Decoupling-based reconfigurable control of linear systems after actuator faults

J. H. Richter*, S. Weiland**, W. P. M. H. Heemels***, and J. Lunze*

***Abstract*— In this paper, we provide the complete solution to the reconfigurable control problem after actuator faults in linear dynamical systems by means of disturbance decoupling approaches. The recovery of the closed-loop internal stability and the exact and approximate recovery of nominal tracking and performance are our main reconfiguration goals. We state necessary and sufficient conditions for the solvability of these problems. The approximate approach broadens the scope of potential applications. A thermofluid process is used to illustrate the exact and approximate methods.**

I. I

In this paper, we present a new fault-tolerant control approach that is based on the virtual actuator, which is designed using linear geometric control theory. Control reconfiguration can be viewed as an approach to create dependable systems by means of appropriate feedback control. Its task is to respond to severe component failures that break the control loop by restructuring the control loop on-line, such that stability and performance are recovered or gracefully degraded [1]. As such, control reconfiguration is an active fault-tolerant control methodology in the sense that it uses an estimate $\hat{f}$ of the fault f, which is obtained from a diagnosis component FDI (Fig. 1). As opposed to passive fault-tolerant control approaches [2], in reconfigurable control the controller is changed to match the faulty plant.

The relevance of reconfigurable control is emphasized by its strong appearance in the literature. Reconfigurable control is, for instance, based on linear model following approaches [3], on the input/output-behaviour [4], [5], on the generalised plant transfer function [6], and on the closed-loop eigenstructure [7]. Optimal control has also been used as a basis for control reconfiguration [8], [9]. An approach based on invariant sets is described in [10]. The use of unfalsified control for deciding on the best reconfigured controller from a list of candidate controllers is a relatively new area [11].

The reconfiguration method presented here is based on the idea of keeping the nominal controller in the loop by inserting a reconfiguration block between the faulty plant and the nominal controller at reconfiguration time (Fig. 2). The reconfiguration block must hide the fault from the controller and at the same time ensure that the faulty plant controlled by the nominal controller and the reconfiguration block meets the nominal performance requirements. This fault-hiding approach leads to the concept of virtual actuators in the actuator fault case [12]–[15].

We assume that the model parameters of the faulty system are provided on-line by a diagnosis component [16]. The assumption of having accurate fault isolation results is not restrictive for applications due to the advent of smart self-diagnosing actuators [17] that report their health status to the supervision level over modern fieldbus systems. If the precise fault model is uncertain, the policy followed in this paper consists in switching off faulty actuators to spare faulty components from further load. The solvability of the reconfiguration problems depends on the presence of analytical redundancy.

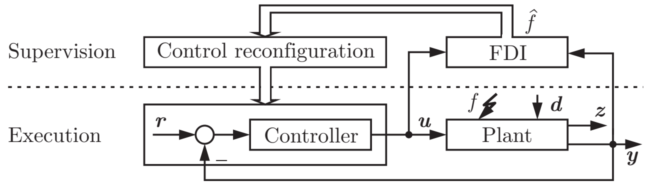

Fig. 1. Active fault-tolerant control scheme.

The novel contributions of this paper are the following. 1) We provide the complete solution to the problem of exactly recovering the closed-loop system trajectory along with necessary and sufficient solvability conditions (Theorem 1). The problem is equivalent to a disturbance decoupling problem with stability for measured disturbance (DDPS'). 2) We solve a relaxed problem that provides trajectory recovery in an approximate sense (Theorem 2). This problem is equivalent to an almost disturbance decoupling problem with stability for measured disturbance (ADDPS'). The latter improvement implies that the approach becomes applicable under less strict conditions. 3) A complex example from process control demonstrates the results.

Prior partial results deferred the solvability test for the crucial stabilization step to the end of the algorithm without giving a priori conditions [13]. Furthermore, the obtained solvability conditions are strict and often not satisfied in practice, necessitating less restrictive approaches. We completely remedy these serious drawbacks in this paper. The derived conditions are relevant also in the context of redundancy assessment, especially during the planning phase of a system prior to its construction. The conditions are useful whether or not virtual actuators are used in reconfiguration, and if they are used, alternative synthesis methods may be used, such as multi-objective optimisation achieving a tradeoff between performance recovery and input effort [18].

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This paper is organised as follows. In Section II, we define our notation. In Section III, we define system descriptions, faults, and reconfiguration problems after actuator faults. The reconfiguration problems for perfect or almost trajectory recovery are solved in Section IV. An example illustrates the feasibility of the approach in Section V. The reader unfamiliar with geometric systems theory might want to consult classical textbooks such as [19], [20], as well as the papers [21]–[23] and the references therein. Generic decoupling problems and their geometric solutions are given in Appendix A.

II. N

The following notation is used throughout. Lower case bold letters denote vectors ($\boldsymbol{x}$), capital bold letters denote matrices ($\boldsymbol{A}$), and script capitals denote vector spaces as well as linear subspaces ($\mathcal{V}$). Linear time-invariant (LTI) dynamical systems are denoted by Σ_P, where the subscript distinguishes different systems. Dynamical operators are denoted by Ω_P. The restriction of a system operator Ω_P with multiple outputs to a specific output $\boldsymbol{y}$ is denoted by $\Omega_P^{\boldsymbol{y}}$. The symbol $\triangleq$ means equal by definition. $\mathbb{R}$ and $\mathbb{C}$ denote the real and complex numbers, and $\mathbb{R}_+ \triangleq [0,\infty)$, $\mathbb{C}_- \triangleq \{s \in \mathbb{C} | \operatorname{Re}(s) < 0\}$. The eigenvalue set of a matrix $\boldsymbol{A}$ is denoted by $\sigma(\boldsymbol{A})$. We say that a controller internally stabilizes (stab.) a plant if the closed-loop eigenvalues lie in $\mathbb{C}_-$. If $\mathcal{X}$ is a vector space, $\mathcal{V} \subset \mathcal{X}$ is a linear subspace, and $\boldsymbol{A} : \mathcal{X} \to \mathcal{X}$ a mapping defined on $\mathcal{X}$, then $\boldsymbol{A}|\mathcal{V}$ denotes the restriction of $\boldsymbol{A}$ to $\mathcal{V}$, and $\mathcal{V}^{\perp}$ denotes the orthogonal complement of $\mathcal{V}$. Given any subspace $\mathcal{K}$, the supremal stabilisability subspace contained in $\mathcal{K}$ and the supremal almost stabilisability subspace contained in $\mathcal{K}$ are respectively denoted by $\mathcal{V}_g^*(\mathcal{K})$ and $\mathcal{V}_{b,g}^*(\mathcal{K})$ [22], [23]. For $1 \leq p \leq \infty$, and for a measurable signal $\boldsymbol{x} : \mathbb{R}_+ \to \mathbb{R}^n$, we say that $\boldsymbol{x} \in \mathcal{L}_p(\mathbb{R}_+, \mathbb{R}^n)$ if $\|\boldsymbol{x}\|_{\mathcal{L}_p} < \infty$, where $\|\boldsymbol{x}\|_{\mathcal{L}_p} \triangleq \left(\int_{\mathbb{R}_+} \|\boldsymbol{x}(t)\|^p \mathrm{d}t\right)^{1/p}$ for $1 \leq p < \infty$ and $\|\boldsymbol{x}\|_{L_\infty} \triangleq \sup_{t\in\mathbb{R}_+} \|\boldsymbol{x}(t)\|$. The space of locally integrable functions is denoted by $\mathcal{L}_1^{loc}$. The H_∞-norm of a transfer function $\boldsymbol{T}(s)$ is denoted by $\|\boldsymbol{T}(s)\|_{H_\infty}$.

III. P

A. Systems

The nominal system Σ_P is modeled in LTI state-space form

$$\Sigma_P : \begin{cases} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}_c(t) + \boldsymbol{B_d}\boldsymbol{d}(t),\ \boldsymbol{x}(0) = \boldsymbol{x}_0 \\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t),\ \boldsymbol{z}(t) = \boldsymbol{C_z}\boldsymbol{x}(t), \end{cases} \tag{1}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$ is the state, $\boldsymbol{u}_c(t) \in \mathbb{R}^m$ the input, $\boldsymbol{d}(t) \in \mathbb{R}^k$ an unknown disturbance, $\boldsymbol{y}(t) \in \mathbb{R}^r$ the measurement, $\boldsymbol{z}(t) \in \mathbb{R}^q$ the output relevant for control performance. $\boldsymbol{A}$ is the state transition matrix, $\boldsymbol{B}$ the input matrix, $\boldsymbol{B_d}$ the disturbance input matrix, $\boldsymbol{C}$ the measurement matrix, and $\boldsymbol{C_z}$ the output matrix. All matrices are of compatible dimensions. Associated with the system Σ_P is the operator $\Omega_P :\ \mathcal{L}_1^{loc}(\mathbb{R}^m) \times \mathcal{L}_1^{loc}(\mathbb{R}^k) \times \mathbb{R}^n \to \mathcal{L}_1^{loc}(\mathbb{R}^r) \times \mathcal{L}_1^{loc}(\mathbb{R}^q)$,

$$(\boldsymbol{y}, \boldsymbol{z}) = \Omega_P(\boldsymbol{u}_c, \boldsymbol{d}, \boldsymbol{x}_0). \tag{2}$$

The linear system (1) is controlled by means of an LTI nominal controller

$$\Sigma_C : \begin{cases} \dot{\boldsymbol{x}}_c(t) &= \boldsymbol{A}_c\boldsymbol{x}_c(t) + \boldsymbol{B}_c\boldsymbol{y}(t) + \boldsymbol{E}_c\boldsymbol{r}(t),\ \boldsymbol{x}_c(0) = \boldsymbol{x}_{c0} \\ \boldsymbol{u}_c(t) &= \boldsymbol{C}_c\boldsymbol{x}_c(t) + \boldsymbol{D}_c\boldsymbol{y}(t) + \boldsymbol{F}_c\boldsymbol{r}(t) \end{cases} \tag{3}$$

with the internal state $\boldsymbol{x}_c \in \mathbb{R}^{n_c}$ and the reference input $\boldsymbol{r} \in \mathbb{R}^r$. We associate with the controller Σ_C the operator $\Omega_C :\ \mathcal{L}_1^{loc}(\mathbb{R}^r) \times \mathcal{L}_1^{loc}(\mathbb{R}^r) \times \mathbb{R}^n \to \mathcal{L}_1^{loc}(\mathbb{R}^m)$,

$$\boldsymbol{u}_c = \Omega_C(\boldsymbol{r}, \boldsymbol{y}, \boldsymbol{x}_{c0}) \tag{4}$$

and assume that the closed-loop operator $\Omega_{CL} :\ \mathcal{L}_1^{loc}(\mathbb{R}^r) \times \mathcal{L}_1^{loc}(\mathbb{R}^k) \times \mathbb{R}^n \times \mathbb{R}^{n_c} \to \mathcal{L}_1^{loc}(\mathbb{R}^q)$,

$$\boldsymbol{z} = \Omega_{CL}(\boldsymbol{r}, \boldsymbol{d}, \boldsymbol{x}_0, \boldsymbol{x}_{c0}) \tag{5}$$

that describes the feedback connection (Σ_P, Σ_C) of the systems (1), (3) is internally stable and satisfies certain requirements regarding the transient and steady-state response from $(\boldsymbol{r}, \boldsymbol{d})$ to $\boldsymbol{z}$. The nominal closed-loop system is shown in Fig. 2a).

B. Actuator faults

Definition 1 (Actuator faults): An *actuator fault* f is an event that changes the nominal input matrix $\boldsymbol{B} \in \mathbb{R}^{(n\times m)}$ to the faulty input matrix $\boldsymbol{B}_f \in \mathbb{R}^{(n\times m)}$ of the same dimensions. In this paper, we assume that faults appear abruptly and remain effective once they have occurred. The pair $(\boldsymbol{A}, \boldsymbol{B}_f)$ is assumed to be stabilizable.

Actuator faults change the nominal system (1) to the faulty system

$$\Sigma_{Pf} : \begin{cases} \dot{\boldsymbol{x}}_f(t) &= \boldsymbol{A}\boldsymbol{x}_f(t) + \boldsymbol{B}_f\boldsymbol{u}_f(t) + \boldsymbol{B_d}\boldsymbol{d}(t),\ \boldsymbol{x}_f(0) = \boldsymbol{x}_0 \\ \boldsymbol{y}_f(t) &= \boldsymbol{C}\boldsymbol{x}_f(t),\ \boldsymbol{z}_f(t) = \boldsymbol{C_z}\boldsymbol{x}_f(t). \end{cases} \tag{6}$$

For example, columns of $\boldsymbol{B}_f$ that correspond to faulty or failed actuators are scaled or set to zero, respectively. Zero columns of the matrix $\boldsymbol{B}_f$ represent actuator blockage at the operating point. To distinguish the faulty system behaviour from the nominal behaviour, all affected signals are labeled by subscript f. As discussed in [24], blockage at arbitrary positions can be represented by additional constant disturbances: $\dot{\boldsymbol{x}}_f(t) = \boldsymbol{A}\boldsymbol{x}_f(t) + \boldsymbol{B}_f\boldsymbol{u}_f(t) + \boldsymbol{B_d}\boldsymbol{d}(t) + \boldsymbol{b}_j\boldsymbol{\delta}(t)$, where $\boldsymbol{b}_j$ is the column of $\boldsymbol{B}$ that corresponds to the blocked actuator, and $\boldsymbol{\delta}(t)$ is a constant that reflects the blocking position. We associate with the faulty plant the operator $\Omega_{Pf} :\ \mathcal{L}_1^{loc}(\mathbb{R}^m) \times \mathcal{L}_1^{loc}(\mathbb{R}^k) \times \mathbb{R}^n \to \mathcal{L}_1^{loc}(\mathbb{R}^r) \times \mathcal{L}_1^{loc}(\mathbb{R}^q)$,

$$(\boldsymbol{y}_f, \boldsymbol{z}_f) = \Omega_{Pf}(\boldsymbol{u}_f, \boldsymbol{d}, \boldsymbol{x}_0). \tag{7}$$

After the fault, the nominal controller (3) using the interconnections $\boldsymbol{y} = \boldsymbol{y}_f$ and $\boldsymbol{u}_c = \boldsymbol{u}_f$ is generally not suitable for controlling the faulty plant (6). In particular, in the case of actuator failures, the loop is partially open. In the next section we seek a reconfigured controller that stabilizes the faulty plant and recovers the nominal closed-loop performance properties.

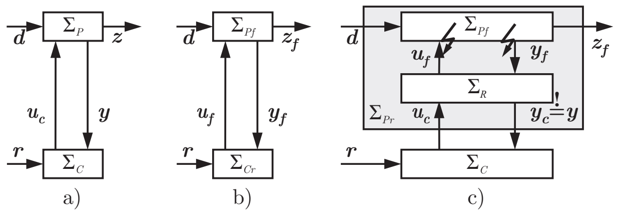

Fig. 2. a) Nominal closed-loop system, b) reconfigured closed-loop system with new controller, c) reconfigured closed-loop system for fault-hiding.

C. Reconfiguration approach

We use a reconfiguration approach where the nominal controller remains part of the reconfigured closed-loop system, which has the practical advantage that minimum-invasive control law adjustments can be formulated as a reconfiguration goal, if so desired. A reconfiguration block Σ_R is placed between the faulty plant (6) and the nominal controller (3) as shown in Fig. 2c). Together with the faulty plant (6), the reconfiguration block Σ_R forms the *reconfigured plant* $\Sigma_{Pr} = (\Sigma_{Pf}, \Sigma_R)$ described by the operator $\Omega_{Pr} : \mathcal{L}_1^{loc}(\mathbb{R}^m) \times \mathcal{L}_1^{loc}(\mathbb{R}^k) \times \mathbb{R}^n \times \mathbb{R}^n \to \mathcal{L}_1^{loc}(\mathbb{R}^r) \times \mathcal{L}_1^{loc}(\mathbb{R}^q)$

$$(\boldsymbol{y}_c, \boldsymbol{z}_f) = \Omega_{Pr}(\boldsymbol{u}_c, \boldsymbol{d}, \boldsymbol{x}_0, \zeta_0) \tag{8}$$

to which the nominal controller (3) is connected via the signal pair $(\boldsymbol{u}_c, \boldsymbol{y}_c)$ (Fig. 2c)). To enable keeping the nominal controller, the following property must be satisfied.

Definition 2 (Strict fault-hiding goal): Consider the nominal system (1) and the faulty system (6). The reconfigured plant Σ_{Pr} meets the strict fault-hiding goal, if there exists a suitable particular initial condition $\hat{\zeta}_0$ of the reconfiguration block Σ_R such that the following relation holds:

$$\Omega_{Pr}^{y_c}(\cdot,\cdot,\cdot,\hat{\zeta}_0) - \Omega_P^{y}(\cdot,\cdot,\cdot) = \boldsymbol{0}.$$

The class of all strictly fault-hiding reconfiguration blocks Σ_R for the nominal plant Σ_P faulty plant Σ_{Pf} is denoted by the set $O_{Rs}(\Sigma_P, \Sigma_{Pf})$.

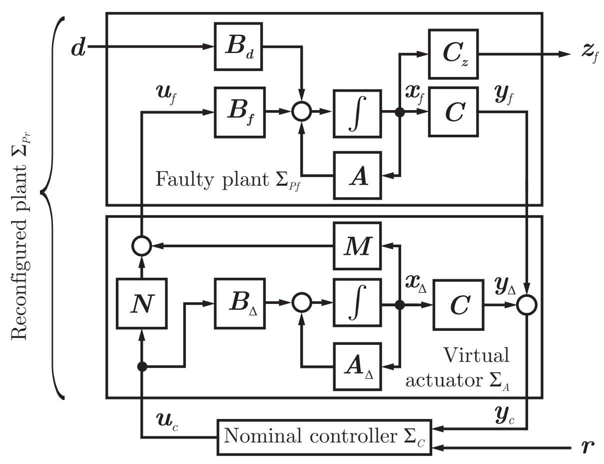

Fig. 3. Virtual actuator in reconfigured closed-loop system.

In this paper, the reconfiguration block Σ_R is a *virtual actuator*

$$\Sigma_A : \begin{cases} \dot{\boldsymbol{x}}_\Delta(t) &= \boldsymbol{A}_\Delta \boldsymbol{x}_\Delta(t) + \boldsymbol{B}_\Delta \boldsymbol{u}_c(t),\ \boldsymbol{x}_\Delta(0) = \boldsymbol{x}_{\Delta 0} \\ \boldsymbol{u}_f(t) &= \boldsymbol{M}\boldsymbol{x}_\Delta(t) + \boldsymbol{N}\boldsymbol{u}_c(t),\ \boldsymbol{y}_c(t) = \boldsymbol{y}_f(t) + \boldsymbol{C}\boldsymbol{x}_\Delta(t) \\ \boldsymbol{A}_\Delta &= \boldsymbol{A} - \boldsymbol{B}_f\boldsymbol{M},\ \boldsymbol{B}_\Delta \triangleq \boldsymbol{B} - \boldsymbol{B}_f\boldsymbol{N} \end{cases} \tag{9}$$

(Fig. 3, $\Sigma_R = \Sigma_A$). The virtual actuator Σ_A, introduced in [12], expresses the difference between nominal and reconfigured dynamics in its state $\boldsymbol{x}_\Delta$ and tries to keep this difference small. The matrices $\boldsymbol{M}$ and $\boldsymbol{N}$ are free design parameters that may be used to affect the virtual actuator behaviour. With Σ_A we associate the operator $\Omega_V : \mathcal{L}_1^{loc}(\mathbb{R}^m) \times \mathcal{L}_1^{loc}(\mathbb{R}^r) \times \mathbb{R}^n \to \mathcal{L}_1^{loc}(\mathbb{R}^m) \times \mathcal{L}_1^{loc}(\mathbb{R}^r)$:

$$(\boldsymbol{u}_f, \boldsymbol{y}_c) = \Omega_V(\boldsymbol{u}_c, \boldsymbol{y}_f, \zeta_0). \tag{10}$$

After application of the state transformation $\boldsymbol{x}_f(t) \to \tilde{\boldsymbol{x}}(t) \triangleq \boldsymbol{x}_f(t) + \boldsymbol{x}_\Delta(t)$, the reconfigured plant (6), (9) is described by

$$\begin{pmatrix} \dot{\tilde{\boldsymbol{x}}}(t) \\ \dot{\boldsymbol{x}}_\Delta(t) \end{pmatrix} = \begin{pmatrix} \boldsymbol{A} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{A}_\Delta \end{pmatrix} \begin{pmatrix} \tilde{\boldsymbol{x}}(t) \\ \boldsymbol{x}_\Delta(t) \end{pmatrix} + \begin{pmatrix} \boldsymbol{B} \\ \boldsymbol{B}_\Delta \end{pmatrix} \boldsymbol{u}_c(t) + \begin{pmatrix} \boldsymbol{B_d} \\ \boldsymbol{0} \end{pmatrix} \boldsymbol{d}(t) \tag{11a}$$

$$\tilde{\boldsymbol{x}}(0) = \boldsymbol{x}_0 + \boldsymbol{x}_{\Delta 0},\ \boldsymbol{x}_\Delta(0) = \boldsymbol{x}_{\Delta 0} \tag{11b}$$

$$\boldsymbol{y}_c(t) = \boldsymbol{C}\tilde{\boldsymbol{x}}(t),\ \boldsymbol{z}_f(t) = \boldsymbol{C_z}\tilde{\boldsymbol{x}}(t) - \boldsymbol{C_z}\boldsymbol{x}_\Delta(t). \tag{11c}$$

Obviously, $\boldsymbol{T}'_{\boldsymbol{u}_c\to\boldsymbol{y}_c}(s) = \boldsymbol{T}_{\boldsymbol{u}_c\to\boldsymbol{y}}(s)$, $\boldsymbol{T}'_{\boldsymbol{d}\to\boldsymbol{y}_c}(s) = \boldsymbol{T}_{\boldsymbol{d}_c\to\boldsymbol{y}}(s)$, and $\boldsymbol{T}'_{\boldsymbol{d}\to\boldsymbol{z}_f}(s) = \boldsymbol{T}_{\boldsymbol{d}\to\boldsymbol{z}}(s)$. Therefore, the strict fault-hiding goal is satisfied for $\boldsymbol{x}_{\Delta 0} = \boldsymbol{0}$ and the virtual actuator defines a candidate set $O_{Rs}(\Sigma_P, \Sigma_{Pf})$ of strictly fault-hiding reconfiguration blocks parameterised over the free matrices $\boldsymbol{M}, \boldsymbol{N}$. The reconfigured closed-loop system exhibits a separation principle, saying that the set of closed-loop eigenvalues is the union of the set of nominal closed-loop eigenvalues (stable by assumption) and the set $\sigma(\boldsymbol{A}_\Delta) \subset \mathbb{C}_g$, where the latter can be made to lie in the good part of the complex plane if and only if (iff) $(\boldsymbol{A}, \boldsymbol{B}_f)$ is stabilisable. We will not repeat proofs for these results, which may be found in [12]. Furthermore, the effect of actuators blocked in arbitrary positions can be compensated if the nominal closed-loop system rejects such constant disturbances [24].

D. Reconfiguration problems

The problem consists in recovering the nominal closed-loop tracking and performance properties, which are summarily denoted as "trajectory recovery".

Problem 1 (Trajectory recovery with stability): Consider the faulty system (6). Find values for the free parameters $\boldsymbol{M}, \boldsymbol{N}$ of the virtual actuator (9) such that the equality $\boldsymbol{T}'_{\boldsymbol{u}_c\to\boldsymbol{z}}(s) - \boldsymbol{T}_{\boldsymbol{u}_c\to\boldsymbol{z}_f}(s) = \boldsymbol{0}$ holds, and such that the unobservable part of Σ_{Pr} is stable.

Problem 2 (Almost trajectory recovery with stability): Consider the faulty system (6). Find values for the free parameters $\boldsymbol{M}, \boldsymbol{N}$ of the virtual actuator (9) such that for given $\varepsilon > 0$, the inequality $\|\boldsymbol{T}'_{\boldsymbol{u}_c\to\boldsymbol{z}_f}(s) - \boldsymbol{T}_{\boldsymbol{u}_c\to\boldsymbol{z}}(s)\|_{H_\infty} \leq \varepsilon$ holds, and such that the unobservable part of Σ_{Pr} is stable.

Problem 1 is equivalent to a disturbance decoupling problem with stabilization for known disturbance (denoted by the acronym DDPS'). Problem 2 is likewise equivalent to an almost decoupling problem with stability (ADDPS'). For these decoupling problems, necessary and sufficient solvability conditions are known, see Appendix A.

It remains to solve the Problems 1 and 2 based on geometric conditions available for solving DDPS' and ADDPS', which is shown in the next section.

IV. P

Consider Problem 1 for the faulty plant (6) and the virtual actuator (9). It is the same problem as considered in [13], which we now completely solve. Problem 1 leaves the relation $\boldsymbol{T}'_{\boldsymbol{u}_c\to\boldsymbol{z}_f}(s) = \boldsymbol{T}_{\boldsymbol{u}_c\to\boldsymbol{z}}(s)$ as our remaining problem. The latter problem is equivalent to finding values for $\boldsymbol{M}$ and $\boldsymbol{N}$ such that the output of the system

$$\begin{aligned}\dot{\boldsymbol{x}}_\Delta(t) &= (\boldsymbol{A} - \boldsymbol{B}_f\boldsymbol{M})\boldsymbol{x}_\Delta(t) + (\boldsymbol{B} - \boldsymbol{B}_f\boldsymbol{N})\boldsymbol{u}_c(t)\\ \boldsymbol{z}_\Delta(t) &\triangleq \boldsymbol{C_z}\boldsymbol{x}_\Delta(t)\end{aligned} \tag{12}$$

vanishes for arbitrary inputs $\boldsymbol{u}_c(t)$ (consider Equations (1) and (11) with $\boldsymbol{x}_\Delta(0) = \boldsymbol{0}$ to see this).

Theorem 1 (Exact trajectory recovery with stability): Consider the faulty system (6). Problem 1 is solvable iff there exists a stabilizability subspace $\mathcal{V}_g^*(\ker\boldsymbol{C_z}) \subseteq \mathbb{R}^n$ that satisfies

$$\operatorname{im}\boldsymbol{B} \subseteq \mathcal{V}_g^*(\ker\boldsymbol{C_z}) + \operatorname{im}\boldsymbol{B}_f, \tag{13}$$

where the virtual actuator initial state must satisfy $\boldsymbol{x}_\Delta(0) \bmod \mathcal{V}_g^*(\ker\boldsymbol{C_z}) = \boldsymbol{0}$.

Proof: Consider the system (11) and the output (12). The exact trajectory recovery problem is easily identified with DDPS' (whose solution is given in Appendix A) by change of variables $\boldsymbol{B} \to \boldsymbol{B_d}$ and $\boldsymbol{B}_f \to \boldsymbol{B}$, proving that the conditions (13) are necessary and sufficient for solvability of Problem 1.

To see that Condition (13) alone is necessary and sufficient for stable trajectory recovery, apply the state transformation $\boldsymbol{x}_\Delta = \boldsymbol{T}\boldsymbol{x}'_\Delta$, where $\boldsymbol{T} = (\boldsymbol{V}\ \ \boldsymbol{V}_o)$, $\operatorname{im}\boldsymbol{V} = \mathcal{V}_g^*(\ker\boldsymbol{C_z})$, and $\operatorname{im}\boldsymbol{V}_o = (\mathcal{V}_g^*(\ker\boldsymbol{C_z}))^\perp$. The transformation splits the difference state $\boldsymbol{x}_\Delta$ into an invariant part $\boldsymbol{x}'_{\Delta,1}$ and an orthogonal part $\boldsymbol{x}'_{\Delta,2}$ as follows:

$$\begin{pmatrix}\dot{\boldsymbol{x}}'_{\Delta,1}\\ \dot{\boldsymbol{x}}'_{\Delta,2}\end{pmatrix} = \begin{pmatrix}\boldsymbol{A}'_{11} - \boldsymbol{B}'_{f,1}\boldsymbol{M}'_1 & \boldsymbol{A}'_{12} - \boldsymbol{B}'_{f,1}\boldsymbol{M}'_2\\ \boldsymbol{A}'_{21} - \boldsymbol{B}'_{f,2}\boldsymbol{M}'_1 & \boldsymbol{A}'_{22} - \boldsymbol{B}'_{f,2}\boldsymbol{M}'_2\end{pmatrix}\begin{pmatrix}\boldsymbol{x}'_{\Delta,1}\\ \boldsymbol{x}'_{\Delta,2}\end{pmatrix} + \begin{pmatrix}\boldsymbol{B}'_1 - \boldsymbol{B}'_{f,1}\boldsymbol{N}'_1\\ \boldsymbol{B}'_2 - \boldsymbol{B}'_{f,2}\boldsymbol{N}'_2\end{pmatrix}\boldsymbol{u}_c.$$

The decoupling is achieved by setting $\boldsymbol{A}'_{21} - \boldsymbol{B}'_{f,2}\boldsymbol{M}'_1 = \boldsymbol{0}$ by suitable choice of $\boldsymbol{M}'_1$, which is possible because $\mathcal{V}_g^*(\ker\boldsymbol{C_z})$ is controlled invariant. As outlined in Section IV-D of [13], the remaining freedom in $\boldsymbol{M}'_1$ may be used to stabilize the internal dynamics of $\mathcal{V}_g^*$. This step always succeeds because $\mathcal{V}_g^*$ is a stabilizability subspace (by definition). Furthermore, Condition (13) ensures that $\boldsymbol{N}'_2$ exists such that $\boldsymbol{B}'_2 - \boldsymbol{B}'_{f,2}\boldsymbol{N}'_2 = \boldsymbol{0}$ holds. Consequently, the external dynamics governed by the system matrix $\boldsymbol{A}'_{22} - \boldsymbol{B}'_{f,2}\boldsymbol{M}'_2$ become autonomous. If $\boldsymbol{x}'_{\Delta,2}(0) = \boldsymbol{0}$, then $\boldsymbol{x}_{\Delta,2}$ is always zero. Hence, the existence of a stabilizability subspace satisfying Condition (13) is a necessary and sufficient condition as claimed in the theorem. If, on the other hand, general initial conditions are admitted, then the additional condition $\{(\boldsymbol{A}, \boldsymbol{B}_f)$ stabilizable$\}$ is necessary and sufficient for achieving external stabilization with relative to $\mathcal{V}_g^*(\ker\boldsymbol{C_z})$ (it does not impose any further restrictions on the internal dynamics). ■

Note that the admissible initializations $\boldsymbol{x}_\Delta(0)$ are refined in the theorem with respect to the first guess $\boldsymbol{x}_\Delta(0) = \boldsymbol{0}$. To conclude stability for arbitrary initial states, the additional condition $(\boldsymbol{A}, \boldsymbol{B}_f)$ stabilisable would be required.

Theorem 2 (Almost trajectory recovery with stability): Consider the faulty system (6). Problem 2 is solvable iff there exists an almost stabilizability subspace $\mathcal{V}_{b,g}^*(\ker\boldsymbol{C_z}) \subseteq \mathbb{R}^n$ that satisfies

$$\{\operatorname{im}\boldsymbol{B} \subseteq \mathcal{V}_{b,g}^*(\ker\boldsymbol{C_z}) + \operatorname{im}\boldsymbol{B}_f\} \wedge \{(\boldsymbol{A}, \boldsymbol{B}_f)\ \text{stab.}\}. \tag{14}$$

Proof: The reconfiguration structure is the same as in the strict case, and the reconfiguration problem is translated into a decoupling problem in the same way. The standard conditions for ADDPS apply. However, unlike in the exact stable trajectory recovery case stated in Theorem 1, the additional stabilizability condition is always necessary, since the difference dynamics are not confined to $\mathcal{V}_{b,g}^*(\ker\boldsymbol{C_z})$ even if so initialised, and the external dynamics relative to $\mathcal{V}_{b,g}^*(\ker\boldsymbol{C_z})$ must be stabilised. ■

Note that the external dynamics with respect to $\mathcal{V}_{b,g}^*(\ker\boldsymbol{C_z})$ always need to be stabilized in this case (see also the proof). Since $\mathcal{V}_{b,g}^* \supseteq \mathcal{V}_g^*$ is always true, the conditions for solving the almost trajectory recovery problem 2 are less strict than the conditions for solving the strict trajectory recovery problem 1, as expected on the basis of the problem definitions. On the other hand, complete stabilizability of the system is a necessary condition for the solvability of almost trajectory recovery, whereas the weaker requirement of internal stabilizability with respect to $\mathcal{V}_g^*(\ker\boldsymbol{C_z})$ holds in the strict case. Finally, almost trajectory-recovering solutions result in high-gain control [22].

Remark 1: If the stability requirement is dropped, necessary and sufficient conditions for trajectory recovery are obtained by the substitutions $\mathcal{V}_g^*(\ker\boldsymbol{C_z}) \to \mathcal{V}^*(\ker\boldsymbol{C_z})$ and $\mathcal{V}_{b,g}^*(\ker\boldsymbol{C_z}) \to \mathcal{V}_b^*(\ker\boldsymbol{C_z})$ in Theorem 1 (recovering the results of [13]) and Theorem 2.

The refinement of Condition (13) over [13] is significant, as the following numerical example shows. The system Σ_P with the parameters $\boldsymbol{A} = [1\ 2; 1\ 3]$, $\boldsymbol{B} = [0.5\ 1; 1\ 0]$, $\boldsymbol{C_z} = [0\ 1]$ and the actuator fault modelled by $\boldsymbol{B}_f = [0.5\ 0; 1\ 0]$ satisfies the conditions $(\boldsymbol{A}, \boldsymbol{B}_f)$ stabilizable and $\operatorname{im}\boldsymbol{B} \subseteq \mathcal{V}^*(\ker\boldsymbol{C_z}) + \operatorname{im}\boldsymbol{B}_f$ given in [13], but not the condition (13). This means that either a stabilizing reconfiguraton solution or a exactly trajectory-recovering solution can be found, but not both at the same time.

The condition (13) means that for each controlled output of the vector $\boldsymbol{z}$, a path from a suitable input to that output must exist in the signal graph of the faulty plant, where all such paths must be independent from each other, and their relative degrees must be no larger than in the nominal case [12]. This interpretation of the geometric conditions emphasizes the necessity of sufficient redundancy. If Condition (13) is not satisfied in reconfigurability analysis, then exact tracking and performance recovery is not achievable. The plant must be fitted with additional redundant components if one insists on the tracking and performance recoverability after the corresponding fault case. Otherwise, performance loss after the actuator fault is inevitable.

V. R

As an example, we use the thermofluid process of the previous case study [24]. The process involves the regulation of fluid level l_{TS}, temperature ϑ_{TS}, and salt concentration ν_{TS} in a reactor container hosting a continuous flow process as shown in Fig. 4. The nonlinear process model is based on the following definitions of the state $\boldsymbol{x} \in \mathbb{R}^7$, the input $\boldsymbol{u} \in \mathbb{R}^6$ and the output $\boldsymbol{y} \in \mathbb{R}^4$: $\boldsymbol{x} = (\vartheta_{TS}, l_{TS}, c_{TS}, \vartheta_{TB}, x_{CW}, x_{el,TB}, x_{el,TS})^T$, $\boldsymbol{u} = (u_{TM}, u_{TB}, u_{el,TB}, u_{el,TS}, u_{CW}, u_{PS})^T$, $\boldsymbol{y} = (\vartheta_{TS}, l_{TS}, \nu_{TS}, \vartheta_{TB})^T$. Instead of the electrical conductivity ν_{TS}, the salt concentration c_{TS} is used as a state variable, which is linked to the conductivity via a static nonlinearity. Further state variables x_{CW}, $x_{el,TB}$ and $x_{el,TS}$ represent the dynamics of actuation systems. Two disturbances are considered: perturbations of the fluid level l_{TS} due to irregularities in the cold water supply, as well as concentration perturbations due to variations in the salinity c_{CW} of the inflowing cold water. The nonlinear model derived in [24] yields the linearised model

$$-\frac{\boldsymbol{A}}{10^{-3}} = \begin{pmatrix} 3.46 & 0 & 0 & -1.47 & 42.23 & 0 & -37.36 \\ 0 & 0.76 & 0 & 0 & -1.41 & 0 & 0 \\ 0 & 0 & 3.16 & 0 & 0.0034 & 0 & 0 \\ 0 & 0 & 0 & 1.34 & 0 & -152.23 & 0 \\ 0 & 0 & 0 & 0 & 270.27 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 37.04 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 15.38 \end{pmatrix} \tag{15}$$

$$\frac{\boldsymbol{B}}{10^{-3}} = \begin{pmatrix} -10.62 & 0 & 0 & 0 & 0 & 0 \\ 7.11 & 8.5 & 0 & 0 & 0 & -1.98 \\ 0.0249 & 0.0235 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 270.27 & 0 \\ 0 & 0 & 37.04 & 0 & 0 & 0 \\ 0 & 0 & 0 & 15.38 & 0 & 0 \end{pmatrix}$$

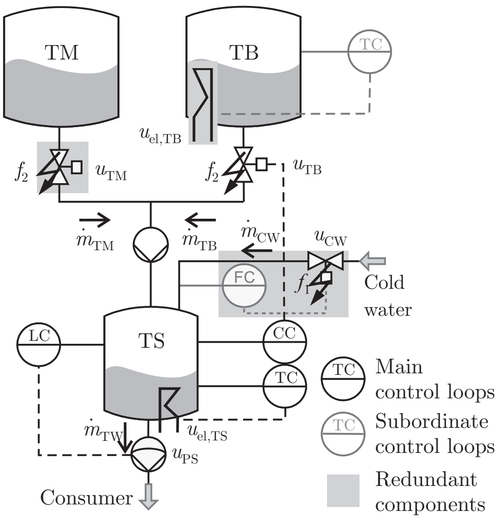

Fig. 4. Thermofluid process.

$$\boldsymbol{C} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2047.7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}, \ \boldsymbol{C_z} = \begin{pmatrix} \boldsymbol{c}_1 \\ \boldsymbol{c}_2 \\ \boldsymbol{c}_3 \end{pmatrix}.$$

The system matrix has the stable eigenvalues shown on the diagonal of matrix $\boldsymbol{A}$ given in Equation (15). It is assumed that a nominal MIMO controller potentially uses all inputs for the regulation task. We ask whether for two actuator fault scenarios a virtual actuator may be found such that the nominal control specifications are met (Problem 1). Of course, stabilization of hidden modes is crucial; the target eigenvalues $\sigma_t = 25\sigma(\boldsymbol{A})$ are used.

A failure of the cold water control input is modeled as

$$f_1 : u_5(t > t_f) = 0, \ \boldsymbol{B}_{f1} = (\boldsymbol{b}_1 \ \boldsymbol{b}_2 \ \boldsymbol{b}_3 \ \boldsymbol{b}_4 \ \boldsymbol{0} \ \boldsymbol{b}_6).$$

The supremal stabilizability subspace is $\mathcal{V}^*_{g1}(\ker \boldsymbol{C_z}) = \mathrm{span}(\boldsymbol{e}_4 \ \boldsymbol{e}_5 \ \boldsymbol{e}_6 \ \boldsymbol{e}_7)$ and it is easily verified that Condition (13) is satisfied. We allow initialization of the virtual actuator at the origin ($\boldsymbol{x}_{\Delta 0} = \boldsymbol{0}$), thus external stabilization with respect to $\mathcal{V}^*_{g1}$ is not needed. A feedback gain that renders $\mathcal{V}^*_{g1}(\ker \boldsymbol{C_z})$ invariant is computed as

$$\boldsymbol{M}_1 = \begin{pmatrix} -8 & 0 & 0 & 0 & 4 & 0 & -4 \\ 8 & 0 & 3227 & 0 & -4 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 7 & -9 & 13854 & 0 & -5 & 0 & 3 \end{pmatrix}. \tag{16}$$

The corresponding feedforward gain that injects the control input into $\mathcal{V}^*_{g1}(\ker \boldsymbol{C_z})$ is obtained as $\boldsymbol{N}_1 = (\boldsymbol{0} \ \ \boldsymbol{0} \ \ \boldsymbol{e}_3 \ \ \boldsymbol{e}_4 \ \ \boldsymbol{0} \ \ \boldsymbol{0})$. It is easily verified that $(\boldsymbol{A} - \boldsymbol{B}_{f1}\boldsymbol{M}_1)\mathcal{V}^*_{g1}(\ker \boldsymbol{C_z}) \subseteq \mathcal{V}^*_{g1}(\ker \boldsymbol{C_z})$ and $\mathrm{im}(\boldsymbol{B} - \boldsymbol{B}_{f1}\boldsymbol{N}_1) \subseteq \mathcal{V}^*_{g1}(\ker \boldsymbol{C_z})$, and $\sigma(\boldsymbol{A} - \boldsymbol{B}_{f1}\boldsymbol{M}_1) \subset \mathbb{C}_-$.

A failure of both control valves (blockage at their operating point) is modeled as

$$f_2 : u_1(t > t_f) = u_2(t > t_f) = 0, \ \boldsymbol{B}_{f2} = (\boldsymbol{0} \ \boldsymbol{0} \ \boldsymbol{b}_3 \ \boldsymbol{b}_4 \ \boldsymbol{b}_5 \ \boldsymbol{b}_6).$$

Unfortunately, Condition (13) is not satisfied for f_2. This fact has a clear physical interpretation. Both control valves influence the salt concentration without delay. The only other possibility of influencing the salt concentration consists in changing the amount of inflowing cold water. This influence passes through a subordinate flow control loop, which has non-negligible first order dynamics. Hence, an added delay in the response of concentration control is unavoidable.

However, the almost stabilizability subspace $\mathcal{V}^*_{b,g2}(\ker \boldsymbol{C_z}) = \mathbb{R}^7$ satisfies Condition (14). A feedback gain that renders $(\boldsymbol{A} - \boldsymbol{B}_{f,2}\boldsymbol{M}_2)$ $\mathcal{V}^*_{b,g2}(\ker \boldsymbol{C_z})$-invariant is

$$\boldsymbol{M}_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -50 & 0 & 74650 & 50 & 50 & 100 & -60 \\ 120 & 0 & -229630 & -80 & -130 & -150 & 150 \\ -10 & 0 & 8460 & 0 & 10 & 10 & -10 \\ 0 & -190 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Since $\mathcal{V}^*_{b,g2}$ covers the entire state space, the feedforward gain may be set to zero: $\boldsymbol{N}_2 = \boldsymbol{0}$. This high-gain solution does not exactly recover the nominal trajectory, but approximates it. The farther away the target eigenvalues are chosen from the imaginary axis, the more accurate the approximate recovery becomes, at the cost of high actuation energy. In practice, high-gain reconfiguration is useful only for plants with strongly dimensioned actuators, since in connection with actuator saturation, high-gain control easily creates an unstable closed-loop system. From this point of view, a passivity-based design of $\boldsymbol{M}$ is more logical [15].

VI. C

This paper completely solves the reconfigurable control problem for exact and approximate tracking and performancer recovery after actuator faults in linear systems based on geometric control theory. Necessary and sufficient conditions for the existence of stabilising solutions to the exact trajectory recovery problem are stated, and the approach is generalized towards approximate trajectory recovery schemes which result in high-gain solutions. The conditions are valuable independent of control for redundancy analysis purposes. As demonstrated by the example, the approximate version requires weaker conditions to be satisfied, hence extending the range of potential applications for the decoupling approach. It is of interest to extend the current framework to sensor faults, also in combination with actuator faults.

VII. A

The first author thanks the German Academic Exchange Service for support through a "DAAD Doktorandenstipendium".

A

A. Decoupling problems and solutions

Consider the system (1) and the following disturbance decoupling problem (DDP). Find a state-feedback controller $\boldsymbol{u} = -\boldsymbol{Kx}$ such that the transfer function $\boldsymbol{T}_{\boldsymbol{d}\to\boldsymbol{z}}(s) : \boldsymbol{d} \mapsto \boldsymbol{z}$, $\boldsymbol{T}_{\boldsymbol{d}\to\boldsymbol{z}}(s) = \boldsymbol{C}_{\boldsymbol{z}}(s\boldsymbol{I} - \boldsymbol{A} + \boldsymbol{BK})^{-1}\boldsymbol{B}_{\boldsymbol{d}}$ is zero: $\boldsymbol{T}_{\boldsymbol{d}\to\boldsymbol{z}}(s) \triangleq \boldsymbol{0}$. DDP is called solvable if such a gain $\boldsymbol{K}$ exists. DDP is solvable iff the supremal $(\boldsymbol{A}, \boldsymbol{B})$-invariant subspace $\mathcal{V}^*(\ker \boldsymbol{C}_{\boldsymbol{z}})$ satisfies $\operatorname{im} \boldsymbol{B}_{\boldsymbol{d}} \subseteq \mathcal{V}^*(\ker \boldsymbol{C}_{\boldsymbol{z}})$. The known disturbance decoupling problem (DDP') admits disturbance feedthrough $\boldsymbol{u} = -(\boldsymbol{Kx} + \boldsymbol{Nd})$ and concerns the transfer function $\boldsymbol{T}_{c,k}(s) \triangleq \boldsymbol{C}_{\boldsymbol{z}}(s\boldsymbol{I} - \boldsymbol{A} + \boldsymbol{BK})^{-1}(\boldsymbol{B}_{\boldsymbol{d}} - \boldsymbol{BN})$. It is solvable iff $\operatorname{im} \boldsymbol{B}_{\boldsymbol{d}} \subseteq \mathcal{V}^*(\ker \boldsymbol{C}_{\boldsymbol{z}}) + \operatorname{im} \boldsymbol{B}$. DDP with stability (DDPS, with $\sigma(\boldsymbol{A} - \boldsymbol{BK}) \subset \mathbb{C}_g$) is solvable iff the supremal stabilizability subspace $\mathcal{V}_g^*(\ker \boldsymbol{C}_{\boldsymbol{z}})$ satisfies $\operatorname{im} \boldsymbol{B}_{\boldsymbol{d}} \subseteq \mathcal{V}_g^*(\ker \boldsymbol{C}_{\boldsymbol{z}})$. DDP' with stability (DDPS') is solvable iff $\operatorname{im} \boldsymbol{B}_{\boldsymbol{d}} \subseteq \mathcal{V}_g^*(\ker \boldsymbol{C}_{\boldsymbol{z}}) + \operatorname{im} \boldsymbol{B}$. Almost disturbance decoupling (ADDP) in the sense that $\forall \varepsilon\ \exists \boldsymbol{K} : \|\boldsymbol{T}_{\boldsymbol{d}\to\boldsymbol{z}}(s)\|_{H_\infty} \leq \varepsilon$ is solvable iff $\operatorname{im} \boldsymbol{B}_{\boldsymbol{d}} \subseteq \mathcal{V}_b^*(\ker \boldsymbol{C}_{\boldsymbol{z}})$, and the disturbance throughput variation ADDP' concerning $\boldsymbol{T}_{c,k}(s)$ is solvable iff the supremal almost $(\boldsymbol{A}, \boldsymbol{B})$-invariant subspace $\mathcal{V}_b^*(\ker \boldsymbol{C}_{\boldsymbol{z}})$ satisfies $\operatorname{im} \boldsymbol{B}_{\boldsymbol{d}} \subseteq \mathcal{V}_b^*(\ker \boldsymbol{C}_{\boldsymbol{z}}) + \operatorname{im} \boldsymbol{B}$. ADDP with stability ($\sigma(\boldsymbol{A} - \boldsymbol{BK}) \subset \mathbb{C}_g$, ADDPS) is solvable iff $\operatorname{im} \boldsymbol{B}_{\boldsymbol{d}} \subseteq \mathcal{V}_{b,g}^*(\ker \boldsymbol{C}_{\boldsymbol{z}})$. Accordingly, ADDPS' is solvable iff $\operatorname{im} \boldsymbol{B}_{\boldsymbol{d}} \subseteq \mathcal{V}_{b,g}^*(\ker \boldsymbol{C}_{\boldsymbol{z}}) + \operatorname{im} \boldsymbol{B}$.

R

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