Event-Triggered Control in Presence of Measurement Noise: A Space-Regularization Approach

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Abstract— **In this paper, general conditions for set stabilization of (distributed) event-triggered control systems affected by measurement noises are presented. It is shown that, under these conditions, both static and dynamic triggers can be designed such that the closed-loop system ensures an input-to-state practical stability property. Additionally, by proper choice of the tuning parameters, the system does not exhibit Zeno behavior. Contrary to various results in the literature, the noises do not have to be differentiable. The general results are applied to point stabilization and consensus problems as particular cases. Simulations illustrate our results.**

I. Introduction

For systems in which the communication energy consumption, communication bandwidth or computation power is constrained, traditional periodic sampling and/or controller updates might require resources that are not available to obtain the desired system performance. To this end, event-triggered control (ETC) can be applied, see, e.g., [1] and the references therein, to reduce computational burden and/or the communication bandwidth of the control strategies, while preserving important stability and performance properties.

In general, most literature on ETC assumes that perfect state or output information is available for control, even though in most physical systems, this is often not the case due to sensor limitations. Since sensors are susceptible to measurement noises, exact state or output information is often not available. It is known that under these circumstances, the design of triggering conditions that do not have Zeno behavior is in general a hard problem, see, e.g., [2]. Several solutions have been proposed in the literature to address this problem, see, e.g., [3], [4]. However, these require differentiability conditions on the noise to obtain input-to-state stability (ISS) or $\mathcal{L}_p$-stability of the closed-loop system with respect to the noise and its time-derivative. When dealing with real sensors, the differentiability condition and global boundedness of the derivative of the noise may not be natural assumptions. The observer-based approaches, see, e.g., [5], overcome this issue, but these results only apply to linear systems and require multiple additional internal models, thereby requiring extra processing power and energy to run. In [6], a periodic event-triggered controller is run simultaneously with a continuous event-triggered controller, and triggering occurs when the triggering conditions of both controllers hold. The downside to this particular method is that close to the origin, periodic sampling is obtained, hence, the communication benefit of ETC is not preserved. This issue is even harder when designing distributed event-triggered controller for consensus [7]. We know of only one paper dealing with measurement noise in this context, [8], where the control input is integrated to estimate an upper-bound for the error. Since a conservative estimate is used and due to the absolute triggering condition, the amount of controller updates (network bandwidth) required is relatively high compared to other ETC consensus algorithms, see, e.g., [9].

In this paper, we present a general framework to address the measurement noise problem, based on space-regularized ETC, in line with classical event generators, such as [10], [11], [12]. For this, we present a new hybrid model, which does not involve the derivative of the noise as opposed to [3], [4]. We then provide general prescriptive conditions, under which both dynamic and static triggering rules are designed to ensure an input-to-state practical stability property, while ruling out Zeno phenomena. In particular, we show that applying space-regularization needs to be done with care to ensure the existence of strictly positive minimum inter-event times. These results are written for the general scenario where N plants, possibly interconnected, are controlled by N event-triggered controllers, hence covering both classical point stabilization problems as in, e.g., [10], [11], [12], [13] and consensus problems as in, e.g., [14] in a unified way. We then demonstrate the relevance of our technique by showing that it can be applied to design event-triggering strategies robust to measurement noise. In particular, we explain how to modify the triggering rules presented in [10], [11] to be applicable in presence of measurement noise. We also apply it to consensus seeking problems, where we show that we can maintain long inter-event times even in the presence of measurement noise. We show this, for instance, in the methods of [9], [15]. Lastly, we use simulations to show the effectiveness of our technique and to demonstrate the implications of applying space-regularization.

II. Preliminaries

A. Notation

The sets of all non-negative and positive integers are denoted $\mathbb{N}$ and $\mathbb{N}_{>0}$, respectively. The field of all reals and all non-negative reals are indicated by $\mathbb{R}$ and $\mathbb{R}_{\geqslant 0}$, respectively. The identity matrix of size $N \times N$ is denoted by I_N, and the vectors in $\mathbb{R}^N$ whose elements are all

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ones or zeros are denoted by $\mathbf{1}_N$ and $\mathbf{0}_N$, respectively. For N vectors $x_i \in \mathbb{R}^{n_i}$, the vector obtained by stacking all vectors into one column vector $x \in \mathbb{R}^n$ with $n = \sum_{i=1}^{N} n_i$ is denoted as $(x_1, x_2, \ldots, x_N)$, i.e., $(x_1, x_2, \ldots, x_N) = \begin{bmatrix} x_1^\top & x_2^\top & \ldots & x_N^\top \end{bmatrix}^\top$. By $\langle \cdot, \cdot \rangle$ and $|\cdot|$ we denote the usual inner product of real vectors and the Euclidean norm, respectively. For a measurable signal $w : \mathbb{R}_{\geqslant 0} \to \mathbb{R}^{n_w}$, we denote by $\|w\|_\infty = \operatorname{ess\,sup}_{t \in \mathbb{R}_{\geqslant 0}} |w(t)|$ its $\mathcal{L}_\infty$-norm, provided it exists and is finite (we then write $w \in \mathcal{L}_\infty$). For any $x \in \mathbb{R}^N$, the distance to a closed non-empty set $\mathcal{A}$ is denoted by $|x|_\mathcal{A} = \inf_{y \in \mathcal{A}} |x - y|$.

A continuous function $\alpha : \mathbb{R}_{\geqslant 0} \to \mathbb{R}_{\geqslant 0}$ is a class-$\mathcal{K}$ function if it is continuous, strictly increasing and $\alpha(0) = 0$ and it is a class-$\mathcal{K}_\infty$ function if, in addition, $\alpha(r) \to \infty$ as $r \to \infty$. A continuous function $\beta : \mathbb{R}_{\geqslant 0} \times \mathbb{R}_{\geqslant 0} \to \mathbb{R}_{\geqslant 0}$ is a class-$\mathcal{KL}$ function if, for each fixed $s \geqslant 0$, the mapping $\beta(\cdot, s)$ is a class-$\mathcal{K}$ function and, for each fixed r, the mapping $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \to 0$ as $s \to \infty$.

B. Hybrid systems

We model hybrid systems using the formalism of [16], [17]. Hence, we consider systems $\mathcal{H}(F, \mathcal{C}, G, \mathcal{D})$ of the form

$$\begin{cases} \dot{\xi} \in F(\xi, w) & (\xi, w) \in \mathcal{C}, \\ \xi^+ \in G(\xi, w) & (\xi, w) \in \mathcal{D}, \end{cases} \tag{1}$$

where $\xi \in \mathbb{R}^{n_\xi}$ denotes the state, $w \in \mathbb{R}^{n_w}$ a disturbance, $\mathcal{C} \in \mathbb{R}^{n_\xi} \times \mathbb{R}^{n_w}$ the flow set, $\mathcal{D} \in \mathbb{R}^{n_\xi} \times \mathbb{R}^{n_w}$ the jump set, $F : \mathbb{R}^{n_\xi} \times \mathbb{R}^{n_w} \rightrightarrows \mathbb{R}^{n_\xi}$ the flow map and $G : \mathbb{R}^{n_\xi} \times \mathbb{R}^{n_w} \rightrightarrows \mathbb{R}^{n_\xi}$ the jump map, where the maps F and G are possibly set-valued. We are mainly interested in systems $\mathcal{H}$ being persistently flowing in the sense that all its maximal solutions are unbounded in t-direction, see [16] for more details on the adopted hybrid terminology. We focus on the following stability definitions in this paper.

Definition 1. *When $\mathcal{H}$ is persistently flowing, we say that a non-empty closed set $\mathcal{A} \subset \mathbb{R}^{n_\xi}$ is* input-to-state practically stable *(ISpS) if there exist $\gamma \in \mathcal{K}$, $\beta \in \mathcal{KL}$ and $d \in \mathbb{R}_{\geqslant 0}$ such that for any solution pair (ξ, w) with $w \in \mathcal{L}_\infty \cap \mathcal{PC}$*[1]

$$|\xi(t, j)|_\mathcal{A} \leqslant \beta(|\xi(0, 0)|_\mathcal{A}, t) + \gamma(\|w\|_\infty) + d, \tag{2}$$

for all $(t, j) \in \operatorname{dom} \xi$. If (2) *holds with $d = 0$, then $\mathcal{A}$ is said to be* input-to-state stable *(ISS) for $\mathcal{H}$.*

To prove that a given non-empty closed set $\mathcal{A}$ is IS(p)S, we will use the following Lyapunov conditions.

Proposition 2. *Consider a persistently flowing system $\mathcal{H}$ and let $\mathcal{A} \subset \mathbb{R}^{n_\xi}$ be a non-empty closed set. If there exist a continuously differentiable $V : \mathbb{R}^{n_\xi} \to \mathbb{R}_{\geqslant 0}$, $\alpha, \underline{\alpha}, \overline{\alpha} \in \mathcal{K}_\infty$, $\gamma \in \mathcal{K}$ and $c \in \mathbb{R}_{\geqslant 0}$ such that*

1) for any $(\xi, w) \in \mathcal{C} \cup \mathcal{D}$,
$$\underline{\alpha}(|\xi|_\mathcal{A}) \leqslant V(\xi) \leqslant \overline{\alpha}(|\xi|_\mathcal{A}),$$

2) for all $(\xi, w) \in \mathcal{C}$,
$$\langle \nabla V(\xi), F(\xi, w) \rangle \leqslant -\alpha(|\xi|_\mathcal{A}) + \gamma(|w|) + c,$$

3) for all $(\xi, w) \in \mathcal{D}$ and any $g \in G(\xi, w)$,
$$V(g) - V(\xi) \leqslant 0,$$

then $\mathcal{A}$ is ISpS, and it is ISS if $c = 0$.

Proof is omitted for space reasons.

[1] A function $w : \mathbb{R}_{\geqslant 0} \to \mathbb{R}^{n_w}$ is said to be piecewise continuous, denoted by $w \in \mathcal{PC}$, if there exists a sequence $\{t_i\}_{i \in \mathbb{N}}$ with $t_{i+1} > t_i > t_0 = 0$ for all $i \in \mathbb{N}$ and $t_i \to \infty$ when $i \to \infty$ such that w is a continuous function on (t_i, t_{i+1}) with $\lim_{t \downarrow t_i} w(t) = w(t_i)$ for each $i \in \mathbb{N}$.

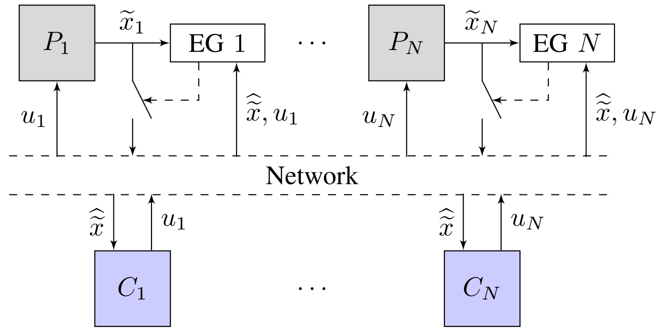

Fig. 1: Networked control setup with event generators (EG).

III. Problem formulation

We consider a collection of $N \in \mathbb{N}_{>0}$ interconnected plants $P_1, P_2, \ldots, P_N$. Each plant P_i, $i \in \mathcal{N} := \{1, 2, \ldots, N\}$, is equipped with a sensor that communicates its state[2] (with measurement noise) to the controllers $C_1, C_2, \ldots, C_N$ via a digital network. Plant P_i, $i \in \mathcal{N}$, has a state $x_i \in \mathbb{R}^{n_x^i}$ with dynamics

$$\dot{x}_i = f_i(x, u_i), \tag{3}$$

where $u_i \in \mathbb{R}^{n_u^i}$ is the control input of P_i, $x := (x_1, x_2, \ldots, x_N)$ is the concatenated state variable, and $f_i : \mathbb{R}^n \times \mathbb{R}^{n_u^i} \to \mathbb{R}^{n_x^i}$ is a continuous function, with $n = \sum_{i \in \mathcal{N}} n_x^i$. Note that f_i may depend on the states of other plants, i.e., physical couplings are allowed. The controllers C_i, $i \in \mathcal{N}$, take the form, in absence of noise,

$$u_i = k_i(x), \tag{4}$$

with $k_i : \mathbb{R}^n \to \mathbb{R}^{n_u^i}$ a continuous map. We assume that plants $P_1, P_2, \ldots, P_N$ in closed loop with the controllers $C_1, C_2, \ldots, C_N$ satisfy desired control objectives in the absence of a network, as formalized in the following.

We investigate the scenario where the values of each state x_i, $i \in \mathcal{N}$, are broadcasted by the corresponding sensors to the controllers $C_1, C_2, \ldots, C_N$, which depend on it, via a digital network, as illustrated in Fig. 1. The corresponding transmissions occur at some time instants t_k^i, $k \in \mathbb{N}$, which are generated by a local triggering condition. Moreover, the measurements are affected by noise. To model the obtained feedback law in this context, we introduce $\widetilde{x}_i$, the noisy measurement of x_i, for $i \in \mathcal{N}$, as

$$\widetilde{x}_i := x_i + w_i, \tag{5}$$

where $w_i \in \mathbb{R}^{n_x^i}$ is an (additive) bounded piecewise continuous measurement noise, which is assumed to satisfy the following assumption.

[2] Extensions to output-feedback control can be directly envisioned.

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Assumption 1. *For each* $i \in \mathcal{N}$, $w_i(t,j) \in \mathcal{W}_i$ *for all* $(t,j) \in \operatorname{dom} w$, *where* $\mathcal{W}_i := \left\{ w_i \in \mathbb{R}^{n_x^i} \mid |w_i| \leqslant \overline{w}_i \right\}$ *for some* $\overline{w}_i \in \mathbb{R}_{\geqslant 0}$.

Because of the packet-based communication over the network, the controllers, which depend on the state of P_i, do not have access to $\widetilde{x}_i$ in (5), but only to its networked version, $\widehat{\widetilde{x}}_i := \widehat{x}_i + \widehat{w}_i$, where

$$\begin{aligned} \widehat{x}_i(t) &= x_i(t_k^i) \qquad \text{for } t \in [t_k^i, t_{k+1}^i),\ k \in \mathbb{N}, \\ \widehat{w}_i(t) &= w_i(t_k^i) \qquad \text{for } t \in [t_k^i, t_{k+1}^i),\ k \in \mathbb{N}. \end{aligned} \tag{6}$$

To keep the definitions consistent with the existing literature, we define the network-induced error $e = (e_1, e_2, \ldots, e_N)$ as the difference between the sampled state $\widehat{x} := (\widehat{x}_1, \widehat{x}_2, \ldots, \widehat{x}_N)$ *without* the measurement noise and the current state *without* measurement noise, i.e.,

$$e := \widehat{x} - x. \tag{7}$$

We also introduce the *measured* network-induced error $\widetilde{e} = (\widetilde{e}_1, \widetilde{e}_2, \ldots, \widetilde{e}_N)$ as the difference between the most recently transmitted state and the currently measured state, which are both affected by noise, i.e.,

$$\widetilde{e} := \widehat{x} + \widehat{w} - x - w = e + \widehat{w} - w. \tag{8}$$

Note that e_i is not known by event generator i, and therefore, cannot be used by the corresponding local triggering condition for determining t_k^i, $k \in \mathbb{N}$. However, the event generators do have access to $\widetilde{e}_i$.

Due to the network and the noisy measured states, the feedback law in C_i applied to plant P_i is, for $i \in \mathcal{N}$,

$$u_i = k_i(x + e + \widehat{w}). \tag{9}$$

Our objective is to determine the transmission times t_k^i, $k \in \mathbb{N}$, for any $i \in \mathcal{N}$, to ensure that:

(i) the combined closed-loop system (3), (9) satisfies an input-to-state practical stability property in the presence of measurement noise;

(ii) there exists a strictly positive time between any two transmissions generated by the triggering condition of plant P_i, i.e., for any initial condition there exists a $T_i > 0$ such that $t_{k+1}^i - t_k^i \geqslant T_i$ for all $k \in \mathbb{N}$, $i \in \mathcal{N}$.

IV. General Results

A. Hybrid model

We model the overall system as a hybrid system $\mathcal{H}$ for which a jump corresponds to the broadcasting of one of the noisy states $\widetilde{x}_i$, $i \in \mathcal{N}$, over the network. We allow the local triggering conditions to depend on a local variable denoted $\eta_i \in \mathbb{R}_{\geqslant 0}$, $i \in \mathcal{N}$, as in the dynamic triggering of [11], [13]. However, we also consider static triggering conditions in the sequel. We define $\eta := (\eta_1, \eta_2, \ldots, \eta_N) \in \mathbb{R}^N_{\geqslant 0}$, and stack the "physical" variables in $\chi := (x, e, \widehat{w})$. The full state for $\mathcal{H}$ becomes $\xi := (\chi, \eta) = (x, e, \widehat{w}, \eta)$ and is defined as

$$\begin{aligned} \dot{\xi} &= F(\xi, w), \qquad (\xi, w) \in \mathcal{C}, \\ \xi^+ &\in G(\xi, w), \qquad (\xi, w) \in \mathcal{D}, \end{aligned} \tag{10}$$

where the flow map is given, for all $(\xi, w) \in \mathbb{X} \times \mathcal{W} := \mathbb{R}^n \times \mathbb{R}^n \times \mathcal{W} \times \mathbb{R}^N_{\geqslant 0} \times \mathcal{W}$, where $\mathcal{W} := \mathcal{W}_1 \times \mathcal{W}_2 \times \ldots \times \mathcal{W}_N$ and $\mathcal{W}_i$ comes from Assumption 1, by

$$F(\xi, w) := (F_\chi(\chi), \Psi(o)), \tag{11}$$

with $\Psi(o) := (\Psi_1, \Psi_2, \ldots, \Psi_N)$ the to-be-designed dynamics of the dynamic variables η and $o := (o_1, o_2, \ldots, o_N)$ collects $o_i \in \mathbb{R}^{n_o^i}$, $i \in \mathcal{N}$, being the information locally available to plant i. In (11),

$$F_\chi(\chi) := (f(x, k(x + e + \widehat{w})), -f(x, k(x + e + \widehat{w})), \mathbf{0}_n), \tag{12}$$

for $\chi \in \mathbb{R}^n \times \mathbb{R}^n \times \mathcal{W}$. Let, for $i \in \mathcal{N}$,

$$\mathcal{C}_i := \{(\xi, w) \in \mathbb{X} \times \mathcal{W} \mid \eta_i + \theta_i \Psi_i(o_i) \geqslant 0\} \tag{13}$$

with $\theta_i \in \mathbb{R}_{\geqslant 0}$ a design parameter. The flow set for the overall system is given by

$$\mathcal{C} := \bigcap_{i \in \mathcal{N}} \mathcal{C}_i. \tag{14}$$

The jump set corresponding to a transmission of $\widetilde{x}_i$ generated by triggering condition $i \in \mathcal{N}$ is defined as,

$$\begin{aligned} \mathcal{D}_i := \big\{ (\xi, w) \in \mathbb{X} \times \mathcal{W} \mid \eta_i + \theta_i \Psi_i(o_i) \leqslant 0 \text{ and} \\ \Psi_i(o_i) \leqslant 0 \big\}. \end{aligned} \tag{15}$$

Note that, with respect to [11], we require the additional condition $\Psi_i(o_i) \leqslant 0$ to ensure that Zeno behavior does not occur when $\theta_i = 0$. By selecting a $\theta_i > 0$, we trigger earlier than the "pure" dynamic case (i.e., when $\theta_i = 0$). Generally, this results in faster convergence but shorter inter-event times, which allows us to tune bandwidth usage versus performance, see [11] for more details. The jump set for the overall system is defined as

$$\mathcal{D} := \bigcup_{i \in \mathcal{N}} \mathcal{D}_i. \tag{16}$$

The jump map for triggering condition i is now defined as

$$G_i(\xi, w) := \begin{cases} \{(G_{\chi,i}(\chi, w), \eta)\}, & \text{if } (\xi, w) \in \mathcal{D}_i \\ \emptyset, & \text{if } (\xi, w) \notin \mathcal{D}_i, \end{cases} \tag{17}$$

where

$$G_{\chi,i}(\chi, w) := (x, \overline{\Gamma}_i e, \overline{\Gamma}_i \widehat{w} + \Gamma_i w), \tag{18}$$

with Γ_i the block diagonal matrix where the i-th block is $I_{n_x^i}$ and all other blocks are $\mathbf{0}_{n_x^j \times n_x^j}$, $j \in \mathcal{N} \setminus \{i\}$, and $\overline{\Gamma}_i := I_n - \Gamma_i$. Map (17) simply means that a jump due to triggering condition i resets e_i to 0 and $\widehat{w}_i$ to w_i (essentially, $\widehat{w}_i^+ \in \mathcal{W}_i$), leaving the other variables unchanged. The complete jump map is given by

$$G(\xi, w) := \bigcup_{i \in \mathcal{N}} G_i(\xi, w). \tag{19}$$

For future use we also define the jump map for $(\chi, w) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathcal{W} \times \mathcal{W}$ as

$$G_\chi(\chi, w) := \bigcup_{i \in \mathcal{N}} G_{\chi,i}(\chi, w). \tag{20}$$

Because of the selected state variables, system (10) does not depend on the time-derivative of w as in [3], [4], which allows us to work under more general and more natural assumptions on the measurement noise.

The goal is to design the dynamics of η_i, $\mathcal{C}_i$ and $\mathcal{D}_i$, i.e., the functions Ψ_i, for all $i \in \mathcal{N}$, such that a given set $\mathcal{A}$ is ISpS, see Definition 1. To formalize objective (ii) stated at the end of Section II, we introduce, for any solution (ξ, w) to $\mathcal{H}$ and $i \in \mathcal{N}$, the set

$$\mathcal{T}_i(\xi, w) := \Big\{(t,j) \in \operatorname{dom}\xi \mid (\xi(t,j), w(t,j)) \in \mathcal{D}_i \text{ and } (\xi(t,j+1), w(t,j+1) \in G_i(\xi(t,j), w(t,j))\Big\}. \tag{21}$$

Hence, $\mathcal{T}^i(\xi, w)$ contains all hybrid times belonging to the hybrid time domain of a solution (ξ, w) at which a jump occurs due to triggering condition i ($\mathcal{D}_i$ and G_i). We introduce the following definition.

Definition 3. *Given a closed set $\mathcal{A} \subset \mathbb{R}^{2n} \times \mathcal{W}$, system* (10) *has a* semi-global individual minimum inter-event time (SGiMIET) *with respect to $\mathcal{A}$, if, for all $\Delta \geqslant 0$ and all $i \in \mathcal{N}$, there exists a $\tau^i_{\text{MIET}} > 0$ such that, for all solutions (ξ, w) with $|\xi(0,0)|_{\mathcal{A}} \leqslant \Delta$, for all $(t,j),(t',j') \in \mathcal{T}_i(\xi, w)$,*

$$t + j < t' + j' \Rightarrow t - t' \geqslant \tau^i_{\text{MIET}}. \tag{22}$$

If τ^i_{MIET} can be chosen independent of Δ for all $i \in \mathcal{N}$, then we say that $\mathcal{H}$ has a global individual minimum inter-event time (GiMIET).

Definition 3 means that the (continuous) time between two successive transmission instants due to a trigger of condition i are spaced by at least τ^i_{MIET} units of times, and that τ^i_{MIET} depends on the size of the initial conditions. Hence, the problem formulation at the end of Section II can be formally stated as for a given set $\mathcal{A}$, synthesize the sets $\mathcal{C}_i$ and $\mathcal{D}_i$, $i \in \mathcal{N}$ such that $\mathcal{A}$ is ISpS and $\mathcal{H}$ has a SGiMIET w.r.t. $\mathcal{A}$.

B. Design and analysis

We assume that controllers C_i, $i \in \mathcal{N}$, are designed such that Assumption 2 below holds. For specific scenarios we show in Section V how this assumption is naturally obtained.

Assumption 2. *There exist $\alpha, \underline{\alpha}, \overline{\alpha} \in \mathcal{K}_\infty$, $\gamma \in \mathcal{K}$, $\beta_i \in \mathcal{K}$ and $\delta_i : \mathbb{R}^{n^i_o} \to \mathbb{R}_{\geqslant 0}$ continuous for all $i \in \mathcal{N}$, a closed non-empty set $\mathcal{A}$ and a continuously differentiable function $V : \mathbb{R}^{2n} \times \mathcal{W} \to \mathbb{R}_{\geqslant 0}$ such that*

i) for any $\chi \in \mathbb{R}^{2n} \times \mathcal{W}$,

$$\underline{\alpha}(|\chi|_{\mathcal{A}}) \leqslant V(\chi) \leqslant \overline{\alpha}(|\chi|_{\mathcal{A}}), \tag{23}$$

ii) for all $\chi \in \mathbb{R}^{2n} \times \mathcal{W}$ and $w \in \mathcal{W}$,

$$\begin{aligned}&\langle \nabla V(\chi), F_\chi(\chi)\rangle \\ &\leqslant -\alpha(|\chi|_{\mathcal{A}}) + \gamma(|w|) + \sum_{i\in\mathcal{N}} \beta_i(|\widetilde{e}_i|) - \delta_i(o_i),\end{aligned} \tag{24}$$

iii) for any $\chi \in \mathbb{R}^{2n} \times \mathcal{W}$, $w \in \mathcal{W}$ and $g \in G_\chi(\chi, w)$,

$$V(g) - V(\chi) \leqslant 0, \tag{25}$$

iv) for any $\Delta > 0$, there exists $M_\Delta \geqslant 0$ such that for any $\chi \in \mathbb{R}^{2n} \times \mathcal{W}$ satisfying $|\chi|_{\mathcal{A}} \leqslant \Delta$,

$$|F_\chi(\chi)| \leqslant M_\Delta. \tag{26}$$

Assumption 2 imposes Lyapunov conditions on the χ-system. Item i) means that V is positive definite and radially unbounded with respect to $\mathcal{A}$. Item (ii) is an input-to-state stability property of set $\mathcal{A}$ for the flow dynamics, but not the desired one as it involves the error $\widetilde{e}_i$. Item iii) implies that the Lyapunov function does not increase at jumps and item iv) imposes boundedness conditions on f_i and k_i. Assumption 2 implies that, in the absence of a digital network (and thus, $\widetilde{e}_i = 0$ and $\widehat{w} = w$), the set $\mathcal{A}$ is input-to-state stable with respect to input w. Again, examples of systems verifying Assumption 2 are provided in Section V.

The next theorem explains how to design Ψ_i, $i \in \mathcal{N}$, arising in the flow map, and the flow and jump set definitions to ensure the desired objectives are met.

Theorem 4. *Consider system* (10) *and suppose Assumptions 1 and 2 hold. We define for all $i \in \mathcal{N}$, $\xi \in \mathbb{X}$ and $w \in \mathcal{W}$*

$$\Psi_i(o_i) := \delta_i(o_i) - \beta_i(|\widetilde{e}_i|) - \epsilon_i\eta_i + c_i, \tag{27}$$

with $c_i > \beta_i(2\overline{w}_i)$ and $\epsilon_i \in \mathbb{R}_{>0}$ tuning parameters. The set $\mathcal{A}^d := \{\xi : \chi \in \mathcal{A} \text{ and } \eta = 0\}$ is ISpS and system (10) *has a SGiMIET.*

Theorem 4 provides the expressions of Ψ_i, $i \in \mathcal{N}$, which ensure that ISS of set $\mathcal{A}$ guaranteed by Assumption 2 in the absence of network is approximately preserved in the presence of the digital network. Moreover, the existence of a strictly positive lower-bound on the inter-event time of each triggering mechanism is guaranteed. The interest of Theorem 4 lies in its simplicity, generality and in revealing the main concepts as a "prescriptive framework".

The expression of Ψ_i in (27) is based on so-called space-regularization, as by introducing c_i, we enlarge the flow set to ensure the existence of a SGiMIET. While space-regularization is well known in the hybrid systems literature, we have to be careful when designing c_i, because a priori the non-Zenoness only holds if c_i satisfies the condition mentioned in Theorem 4. The consequence of this is that we obtain practical stability, i.e., the constant d in (2) will be non-zero, see Remark 2 below for more details. On the other hand, Theorem 4 does not require to make assumptions on the differentiability of w_i, and a fortiori on boundedness properties of $\dot{w}_i$, as in various works in ETC considering measurement noise, see, e.g., [3], [4]. Additionally, we may exploit the structure present in specific scenarios or ETC mechanisms to obtain less conservative bounds for the parameters c_i and, in some cases, a GiMIET, as opposed to semiglobal one in Theorem 4, as will be illustrated in Section V.

Proof of Theorem 4. The first part of the proof consists in showing that the conditions of Proposition 2 hold. To this end, we introduce a Lyapunov candidate U, defined as

$$U(\xi) := V(\chi) + \sum_{i\in\mathcal{N}} \eta_i, \text{ for } \xi \in \mathbb{X}. \tag{28}$$

Lyapunov conditions. Due to item i) of Assumption 2, there exist class-$\mathcal{K}_\infty$ functions α_1, α_2 such that $\alpha_1(|\xi|_{\mathcal{A}^d}) \leqslant U(\xi) \leqslant \alpha_2(|\xi|_{\mathcal{A}^d})$ for all $\xi \in \mathcal{C} \cup \mathcal{D}$, and thus item 1) of Proposition 2 holds. Next, let $(\xi, w) \in \mathcal{C}$, in view of (24) and (27),

$$\begin{aligned}\langle \nabla U(\xi), F(\xi,w)\rangle &= \langle \nabla V(\chi), F_\chi(\chi)\rangle + \sum_{i\in\mathcal{N}} \Psi_i(o_i) \\ &\leqslant -\alpha(|\chi|_{\mathcal{A}}) + \gamma(|w|) + \sum_{i\in\mathcal{N}} \beta_i(|\widetilde{e}_i|) - \delta_i(o_i) + \Psi_i(o_i) \\ &= -\alpha(|\chi|_{\mathcal{A}}) + \gamma(|w|) + \sum_{i\in\mathcal{N}} c_i - \epsilon_i\eta_i \\ &\leqslant -\alpha^d(|\xi|_{\mathcal{A}^d}) + \gamma(|w|) + c\end{aligned} \tag{29}$$

with $c := \sum_{i\in\mathcal{N}} c_i$ and for some $\alpha^d \in \mathcal{K}_\infty$. Hence, item 2) holds. Since $\eta^+ = \eta$ and due to item iii) of Assumption 2, we note that for any $(\xi, w) \in \mathcal{D}$ and all $g \in G(\xi, w)$,

$$U(g) - U(\xi) \leqslant 0, \tag{30}$$

thus, item 3) also holds. Hence, we are left with proving that $\mathcal{H}$ is persistently flowing.

Completeness of maximal solutions. Even though Proposition 2.10 in [16] does not directly apply to hybrid systems with inputs, similar arguments can be followed to establish completeness of maximal solutions for (10), exploiting the piecewise continuity of w_i.

Semi-global individual minimum inter-event time. We prove t-completeness by showing that system (10) has the SGiMIET property. To this end, we examine the time between two successive jumps generated by triggering condition $i \in \mathcal{N}$. Recall that we trigger when $\eta_i + \theta_i\Psi_i(o_i) \leqslant 0$ and $\Psi_i(o_i) \leqslant 0$. By [11, Prop. 2.3], we know that, after a first triggering instant has occurred,

$$\delta_i(o_i) + c_i - \beta_i(|\widetilde{e}_i|) \leqslant 0 \text{ and } \eta_i \geqslant 0 \tag{31}$$

is always satisfied before $\eta_i + \theta_i\Psi_i(o_i) \leqslant 0$, $\Psi_i(o_i) \leqslant 0$ and $\eta_i \geqslant 0$ is. Hence, we can analyze when (31) holds to obtain a lower-bound for the inter-event times. Since δ_i takes non-negative values, we can under-estimate the inter-event times for triggering condition i by analyzing when

$$c_i = \beta_i(|\widetilde{e}_i|). \tag{32}$$

Rewriting this, we obtain the condition

$$\beta_i^{-1}(c_i) = |\widetilde{e}_i|. \tag{33}$$

Note that we can upper-bound the right-hand side of (33) as, in view of Assumption 1,

$$\beta_i^{-1}(c_i) = |\widetilde{e}_i| \leqslant |e_i| + |\widehat{w}_i| + |w_i| \leqslant |e_i| + 2\overline{w}_i. \tag{34}$$

Hence, we can under-estimate the triggering times by analyzing when

$$\beta_i^{-1}(c_i) - 2\overline{w}_i = |e_i|. \tag{35}$$

Recall that, by the condition on c_i in Theorem 4, we have $c_i > \beta_i(2\overline{w}_i)$, thus, the left-hand side of (35) is always positive. In view of (35), we define

$$\overline{c}_i := \beta_i^{-1}(c_i) - 2\overline{w}_i > 0. \tag{36}$$

Since $|e_i|$ is 0 after a transmission due to triggering rule i, the inter-event time for triggering rule i is lower bounded by the time it takes for $|e_i|$ to grow from 0 to $\overline{c}_i$ in view of (35). Note that the bound in (36) is *not* dependent on actual values of w_i, only on the upper-bounds presented in $w_i \in \mathcal{W}_i$, $i \in \mathcal{N}$. In the following, we provide a lower-bound on this inter-event time. Let $\mu > 0$ and consider (ξ, w) such that $|\xi(0,0)|_{\mathcal{A}^d} \leqslant \mu$. Note that by (29), (30) and the satisfaction of item (i) of Assumption 2, $|\xi(t,j)|_{\mathcal{A}^d} \leqslant \Delta$ for some $\Delta > 0$ (dependent on μ but not on $|\xi(0,0)|_{\mathcal{A}^d}$) and any $(t,j) \in \operatorname{dom}\xi$. Hence, in view of item iv) of Assumption 2, $|F_\chi(\chi(t,j))| \leqslant M_\Delta$. Thus, for almost all $j \in \mathbb{N}_{\geqslant 0}$ and almost all $t \in I^j$ where $I^j = \{t : (t,j) \in \operatorname{dom}\xi\}$, $\frac{d|e_i(t)|}{dt} \leqslant M_\Delta$. Consequently, the time between any two transmissions generated by triggering rule i is larger than or equal to $\overline{c}_i/M_\Delta$. Hence, $\mathcal{H}$ has the SGiMIET property and thus solutions are persistently flowing.

Since the system is persistently flowing, we also have that $\mathcal{H}$ is ISpS w.r.t. the set $\mathcal{A}^d$. ∎

We can derive similar results when the triggering conditions are static, i.e., when no variable η_i is introduced to define the transmission instants. In this case, we obtain the hybrid system $\mathcal{H}^s$ defined as

$$\begin{aligned}\dot{\chi} &= F_\chi(\chi), && (\chi, w) \in \mathcal{C}^s, \\ \chi^+ &\in G_\chi(\chi, w), && (\chi, w) \in \mathcal{D}^s,\end{aligned} \tag{37}$$

where

$$\mathcal{C}^s := \bigcap_{i\in\mathcal{N}} \mathcal{C}_i^s, \quad \mathcal{D}^s := \bigcup_{i\in\mathcal{N}} \mathcal{D}_i^s, \tag{38}$$

with the sets $\mathcal{C}_i^s, \mathcal{D}_i^s$ as

$$\begin{aligned}\mathcal{D}_i^s &:= \{(\chi, w) \in \mathbb{R}^{2n} \times \mathcal{W} \times \mathcal{W} \mid \Psi_i^s(o_i) \leqslant 0\}, \\ \mathcal{C}_i^s &:= \{(\chi, w) \in \mathbb{R}^{2n} \times \mathcal{W} \times \mathcal{W} \mid \Psi_i^s(o_i) \geqslant 0\},\end{aligned} \tag{39}$$

where $\Psi_i^s(o_i)$ is a static triggering condition, which is designed according to the next result.

Corollary 5. *Consider system* (37) *and suppose Assumptions 1 and 2 hold. We define for all $i \in \mathcal{N}$, $\chi \in \mathbb{R}^{2n} \times \mathcal{W}$ and $w \in \mathcal{W}$*

$$\Psi_i^s(o_i) := \delta_i(o_i) + c_i - \beta_i(|\widetilde{e}_i|) \tag{40}$$

with $c_i > \beta_i(2\overline{w}_i)$ tuning parameters. The set $\mathcal{A}$ is ISpS and system (37) *has a SGiMIET.*

The proof of Corollary 5 follows similar steps as the proof of Theorem 4, and is therefore omitted for space reasons.

Remark 1. *Assumption 2 and Corollary 5 allow us to consider the case where any δ_i is equal to zero. In this case, Ψ_i^s is given by $\Psi_i^s(o_i) := c_i - \beta_i(|\widetilde{e}_i|)$, with $c_i > \beta_i(2\overline{w}_i)$ tuning parameters. Note that triggering conditions of this form are often called absolute triggering conditions in the event-triggered control literature, see e.g. [2], [18], [19].*

Remark 2. *The parameters c_i in Theorem 4 and Corollary 5 are directly related to the constant d in the ISpS definition* (2)*. Note that* (2) *holds with $d = \theta(c)$ for some $\theta \in \mathcal{K}_\infty$, where $c = \sum_{i\in\mathcal{N}} c_i$. Hence, for a tighter bound on $|\xi(t,j)|_{\mathcal{A}^d}$, we require that c_i is as small as possible. Note, however, that due to Theorem 4, c_i is lower-bounded by $\beta_i(2\overline{w}_i)$, and thus the minimum value of d is $d_{\min} = \theta(\sum_{i\in\mathcal{N}} \beta_i(2\bar{w}_i))$ to ensure proper SGiMIET and ISpS properties. On the other hand, selecting a small c_i implies a small lower-bound on the SGiMIET in view of* (36)*. Hence, there exists a trade-off between large lower-bounds on the inter-event times and "asymptotic closeness" to $\mathcal{A}^d$ in terms of d, which is tunable through selection of c_i, $i \in \mathcal{N}$.*

V. Case studies

In this section, we investigate several existing event-triggering techniques in the literature and show how to modify these to handle measurement noise. We want to stress that this is a non-exhaustive sample of techniques which can be addressed by this method. We prove for this purpose that Assumption 2 is verified, which allows to directly apply Theorem 4 or Corollary 5.

A. Stabilization of a single system [10], [11]

A single plant P and a single controller C are considered here. In particular, the plant is given by

$$\dot{x} = f(x,u), \tag{41}$$

and the feedback controller by

$$u = k(x). \tag{42}$$

As in [10], [11], we assume the following properties,

Assumption 3. *Maps f and k are Lipschitz continuous on compacts. Additionally, there exist $\underline{\alpha}, \overline{\alpha}, \alpha, \gamma \in \mathcal{K}_\infty$ and a continuously differentiable Lyapunov function $V : \mathbb{R}^n \to \mathbb{R}$ satisfying, for any $x, v \in \mathbb{R}^n$,*

$$\begin{aligned} \underline{\alpha}(|x|) \leqslant V(x) \leqslant \overline{\alpha}(|x|), \\ \langle \nabla V(x), f(x,k(x+v))\rangle \leqslant -\alpha(|x|) + \gamma(|v|), \end{aligned} \tag{43}$$

implying that the origin of $\dot{x} = f(x,k(x+v))$ is ISS with respect to v.

We derive the following result from Assumption 3.

Proposition 6. *Consider system* (41) *with controller* (42) *and suppose Assumption 3 holds. Then all conditions of Assumption 2 are met for $\mathcal{A} = \{\chi : x = \mathbf{0}\}$ with $\beta(s) = \gamma(2s)$ for any $s \geqslant 0$, $\delta(o) = \sigma\alpha(\frac{1}{2}|\widetilde{x}|)$ for any $\widetilde{x} \in \mathbb{R}^n$ and V as in Assumption 3, with $\sigma \in (0,1)$ a tuning parameter.*

Proposition 6 implies that, for any bounded measurement noise as defined by Assumption 1, the triggers defined in Theorem 4 and Corollary 5 render the origin of the closed-loop system ISpS with the SGiMIET property.

Proof. By Assumption 3, item i) of Assumption 2 holds trivially. Let $x, e, \widehat{w} \in \mathbb{R}^{2n} \times \mathcal{W}$. In view of Assumption 3, by substituting v with $e + \widehat{w}$, we obtain

$$\langle \nabla V(x), f(x,k(x+e+\widehat{w}))\rangle \leqslant -\alpha(|x|) + \gamma(|e+\widehat{w}|). \tag{44}$$

By using (8), i.e., $e + \widehat{w} = \widetilde{e} + w$, we obtain

$$\langle \nabla V(x), f(x,k(x+e+\widehat{w}))\rangle \leqslant -\alpha(|x|) + \gamma(|\widetilde{e}+w|). \tag{45}$$

Next, we use the weak triangle inequality, $\gamma(a+b) \leqslant \gamma(2a) + \gamma(2b)$, see [20], to obtain

$$\langle \nabla V(x), f(x,k(x+e+\widehat{w}))\rangle \leqslant -\alpha(|x|) + \gamma(2|\widetilde{e}|) + \gamma(2|w|).$$

Then, for any $\sigma \in (0,1)$,

$$\begin{aligned} &\langle \nabla V(x), f(x,k(x+e+\widehat{w}))\rangle \\ &\leqslant -(1-\sigma)\alpha(|x|) - \sigma\alpha(|x|) - \sigma\alpha(|w|) + \sigma\alpha(|w|) \\ &\quad + \gamma(2|\widetilde{e}|) + \gamma(2|w|) \\ &\leqslant -(1-\sigma)\alpha(|x|) - \sigma\alpha\left(\tfrac{1}{2}(|x|+|w|)\right) + \gamma(2|\widetilde{e}|) \\ &\quad + \gamma(2|w|) + \sigma\alpha(|w|) \\ &\leqslant -(1-\sigma)\alpha(|x|) - \sigma\alpha\left(\tfrac{1}{2}|\widetilde{x}|\right) + \gamma(2|\widetilde{e}|) \\ &\quad + \gamma(2|w|) + \sigma\alpha(|w|) \\ &\leqslant -(1-\sigma)\alpha(|x|) + \zeta(|w|) + \gamma(2|\widetilde{e}|) - \sigma\alpha(\tfrac{1}{2}|\widetilde{x}|), \end{aligned} \tag{46}$$

for some $\zeta \in \mathcal{K}$, hence item ii) of Assumption 2 holds. Since the Lyapunov function V does not depend on e or $\widehat{w}$, for all $(\xi, w) \in \mathcal{D}$ and $g \in G(\xi, w)$, $V(g) - V(\xi) = 0$, and item iii) holds. Since f and k are Lipschitz continuous on compacts, for any $|\xi| \leqslant \Delta$ there exists a constant $L > 0$ such that $|f(x,k(x+e+\widehat{w}))| \leqslant L|\xi| = L\Delta$ and item iv) holds. ■

B. Consensus for multi-agent systems

A specific field of interest for ETC is consensus of multi-agent systems. We study several event-triggering control schemes in this context next. Due to limited space, we omit commonly used definitions on consensus and the interested reader is referred to [7]. We focus here on single integrator systems, where each plant P_i, which we call agent in this section, has single integrator dynamics, i.e., $\dot{x}_i = u_i$, with $x_i, u_i \in \mathbb{R}$. However, the ideas of this work apply in more general settings.

For a connected undirected graph $\mathcal{G}$ with Laplacian L, it is known that agents achieve consensus when the control law

$$u_i = \sum_{j\in\mathcal{V}_i^{\text{in}}} (x_i - x_j), \tag{47}$$

with $\mathcal{V}_i$ the neighbors of agent i, is applied, see [21]. In vector notation, this is written as $u = -Lx$, where $u = (u_1, u_2, \ldots, u_N)$. We use the noisy sampled states for each agent instead of the actual states, resulting in the control law

$$u = -L(x + e + \widehat{w}). \tag{48}$$

Hence, the closed-loop system dynamics are

$$\dot{x} = -Lx - Le - L\widehat{w}, \tag{49}$$

which results in the dynamics for the hybrid system as

$$F_\chi(\chi) = (-Lx - Le - L\widehat{w}, Lx + Le + L\widehat{w}, \mathbf{0}_N). \quad (50)$$

We are interested in stability properties of the consensus set

$$\mathcal{A} := \left\{\chi \in \mathbb{R}^{2n} \times \mathcal{W} \mid x_1 = x_2 = \ldots = x_N\right\}. \quad (51)$$

We show that our results extend the works of [15] and [9], to render the ETC schemes robust to measurement noise.

1) Decentralized triggering of [15]: We consider a similar triggering style as in [15] first. This event generator is of particular interest, since the original paper does *not* have a non-Zeno proof, as also noted in [7]. By applying our results, we can design two robust triggers, one static and one dynamic, that have the SGiMIET property and thus no Zeno behavior.

The proposition below contains the functions required to design a trigger for (49) such that Assumption 2 holds.

Proposition 7. *Assumption 2 holds for F_χ defined as in* (50) *and G_χ as in* (20) *with $\beta_i(s) = \frac{1}{a}N_i s^2$ and $\delta_i(o_i) = \sigma_i(1-2aN_i)u_i^2$, where N_i denotes the number of neighbors of agent i and $a \in (0, \frac{1}{2N_i})$, $\sigma_i \in (0,1)$ are tuning parameters.*

Proposition 7 implies that, for any bounded measurement noise as defined by Assumption 1, the triggers defined in Theorem 4 and Corollary 5 render the hybrid system (10) ISpS w.r.t. $\mathcal{A}^d$ with the SGiMIET property.

Proof is omitted for space reasons.

2) Decentralized dynamic trigger of [9]: Next we analyze the trigger designed in [9] *without* transmission delays to avoid blurring the exposition with too many technicalities. For the scheme of [9] we require that each agent has an internal clock, $\tau_i \in \mathbb{R}_{\geqslant 0}$, such that $\dot{\tau}_i = 1$ on flows and $\tau_i^+ = 0$ at any triggering instant of agent i, i.e., we reset the clock if agent i transmits its state. We denote the hybrid system in which these clocks are integrated in $\mathcal{H}$ ((10)-(20)) with $\mathcal{H}_{\text{clock}}$.

The proposition below contains the functions required to design a trigger for system (49) such that Assumption 2 holds. Its proof is omitted for space reasons.

Proposition 8. *Assumption 2 holds for* (20) *and* (50) *with $\mathcal{A} = \left\{\chi \in \mathbb{R}^{2n} \times \mathcal{W} \mid x_i = x_j \text{ for all } i,j \in \mathcal{N}, e = \mathbf{0}\right\}$, $\beta_i(\widetilde{e}_i, \tau_i) = (1-\omega_i(\tau_i))\gamma_i^2\left(\frac{1}{\alpha_i\sigma_i}\lambda_i^2 + 1\right)\widetilde{e}_i^2$ and $\delta_i(o_i) = (1-\alpha_i)\sigma_i u_i^2$, where $\sigma_i := (1-\varrho)(1-2aN_i)$, $\gamma_i := \sqrt{\frac{1}{a}N_i + \mu_i}$, $d_i := \varrho(1-2aN_i)$,*

$$\omega_i(\tau_i) := \begin{cases} \{1\}, & \text{when } \tau_i \in [0, \tau_{\text{MIET}}^i), \\ [0,1], & \text{when } \tau_i = \tau_{\text{MIET}}^i, \\ \{0\}, & \text{when } \tau_i > \tau_{\text{MIET}}^i, \end{cases}$$

$$\tau_{\text{MIET}}^i := -\frac{\sqrt{\alpha_i\sigma_i}}{\gamma_i}\arctan\left(\frac{(\lambda_i^2-1)\sqrt{\alpha_i\sigma_i}}{\lambda_i(\alpha_i\sigma_i+1)}\right),$$

with $\alpha_i \in (0,1)$, $\varrho \in (0,1)$, $\mu_i \in \mathbb{R}_{>0}$ and $\lambda_i \in (0,1)$ tuning parameters.

Proposition 8 implies that, for any bounded measurement noise as defined by Assumption 1, the triggers defined in Theorem 4 and Corollary 5 render the hybrid system (10) ISpS w.r.t. $\mathcal{A}^d_{\text{clock}} := \{(\xi, \tau) : \xi \in \mathcal{A}^d \text{ and } \tau \in \mathbb{R}^N_{\geqslant 0}\}$. Let us note that, due to the inclusion of the timer-dependent function ω_i in the triggers, the system has a GiMIET (instead of a SGiMIET) in this particular case. Additionally, there is no requirement (i.e., no lower bound) on the space-regularization constants c_i, and, in fact, if $c_i = 0$ for all $i \in \mathcal{N}$, we obtain ISS w.r.t. $\mathcal{A}^d$ (instead of ISpS).

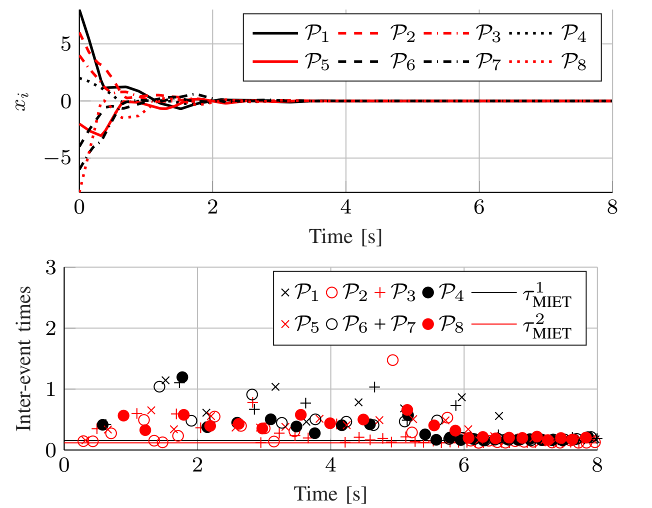

Fig. 2: Evolution of the states (top) and inter-event times (bottom) of the MAS using the dynamic trigger obtained by applying Theorem 4 to Proposition 8 with $c_i = 0$ and initial condition $x(0) = (8, 6, 4, 2, -2, -4, -8)$.

VI. Numerical Examples

In this section, we illustrate the results of Section V-B.2 with $N = 8$ agents that are connected as described by a graph $\mathcal{G}$ with undirected edges $(1,2)$, $(1,8)$, $(2,3)$, $(2,7)$, $(3,4)$, $(3,6)$, $(4,5)$, $(5,6)$, $(5,8)$ and $(7,8)$. Due to space limitations, we only show simulation results for the modified algorithm of [9]. The tuning parameters of [9] are used, i.e., $\delta = \mu_i = \epsilon_{\eta,i} = 0.05$, $a = 0.1$ and $\alpha_i = 0.5$ for all $i \in \mathcal{N}$. Given these tuning parameters, we obtain $\gamma_i = 4.478$ and $\sigma_i = 0.76$ for agents $i \in \mathcal{N}$ with two neighbors (i.e., $N_i = 2$, thus agents P_1, P_4, P_6 and P_7) and $\gamma_i = 5.482$ and $\sigma_i = 0.665$ for agents $i \in \mathcal{N}$ with three neighbors (i.e., $N_i = 3$, thus agents P_2, P_3, P_5 and P_8). We choose $\lambda_i = 0.2$ for all agents. For these values, we obtain $\tau^i_{\text{MIET}} = 0.1562$ for agents $i \in \mathcal{N}$ for which $N_i = 2$ and $\tau^i_{\text{MIET}} = 0.1180$ for agents $i \in \mathcal{N}$ for which $N_i = 3$.

We include uniformly distributed noise in the interval $[-1 \cdot 10^{-3}, 1 \cdot 10^{-3}]$ as measurement noise, hence, $\overline{w}_i = 1 \cdot 10^{-3}$ for all $i \in \mathcal{N}$. The noise is sampled at a rate of $1 \cdot 10^4 Hz$ and a zero-order-hold is applied between samples. We demonstrate the results of Theorem 4, i.e., we apply dynamic triggering. Two cases are simulated, first with no space-regularization for all $i \in \mathcal{N}$, for which we obtain ISS w.r.t. the consensus set, second with space-regularization constant $c_i = 1 \cdot 10^{-5}$ for all $i \in \mathcal{N}$, for which we have ISpS w.r.t. the consensus set. To compare the results to [9] (not considering measurement noise), in all cases we select $\theta_i = 0$. In Fig. 2, the evolution of the states x_i, $i \in \mathcal{N}$, with

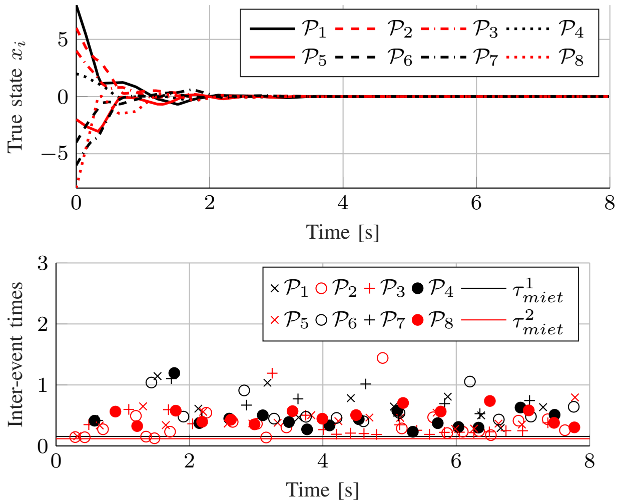

Fig. 3: Evolution of the states (top) and inter-event times (bottom) of the MAS using the dynamic trigger obtained by applying Theorem 4 to Proposition 8 with $c_i = 1 \cdot 10^{-5}$ and initial condition $x(0) = (8, 6, 4, 2, -2, -4, -8)$.

$c_i = 0$ and the corresponding inter-event times are shown for the initial condition $x(0) = (8, 6, 4, 2, -2, -4, -6, -8)$. Fig. 3 depicts the same simulations for $c_i = 1 \cdot 10^{-5}$.

From the simulations we can make a few observations. Note that, for $c_i = 0$, close to the consensus set the inter-event times are generally close to τ^i_{MIET}. This can be explained from the observation that, in these cases, $\eta_i^+ = 0$ and u_i is generally small, and consequently, the increase in η_i for $\tau \in [0, \tau^i_{\text{MIET}})$ is limited. Additionally, we observe that by selecting a $c_i > 0$, the inter-event times are generally significantly larger than the enforced minimum inter-event time. Moreover, because there is no lower-bound on c_i, a relatively small c_i is often sufficient to obtain desirable average inter-event times. We want to stress that this is a beneficial aspect of this particular scheme, since in general there are constraints on the minimum size of the space-regularization constants c_i to ensure non-Zenoness.

Even though the inclusion of c_i leads to ISpS instead of ISS properties, applying space-regularization leads to triggering conditions that are not only robust to measurement noise, but also have, on average, larger inter-event times. Since ISS only leads to asymptotic behavior of the consensus set for vanishing noise, and since most measurement noise is non-vanishing, practical stability or ISpS with larger inter-event times may be more desirable when having communication limitations in mind.

VII. Conclusions

We presented a general "prescriptive" framework for set stabilization of event-triggered control systems affected by measurement noise. It is shown that, by careful design, we obtain both *dynamic* and *static* triggering conditions that render the closed loop input-to-state (practically) stable with a guaranteed positive (semi-)global individual minimum inter-event time. The strengths and generality of the framework are demonstrated on several interesting event-triggered control problems including consensus problems for multi-agent systems robustifying them for measurement noise.

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