain of attraction of the desired equilibrium point includes the surge limit cycle. Note that for the linearized system the equilibrium point will be reached within one cycle. In case of $K = -9.8$ the closed-loop poles are complex, which causes the system to switch frequently between the controlled and uncontrolled mode as expected from the proof of Theorem 2.2.

6. Experimental result

The proposed controller is implemented on the experimental set-up of Section 3 as described in Willems and de Jager (2000). Fig. 6 shows an experimental result. The left-hand figure shows the recorded time trace of the plenum pressure whereas a time trace of the applied control signal u is shown in the right-hand figure. The *uncontrolled* system ($u = 0$) is initially operated in surge, as illustrated by the large amplitude pressure oscillation in Fig. 6. After 0.3 s, the controller is switched on and the system stabilizes from surge after three cycles. Stabilization in the desired equilibrium point requires small average control authority: $\bar{u} = 0.047$. This corresponds with relatively small average control valve mass flow, so this control strategy is efficient. In the examined case the surge point mass flow is reduced by 7% (Willems, 2000). From the stability analysis of the linearized compression system, it is seen that the stable operating range can be further increased. In the examined operating point we expect the closed-loop system to be unstable for the applied control gain ($K = -2$) on the basis of the stability analysis of the linearized system, while this experiment shows stable behavior. Additional experiments show that this may be due to an unmodeled two-dimensional aero-

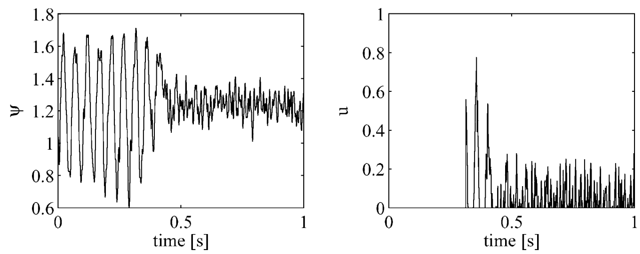

Fig. 6. Experimental result using (8) with $N \approx 19,000$ rpm, $u_{t0} = 0.271$ and $K = -2$.

dynamic flow instability: *rotating stall* (Willems, 2000). This effectively changes the value of m_c leading to another range of stabilizing K.

7. Conclusions

This paper combines a nice theoretical result on positive feedback stabilization with an interesting application: the stabilization of compressor surge.

Although the developed theory on positive feedback stabilization is only applicable to a limited class of linear systems, the presented result has many advantages due to the simple implementation and design of the feedback controller (certainly in comparison with Smirnov (1996) and Zaslavsky (1990)) and to the available parameters for shaping the closed-loop behavior.

A positive static output feedback controller has been applied to stabilize surge in a compression system with a centrifugal compressor. Experiments with this new surge control strategy have generated promising results; surge can be suppressed and, so far, a 7% reduction in surge point mass flow is realized. Moreover, this strategy is efficient, since the control valve is nearly closed in the stabilized equilibrium point, so stationary bleed losses are avoided.

Acknowledgements

The authors would like to acknowledge Harm van Essen, Rick de Lange, and Corine Meuleman for their practical advice and assistance during experiments.

References

Botros, K. K., Campbell, P. J., & Mah, D. B. (1991). Dynamic simulation of compressor station operation including centrifugal compressor and gas turbine. *ASME Journal of Engineering for Gas Turbines and Power*, *113*(2), 300–311.

Boyd, S., & Yang, Q. (1989). Structured and simultaneous Lyapunov functions for system stability problems. *International Journal on Control*, *49*(6), 2215–2240.

Brammer, R. F. (1972). Controllability in linear autonomous systems with positive controllers. *SIAM Journal on Control*, *10*(2), 339–353.

Branicky, M. S. (1994). Analyzing continuous switching systems: Theory and examples. *Proceedings of the American Control Conference* (pp. 3110–3114).

Branicky, M. S. (1998). Stability theory for hybrid dynamical systems. *IEEE Transactions on Automatic Control*, *43*(4), 475–482.

DiLiberti, J.-L., Van den Braembussche, R. A., Konya, P., & Rasmussen, S. (1996). *Active control of surge in centrifugal compressors with inlet pipe resonance.* ASME Paper No. 96-WA/PID-1.

Epstein, A. H., Ffowcs Williams, J. E., & Greitzer, E. M. (1989). Active suppression of aerodynamic instabilities in turbomachines. *Journal of Propulsion*, *5*(2), 204–211.

Farina, L., & Benvenutti, L. (1997). Polyhedral reachable sets with positive controls. *Mathematics of Control, Signals, and Systems*, *10*, 364–380.

Gravdahl, J. T., & Egeland, O. (1998). *Compressor surge and rotating stall: Modeling and control*. Berlin: Springer.

Greitzer, E. M. (1976). Surge and rotating stall in axial flow compressors. Part I: Theoretical compression system model. *ASME Journal of Engineering for Power*, *98*(2), 191–198.

Greitzer, E. M. (1981). The stability of pumping systems—The 1980 Freeman scholar lecture. *ASME Journal of Fluid Dynamics*, *103*(2), 193–242.

Heemels, W. P. M. H., Schumacher, J. M., & Weiland, S. (2000). Linear complementarity systems. *SIAM Journal of Applied Mathematics*, *60*(4), 1234–1269.

Heemels, W. P. M. H., & Stoorvogel, A. A. (1998). *Positive stabilizability of a linear continuous-time system*. Technical Report 98I-01, Measurement and Control, Elect. Eng., Eindhoven Univ. Technology.

Heemels, W. P. M. H., Van Eijndhoven, S. J. L., & Stoorvogel, A. A. (1998). Linear quadratic regulator problem with positive controls. *International Journal on Control*, *70*(4), 551–578.

Heymann, M., & Stern, R. J. (1975). Controllability of linear systems with positive controls: Geometric considerations. *Journal of Mathematical Analysis and Applications*, *52*, 36–41.

Johansson, M., & Rantzer, A. (1988). Computation of piecewise quadratic Lyapunov functions for hybrid systems. *IEEE Transactions on Automatic Control*, *43*(4), 555–559.

Khalil, H. K. (1992). *Nonlinear Systems*. New York: MacMillan.

Liberzon, D., & Morse, A. S. (1999). Benchmark problems in stability and design of switched systems. *Nonlinear Control Abstracts*, *10*.

Lütkepohl, H. (1996). *Handbook of Matrices*. New York: Wiley.

Meuleman, C., Willems, F., de Lange, R., & de Jager, B. (1998). *Surge in a low-speed radial compressor*. ASME Paper No. 98-GT-426.

Narendra, K. S., & Balakrishnan, J. (1994). A common Lyapunov function for stable LTI systems with commuting A-matrices. *IEEE Transactions on Automatic Control*, *39*(12), 2469–2471.

Pachter, M. (1980). The linear-quadratic optimal control problem with positive controllers. *International Journal on Control*, *32*(4), 589–608.

Simon, J. S., Valavani, L., Epstein, A. H., & Greitzer, E. M. (1993). Evaluation of approaches to active compressor surge stabilization. *ASME Journal of Turbomachinery*, *115*(1), 57–67.

Sissaoui, H., Collins, W. D., & Harley, P. J. (1988). The numerical solution of the linear regulator problem with positive controls. *IMA Journal of Mathematical Control and Information*, *5*, 191–201.

Smirnov, G. V. (1996). Stabilization by constrained controls. *SIAM Journal of Control and Optimization*, *34*(6), 1616–1649.

Sontag, E. D. (1981). Nonlinear regulation: The piecewise linear approach. *IEEE Transactions on Automatic Control*, *26*(2), 346–357.

van der Schaft, A. J., & Schumacher, J. M. (1996). The complementary-slackness class of hybrid systems. *Mathematics of Control, Signals and Systems*, *9*, 266–301.

Willems, F., & de Jager, B. (1999). Modeling and control of compressor flow instabilities. *IEEE Control Systems Magazine*, *19*(5), 8–18.

Willems, F., & de Jager, B. (2000). *One-sided control of surge in a centrifugal compressor system*. ASME Paper No. 2000-GT-527.

Willems, F. (2000). *Modeling and bounded feedback stabilization of centrifugal compressor surge*. Ph.D. thesis, Mechanical Engineering, Eindhoven Univ. Technology.

Zaslavsky, B. G. (1990). Positive stabilizability of control processes. *Automation and Remote Control USSR*, *51*(3), 291–294.

Frank Willems received the M.Sc. and Ph.D. degrees in Mechanical Engineering from Eindhoven University of Technology in 1995 and 2000, respectively. The work presented in this article was performed during his Ph.D. study. It forms a part of the Compressor Surge Project at the Faculty of Mechanical Engineering. This research project is a cooperation between the Systems and Control Group and the Energy Technology Group. Currently, he is with the Netherlands Organisation for Applied Scientific Research (TNO). His main research interests are modeling and control of powertrains, with a focus on the development of EGR control for heavy-duty diesel engines.

Maurice Heemels was born in St. Odilienberg, The Netherlands, in 1972. He received the M.Sc. degree (with honours) from the Department of Mathematics and the Ph.D. degree (cum laude) from the Department of Electrical Engineering of the Eindhoven University of Technology (The Netherlands) in 1995 and 1999, respectively. He was awarded the ASML price for the best Ph.D. Thesis of the Eindhoven University of Technology in 1999/2000 in the area of fundamental research. Currently, he is working as an assistant professor in the Control Systems group of the Department of Electrical Engineering of the Eindhoven University of Technology. His research interests include modeling, analysis and control of hybrid systems and dynamics under inequality constraints (especially complementarity problems and systems).

Bram de Jager. For a biography and photograph see the April 2001 issue (volume 37, number 4) of *Automatica*.

Anton A. Stoorvogel received the M.Sc. degree in Mathematics from Leiden University in 1987 and the Ph.D. degree in Mathematics from Eindhoven University of Technology, the Netherlands in 1990. He has been associated with Eindhoven University of Technology since 1987. In 2000, he was also appointed as professor in the Department of Information Technology and Systems of Delft University of Technology and as professor and adjunct faculty in the Department of Electrical Engineering and Computer Science of Washington State University. In 1991 he visited the University of Michigan.

From 1991 till 1996 he was a researcher of the Royal Netherlands Academy of Sciences. Anton Stoorvogel is the author of three books and numerous articles.