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Reconfigurable control of piecewise affine systems with actuator and sensor faults: Stability and tracking ☆

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ABSTRACT

A reconfigurable control approach for continuous-time piecewise affine (PWA) systems subject to actuator and sensor faults is presented. The approach extends the concept of virtual actuators and virtual sensors from linear to PWA systems on the basis of the fault-hiding principle that provides the underlying conceptual idea: the fault is hidden from the nominal controller and the fault effects are compensated. Sufficient linear matrix inequality (LMI) conditions for the existence of virtual actuators and virtual sensors are given that guarantee the recovery of closed-loop stability and the tracking of constant reference inputs. Since LMIs are efficiently solvable, this solution leads to a tractable computational algorithm that solves the reconfiguration problem. The approach is proven to be robust against model uncertainties and inaccurate fault diagnosis, and is evaluated using an example system of interconnected tanks.

1. Introduction

In this paper, a reconfigurable control strategy for continuous-time piecewise affine systems is presented. Reconfigurable control is an important technology for building truly autonomous dependable systems. Reconfigurable control is designed to respond to severe component faults (such as failures) that would otherwise break the control loop by restructuring the controller on line (Blanke, Kinnaert, Lunze, & Staroswiecki, 2006). Fig. 1 shows the role of control reconfiguration in an active fault-tolerant control context. The reconfiguration component obtains an estimate $\hat{f}$ of the fault f from a diagnosis component (fault detection and isolation, FDI) and changes the controller to match the faulty plant once the fault has been isolated. Numerous reconfigurable control methods for linear systems and restricted classes of nonlinear systems have been developed, of which a short review will be given in Section 2.

The method presented here is based on the idea of keeping the nominal controller in the loop by inserting a reconfiguration block between the faulty plant and the nominal controller after a fault has occurred. The reconfiguration block is chosen to "hide" the fault from the controller and at the same time to ensure that the faulty plant controlled by the nominal controller together with the reconfiguration block remains globally input-to-state stable with respect to reference inputs and disturbances, and recovers the nominal closed-loop tracking properties. The fault-hiding approach has previously been developed for linear and Hammerstein systems and leads to the use of virtual actuators for the actuator fault case and to virtual sensors for the sensor fault case (Quevedo, Puig, & Serra, 2007; Richter & Lunze, 2010; Richter, Schlage, & Lunze, 2007). Until now, the fault-hiding approach was not available for PWA systems.

Our motivation for studying PWA systems is at least twofold. Firstly, PWA systems are receiving wide attention due to the fact that the PWA framework (Sontag, 1981) provides a way to describe dynamic systems exhibiting switching between a multitude of linear dynamic regimes, see also Carmona, Freire, Ponce, and Torres (2002), Heemels, de Schutter, and Bemporad (2001) and Johansson (2003). The switching can be due to piecewise-linear characteristics such as dead-zone, saturation, hysteresis or relays. Secondly, PWA systems may result from piecewise linear approximations of complex nonlinear dynamics (Johansson, 2003). It has been recognised that many standard control-related analysis and synthesis problems for PWA systems are hard, in fact many of them are undecidable in the general case (Blondel & Tsitsiklis, 1999, 2000). Therefore, special subclasses of PWA systems are

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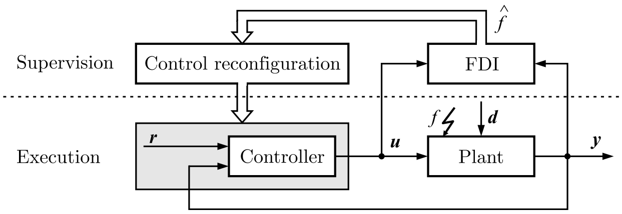

Fig. 1. Active fault-tolerant control scheme.

frequently considered in the literature (Camlibel, Heemels, & Schumacher, 2008; Juloski, Heemels, & Weiland, 2007; Lunze & Lamnabhi-Lagarrigue, 2009; Pavlov, Pogromsky, van de Wouw, & Nijmeijer, 2007). This is also true for this paper, where we address continuous PWA systems with constant input matrix. The undecidability of many problems for arbitrary PWA systems motivates the choice for continuous PWA systems. In particular, the control reconfiguration problem that we aim to solve in this paper leads to incremental stabilisation problems, which are unsolved for PWA systems with non-constant input matrix, see Pavlov et al. (2007) for a discussion and a counterexample. For the continuous subclass of PWA systems, we will provide a constructive and robust way of finding reconfigured controllers after actuator and sensor faults. No methods for reconfigurable control of multimodal PWA systems have been reported to our best knowledge. Furthermore, several practically important systems are well modelled with constant input matrix, such as mechanical systems with one-sided restoring characteristics (Bonsel, Fey, & Nijmeijer, 2004; van de Wouw, Pavlov, & Nijmeijer, 2006), mechanical motion systems with friction, (van de Wouw & Pavlov, 2008), a controlled pendulum, etc.

In this paper, we extend the fault-hiding approach to reconfigurable control from linear to PWA systems and solve the problem of recovering the nominal closed-loop stability and output tracking properties for the faulty system. It is assumed here that the fault isolation task has been solved and that the model of the faulty system is available to the reconfiguration component.[1] The contributions of this paper are as follows:

1. we design a PWA reconfiguration block satisfying the fault-hiding principle for PWA systems after actuator and sensor faults;
2. we describe a systematic computationally tractable approach to finding stabilising gains in the reconfiguration block that guarantee the recovery of the nominal tracking properties for constant reference inputs and constant disturbances;
3. we prove that the approach is robust against model uncertainties and against time-varying disturbances; and
4. we apply the approach to an example.

The presented computational approach relies on sufficient stability conditions formulated as linear matrix inequalities. The main results are a characterisation of a reconfigured closed-loop system that recovers the fault-free closed-loop stability property and the setpoint tracking properties in the presence of constant disturbances (Theorem 4), robustness properties (Theorem 5), and a corresponding synthesis algorithm (Algorithm 1).

This paper is organised as follows. The literature on reconfigurable control is discussed in Section 2. Notations, PWA systems and preliminary stability concepts are briefly introduced in Section 3. Actuator and sensor faults as well as related reconfiguration problems concerning both stability and tracking are defined in Section 4. The solution to the reconfiguration problem with stability and tracking recovery for constant inputs and constant disturbances is described in Section 5. Robustness against model approximation errors and time-varying disturbances is shown in Section 6. An example illustrates the applicability of the method in Section 7. The paper concludes in Section 8. Technical proofs are collected in Appendix.

2. Literature discussion

In this section, we discuss recent developments in fault-tolerant control. A detailed tutorial can be found in Lunze and Richter (2008) and comprehensive literature surveys are available in Zhang and Jiang (2003, 2006). Passive fault-tolerant control works with a fixed fault-tolerant controller that provides robust properties such as stability or performance for every expected fault. A passive fault-tolerant control approach for linear systems based on simultaneous stabilisation is reported in Stoustrup and Blondel (2004). For nonlinear systems, an approach that does not explicitly distinguish diagnosis and reconfiguration was presented in Bonivento, Isidori, Marconi, and Paoli (2004) and extended in Benosman and Lum (2008), which is based on the use of a control Lyapunov function. An idea that complements fault-tolerant control techniques is the development of high-redundancy actuators that are designed for graceful performance degradation (Steffen, Davies, Dixon, Goodall, Pearson, & Zolotas, 2008).

Passive fault-tolerant control has limited post-fault performance; therefore, active approaches have been developed that replace the nominal controller with a new controller tailored to the faulty plant. The synthesis (whether online or offline) is usually based on perfect model following (Gao & Antsaklis, 1992), on linear eigenstructure assignment (Ashari, Sedigh, & Yazdanpanah, 2005), on extensions of the classical linear pseudo-inverse method (Staroswiecki, Yang, & Jiang, 2006), and on adaptive control principles (Bodson & Groszkiewicz, 1997; Bošković & Mehra, 2006; Chen & Saif, 2007). Another widely studied method consists in the design of actuator fault compensators, where a fault-compensation input is superimposed on the nominal control input, see, for example, Zhang and Qin (2008). In the case of sensor faults, it is common practice to replace lost measurements with estimates obtained from a state estimator. This idea has been widely used and is sometimes called "sensor masking", see, for example, Wu, Thavamani, Zhang, and Blanke (2006).

Approaches that take into account uncertain diagnosis results are rare in the literature to date. A probabilistic approach that accounts for missed detections and false alarms has been presented in Mahmoud, Jiang, and Zhang (2003), which comes at the cost of a high computational complexity that limits the applicability to offline synthesis and to the usage within controller banks. Reconfiguration approaches for actuator and sensor faults based on invariant set theory and controller banks are described in Martínez, Seron, and de Doná (2008), Ocampo-Martínez, de Doná, and Seron (2008) and Olaru, de Doná, and Seron (2008). In the latter approach, it is required that any combination of faulty plants and fault-case controllers yields a stable reconfigured closed-loop system. In Seron and de Doná (2009), it is shown in discrete time that linear virtual actuator-based control reconfiguration can be well combined with fault isolation. The use of unfalsified control

[1] Actuators and sensors are increasingly equipped with self-diagnosing capabilities that are communicated to supervisory control levels over digital fieldbus networks (Discenzo, Unsworth, Loparo, & Marcy, 1999). It is therefore reasonable to consider the control reconfiguration problem separately from the fault diagnosis problem. Furthermore, identification methods for PWA systems can be employed for the detection and identification of faults and the delivery of a model of the faulty plant (Ferrari-Trecate, Muselli, Liberati, & Morari, 2003; Juloski, Weiland, & Heemels, 2005; Nakada, Takaba, & Katayama, 2005; Wen, Wang, Jin, & Ma, 2007). General overviews of the fault diagnosis problem and solution approaches are available in Blanke et al. (2006), Gertler (1998), Isermann (2006), and further interesting ideas are provided in Aßfalg and Allgöwer (2006), Selmic, Polycarpou, and Parisini (2009), Stoustrup and Niemann (2006), Wolff, Krutina, and Krebs (2008) and Zhang, Polycarpou, and Parisini (2002).

for fault-tolerant control is a relatively new area that is still in its infancy (Ingimundarson & Sánchez Peña, 2008). It provides a systematic means for selecting one controller from a set of finitely many candidate controllers. The approach interoperates with any synthesis method for determining the controllers in the candidate set. Consequently, unfalsified control could be used as a decision tool with our new approach if applied offline, although online applications are the intended use of the approach described in this paper.

For nonlinear systems, a fault accommodation approach is described, for example, in Jiang, Staroswiecki, and Cocquempot (2006). Model predictive control has been used as a basis for reconfigurable control (Maciejowski & Jones, 2003; Rosich, Puig, & Quevedo, 2006), and an optimisation technique based on hybrid automata has been proposed (Tran, Stursberg, & Engell, 2007). However, these methods require considerable online computational power. For switched and hybrid systems, adaptive schemes (Yang, Jiang, Cocquempot, & Staroswiecki, 2006a), observer-based switching schemes based on multiple Lyapunov functions (Yang, Jiang, & Staroswiecki, 2007), and output feedback controller redesign (Rodrigues, Theilliol, & Sauter, 2006) have been developed, however based on additive fault models. In Yang, Cocquempot, and Jiang (2008), fault-tolerance analysis based on global passivity is addressed, however without synthesis procedures. Periodic systems were considered in Yang, Jiang, and Cocquempot (2009). A hybrid controller approach based on hybrid automata models and verification techniques has been described in Parisini and Sacone (1998) and Yang (2000). A sensor fault accommodation technique based on bond graphs is available in Yang, Mao, and Jiang (2006b). For PWA systems, model-predictive control has been proposed for fault-tolerant control purposes (Ocampo-Martinez & Puig, 2008; Tsuda, Mignone, Ferrari-Trecate, & Morari, 2001), however in discrete time. Fault-tolerant control of bimodal continuous-time PWA systems is addressed in Nayebpanah, Rodrigues, and Zhang (2009).

Of all methods described in the literature, the fault-hiding idea followed in this paper is closest to the sensor masking idea (Wu et al., 2006) in the case of sensor faults, and closest to fault compensators in the case of actuator faults (Zhang & Qin, 2008). Both references focus on the stability recovery but do not address the tracking recovery. Furthermore, no reconfigurable control approach tailored to continuous-time PWA systems with large numbers of modes has been developed yet that permits us to keep the nominal controller in the closed-loop system, that explicitly addresses the regulation of the difference between the nominal and the reconfigured dynamics, and that studies actuator faults and sensor faults together. Such an approach is developed in this paper and evaluated using an example with 22 modes. Partial and preliminary results of this paper are available in Richter, Heemels, van de Wouw, and Lunze (2008).

3. Preliminaries

The notation $\triangleq$ means equal by definition. Lower case bold letters denote vectors ($\boldsymbol{x}$), capital bold letters denote matrices ($\boldsymbol{A}$), and script capitals denote spaces ($\mathcal{L}$). Systems are denoted by $\Sigma_1, \Sigma_2, \dots$, where the subscripts distinguish different systems. The interconnection of two systems through common input/output variables is denoted by (Σ_1, Σ_2). $\mathbb{R}$ denotes the reals, and $\mathbb{R}_+ \triangleq [0, \infty)$. We use the Euclidian vector norm $\|\cdot\|_2$, and by convention, the abbreviated notation $\|\cdot\|$ always refers to the Euclidian norm. For $1 \le p \le \infty$, and for a measurable signal $\boldsymbol{x} : \mathbb{R}_+ \to \mathbb{R}^n$, we say that $\boldsymbol{x} \in \mathcal{L}_p(\mathbb{R}_+, \mathbb{R}^n)$ if $\|\boldsymbol{x}\|_{\mathcal{L}p} < \infty$, where $\|\boldsymbol{x}\|_{\mathcal{L}p} \triangleq \left(\int_{\mathbb{R}_+} \|\boldsymbol{x}(t)\|^p \mathrm{d}t\right)^{1/p}$ for $1 \le p < \infty$ and $\|\boldsymbol{x}\|_{\mathcal{L}\infty} \triangleq \operatorname{ess\,sup}_{0\le\tau\le t} \|\boldsymbol{x}(\tau)\|$. The space of locally integrable signals is denoted by $\mathcal{L}_1^{loc}$. The notation $\boldsymbol{x} \equiv \boldsymbol{0}$ means $\forall t : \boldsymbol{x}(t) = \boldsymbol{0}$. The pseudoinverse of a matrix $\boldsymbol{A}$ satisfying all four Moore–Penrose conditions (Ben-Israel & Greville, 2003) is denoted by $\boldsymbol{A}^+$. The set of eigenvalues of a matrix $\boldsymbol{A}$ is denoted by $\sigma(\boldsymbol{A})$. The notation $\boldsymbol{A} \prec 0$ ($\boldsymbol{A} \succ 0$) means that the matrix $\boldsymbol{A}$ is negative (positive) definite. A polyhedron is a set Λ defined by a finite number of strict or nonstrict linear inequalities (Ziegler, 1998). Its interior is denoted by $\mathrm{int}(\Lambda)$. The unit step function $\rho(\alpha)$ is defined as $\rho(\alpha) = 0$ for $\alpha < 0$ and $\rho(\alpha) = 1$ for $\alpha \ge 0$.

We base the methods presented in this paper on the well-known concepts of 0-global uniform stability (Khalil, 2002), globally uniformly asymptotically stable solutions (Pavlov, van de Wouw, & Nijmeijer, 2006), input-to-state (practical) stability (ISS, ISpS) (Sontag, 1989), input-to-output stability (IOS) (Sontag, 2001), and (uniform) convergence (Demidovich, 1967; Pavlov et al., 2006). We will also use interconnection theorems for ISS and IOS systems and corresponding small-gain theorems (Jiang, Teel, & Praly, 1994).

Uniform convergence[2] implies the existence of a unique and bounded on $\mathbb{R}$ steady-state solution $\bar{\boldsymbol{x}}_{\boldsymbol{u}}$, which depends only on the input signal $\boldsymbol{u}$. In other words, a uniformly convergent system "forgets" its initial condition. If the input $\boldsymbol{u}$ to a uniformly convergent system is periodic with the period T, then its steady-state solution $\bar{\boldsymbol{x}}_{\boldsymbol{u}}$ is periodic with the period T (Pavlov et al., 2006). In particular, if the input is constant, then the steady-state solution is constant. This fact will be used to obtain tracking recovery in Section 5.

In this paper, we consider nominal systems Σ_P that are modeled as PWA systems:

$$\Sigma_P : \begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{a}_i + \boldsymbol{B}\boldsymbol{u}_c(t) + \boldsymbol{B_d}\boldsymbol{d}(t) \\ \quad \text{for } \boldsymbol{x}(t) \in \Lambda_i,\ i \in \{1,\dots,p\} \\ \boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) \\ \boldsymbol{z}(t) = \boldsymbol{C_z}\boldsymbol{x}(t), \end{cases} \tag{1}$$

$\boldsymbol{x}(0) = \boldsymbol{x}_0$, with the state $\boldsymbol{x}(t) \in \mathbb{R}^n$, the control input $\boldsymbol{u}_c(t) \in \mathbb{R}^m$, the disturbance $\boldsymbol{d}(t) \in \mathbb{R}^k$, the measured output $\boldsymbol{y}(t) \in \mathbb{R}^r$ and the relevant output $\boldsymbol{z}(t) \in \mathbb{R}^q$ at time $t \in \mathbb{R}_+$. $\boldsymbol{A}_i$, $i \in \{1,\dots,p\}$, forms a family of system matrices, $\boldsymbol{a}_i$, $i \in \{1,\dots,p\}$ a family of affine terms, $\boldsymbol{B}$ the input matrix and $\boldsymbol{B_d}$ the disturbance input matrix that defines the structure of disturbance influence. $\boldsymbol{C}$ is the measurement matrix, and $\boldsymbol{C_z}$ the relevant output matrix. All matrices are of constant compatible dimensions. Each of the pairwise disjoint sets Λ_i corresponds to a *mode* of the PWA system (1) in the sense that if $\boldsymbol{x}(t) \in \Lambda_i$, then at time t the system is described by the ith affine system represented by the tuple $(\boldsymbol{A}_i, \boldsymbol{a}_i, \boldsymbol{B}, \boldsymbol{B_d}, \boldsymbol{C}, \boldsymbol{C_z})$. The sets $\Lambda_i, i \in \{1,\dots,p\}$, are described by polyhedra such that $\forall i \ne j : \mathrm{int}(\Lambda_i) \cap \mathrm{int}(\Lambda_j) = \emptyset$ and $\bigcup_{i=1}^p \Lambda_i = \mathbb{R}^n$, and switching is triggered when the state trajectory crosses a boundary between two polyhedra. The input matrix is not allowed to be mode-dependent in this model, since we need the following technical assumption.

Assumption 1. The right-hand side of the system (1) is assumed to be a continuous function of $\boldsymbol{x}$, $\boldsymbol{u}_c$ and $\boldsymbol{d}$.

The PWA system (1) is automatically continuous for $\boldsymbol{x} \in \mathrm{int}(\Lambda_i)$, $i \in \{1,\dots,p\}$. Discontinuities may occur at the boundaries between adjacent polyhedra, and consequently, Assumption 1 imposes conditions at the boundaries. Typically, when the PWA model is obtained as an approximation of a continuous nonlinear system, this assumption is normally satisfied by construction. Note that continuity is lost if mode-dependent input matrices $\boldsymbol{B}_i$ are

[2] For a formal definition of uniform convergence we refer to Demidovich (1967) and Pavlov et al. (2006, 2007).

admitted in the model. Nevertheless, the right-hand side of the PWA system is typically not smooth. Assumption 1 guarantees that the system (1) is Lipschitz continuous. For any $\boldsymbol{u}_c \in \mathcal{L}_1^{loc}(\mathbb{R}^m)$, $\boldsymbol{d} \in \mathcal{L}_1^{loc}(\mathbb{R}^k)$, and $\boldsymbol{x}_0 \in \mathbb{R}^n$, it has a unique and globally defined solution that is locally absolutely continuous. Also, sliding modes cannot occur as solutions of the PWA system (1).

The following proposition, which is central to most of the subsequent proofs, states prior results on incremental stability, ISS, and exponential convergence of continuous PWA systems (Pavlov et al., 2006, 2007).

Proposition 1 (*PWA ISS and Convergence*)**.** *Consider the PWA system (1) with the right-hand side* $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}_c, \boldsymbol{d}) \triangleq \boldsymbol{A}_i\boldsymbol{x}+\boldsymbol{a}_i+\boldsymbol{B}\boldsymbol{u}_c+\boldsymbol{B_d}\boldsymbol{d}$ *for* $\boldsymbol{x} \in \Lambda_i$, $i \in \{1, \dots, p\}$, *and suppose that Assumption 1 holds. If there exists a matrix* $\boldsymbol{X} \in \mathbb{R}^{(n\times n)}$, $\boldsymbol{X} = \boldsymbol{X}^T \succ 0$ *that satisfies the LMIs*

$$\boldsymbol{X}\boldsymbol{A}_i + \boldsymbol{A}_i^T\boldsymbol{X} \prec 0, \quad i = 1, \dots, p, \tag{2}$$

then the system (1) is 0-GES for $\boldsymbol{u}_c, \boldsymbol{d} \equiv \boldsymbol{0}$, *globally ISS w.r.t.* $(\boldsymbol{u}_c, \boldsymbol{d})$, *and there exists* $\beta > 0$ *such that for any two points* $\boldsymbol{x}_1, \boldsymbol{x}_2 \in \mathbb{R}^n$ *the algebraic inequality*

$$(\boldsymbol{x}_1 - \boldsymbol{x}_2)^T\boldsymbol{X}\left(\boldsymbol{f}(\boldsymbol{x}_1, \boldsymbol{u}_c, \boldsymbol{d}) - \boldsymbol{f}(\boldsymbol{x}_2, \boldsymbol{u}_c, \boldsymbol{d})\right) \\ \le -\beta(\boldsymbol{x}_1 - \boldsymbol{x}_2)^T\boldsymbol{X}(\boldsymbol{x}_1 - \boldsymbol{x}_2) \tag{3}$$

holds, meaning that the system is quadratically incrementally stable.[3] *The number* $\beta > 0$ *depends only on the matrix* $\boldsymbol{X}$. *Furthermore, the system (1) is globally exponentially convergent.*

The system (1) is operated in feedback interconnection with a PWA nominal controller Σ_C of the form (1),

$$\boldsymbol{u}_c(t) = \Sigma_C(\boldsymbol{r}(t), \boldsymbol{y}(t), \boldsymbol{x}_{c0}), \tag{4}$$

with the reference signal $\boldsymbol{r}$ and the controller initial condition $\boldsymbol{x}_{c0} \in \mathbb{R}^{n_c}$. Note that the nominal closed-loop system (Σ_P, Σ_C) is also a PWA system (see Johansson (2003)). The following assumption for the nominal (fault-free) closed-loop system will be in place for solving the reconfigurable control problem for stabilisation and tracking.

Assumption 2 (*Stabilising and tracking nominal control*)**.** The feedback interconnection (Σ_P, Σ_C) of the nominal PWA system (1) with bounded intermittent measurement noise $\boldsymbol{n_y}$ ($\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{n_y}(t)$ where $\lim_{t\to\infty} \boldsymbol{n_y}(t) = \boldsymbol{0}$) and the nominal controller (4) is ISS w.r.t. the input $(\boldsymbol{r}, \boldsymbol{d}, \boldsymbol{n_y})$ and IOS w.r.t. the input $(\boldsymbol{r}, \boldsymbol{d}, \boldsymbol{n_y})$ and the output $(\boldsymbol{x}, \boldsymbol{u}_c)$. Furthermore, constant reference commands $\boldsymbol{r}(t) = \bar{\boldsymbol{r}}\rho(t)$, $\bar{\boldsymbol{r}} \in \mathbb{R}^q$, are asymptotically tracked to precision $K \ge 0$ in the presence of constant disturbances $\boldsymbol{d}(t) = \bar{\boldsymbol{d}}\rho(t)$, $\bar{\boldsymbol{d}} \in \mathbb{R}^k$ and intermittent measurement noise $\boldsymbol{n_y}(t)$ with constant steady-state control input $\bar{\boldsymbol{u}}_c \in \mathbb{R}^m$ in the sense that for all $\boldsymbol{x}_0, \boldsymbol{x}_{c0}$

$$\left\{\boldsymbol{d}(t) = \bar{\boldsymbol{d}}\rho(t), \boldsymbol{r}(t) = \bar{\boldsymbol{r}}\rho(t)\right\} \Rightarrow \begin{cases} \limsup_{t\to\infty} \|\boldsymbol{r}(t) - \boldsymbol{z}(t)\| \le K \\ \lim_{t\to\infty} \boldsymbol{u}_c(t) = \bar{\boldsymbol{u}}_c. \end{cases}$$

The previous assumption is realistic in many cases, since approaches for the tracking control of PWA systems have recently been reported (Pavlov et al., 2006; van de Wouw & Pavlov, 2008). These approaches can be used to satisfy Assumption 2. The rejection of intermittent measurement noise is not restrictive and needed in several of the subsequent proofs.

4. Reconfiguration problem

4.1. Model of the faulty plant

A *fault* f in actuators and sensors is modeled as a change of the corresponding input and output matrices of the system (1).

Definition 1 (*Actuator and Sensor Faults*)**.** An *actuator fault* is an instantaneous change at time $t_f < 0$ of the nominal input matrix $\boldsymbol{B} \in \mathbb{R}^{(n\times m)}$ and the nominal affine terms $\boldsymbol{a}_i$ to the faulty input matrix $\boldsymbol{B}_f \in \mathbb{R}^{(n\times m)}$ and the faulty affine terms $\boldsymbol{a}_{f,i}$ of the same dimensions. A *sensor fault* is an instantaneous change of the nominal measurement matrix $\boldsymbol{C} \in \mathbb{R}^{(r\times n)}$ to the faulty measurement matrix $\boldsymbol{C}_f \in \mathbb{R}^{(r\times n)}$ of the same dimensions.

In particular, the blockage of actuators with the indices J in given positions $\bar{u}_k$, $k \in J$, is modeled by means of a changed affine term:

$$\boldsymbol{a}_{f,i} = \boldsymbol{a}_i + \sum_{k\in J} \boldsymbol{b}_k\bar{u}_k, \quad i \in \{1, \dots, p\} \tag{5}$$

and corresponding zero columns in the matrix $\boldsymbol{B}_f$. In this paper, we assume that faults appear abruptly and persist once they have occurred. Without loss of generality, all signal dimensions remain constant after faults. The nominal model must completely describe all redundant components of the plant, and failures typically reduce the rank of the system parameter matrices.

Typical technological examples for faults are stuck valves, failed motors, or failed sensors. The above fault definition includes partial component degradation and complete failures, and every single fault may affect more than one component. As an example, a single actuator degradation might be modeled by scaling the corresponding input matrix column, whereas a complete single actuator failure requires setting the respective column to zero. However, our fault model allows for arbitrary changes in the input and measurement matrices.[4] The fault event abruptly changes the nominal PWA system (1) to the faulty PWA system:

$$\Sigma_{Pf}: \begin{cases} \dot{\boldsymbol{x}}_f(t) = \boldsymbol{A}_i\boldsymbol{x}_f(t) + \boldsymbol{a}_{f,i} + \boldsymbol{B}_f\boldsymbol{u}_f(t) + \boldsymbol{B_d}\boldsymbol{d}(t) \\ \quad \text{for } \boldsymbol{x}_f(t) \in \Lambda_i, \ i \in \{1, \dots, p\} \\ \boldsymbol{y}_f(t) = \boldsymbol{C}_f\boldsymbol{x}_f(t) \\ \boldsymbol{z}_f(t) = \boldsymbol{C_z}\boldsymbol{x}_f(t), \end{cases} \tag{6}$$

$\boldsymbol{x}_f(0) = \boldsymbol{x}_0$, where the matrices $\boldsymbol{B}_f$, $\boldsymbol{C}_f$ and the vectors $\boldsymbol{a}_{f,i}$ reflect the fault, whereas all other matrices remain unchanged. Since the behaviour of the faulty system differs from the nominal behaviour, all signals except the disturbance are distinguished from the nominal case by means of subscript f. It is clear from Eq. (5) that continuity is preserved under the considered faults, also under blockage. It is thus not restrictive to assume the following.

Assumption 3. The right-hand side of the faulty system (6) is assumed to be a continuous function of $\boldsymbol{x}_f$, $\boldsymbol{u}_f$ and $\boldsymbol{d}$.

The nominal controller (4) with $\boldsymbol{y} = \boldsymbol{y}_f$ and $\boldsymbol{u}_c = \boldsymbol{u}_f$ is generally not suitable for controlling the faulty plant (6). In particular, in the case of actuator or sensor failures at time t_f, typically some

[3] Incremental stability refers to the stability of solutions with respect to each other; see Angeli (2002) for an exact definition of incremental stability as well as Lyapunov-characterisations thereof. ISS results for locally Lipschitz systems were published in Sontag (1995).

[4] Additive fault models represent an alternative to the multiplicative fault models used here. However, the class of severe faults such as component failure is better represented by multiplicative models as used here, see Niemann and Stoustrup (2005). In Niemann and Stoustrup (2005), it was pointed out that additive faults can never destabilise a stable linear closed-loop system, whereas actuator or sensor failures can very well destabilise the loop for an open-loop unstable plant in reality. This consideration shows that additive fault models do not capture the entire nature of severe faults.

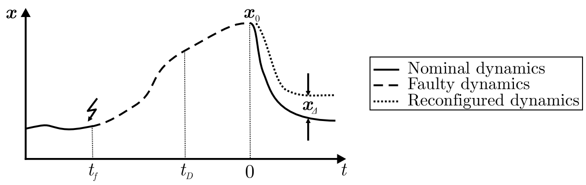

Fig. 2. State of the system undergoing fault, diagnosis, and reconfiguration.

state variables of Σ_{Pf} operate in open loop. It is assumed that a fault diagnosis component provides the model of the faulty PWA plant (6) at time t_D, where $t_D > t_f$. Without loss of generality, the reconfiguration step is assumed to happen at time $t = 0$ (otherwise apply a time shift), and thus $t_f < t_D < 0$. The initial conditions of the nominal system (1) and faulty system (6) refer to time $t = 0$ and thus represent the system state after the fault occurrence at the time where the reconfiguration takes place.

Fig. 2 emphasises that the system state starts deviating from its nominal trajectory as soon as the fault occurs at time t_f (dashed curve). This deviation is unavoidable and brings the system to the state $\boldsymbol{x}_0$ at reconfiguration time. At time $t = 0$ starting from the state $\boldsymbol{x}_0$, the system can either be governed by nominal dynamics (solid curve, corresponding to system repair or ideal reconfiguration) or by reconfigured dynamics with somewhat degraded performance (dotted curve). In the case of reconfiguration, the goal is to limit the performance degradation in the sense of recovering certain properties such as stability, disturbance rejection, and tracking.

4.2. Fault-hiding approach

The reconfiguration problem consists in finding a new controller Σ_{Cr} based on the model (6) such that the reconfigured closed-loop system $(\Sigma_{Pf}, \Sigma_{Cr})$ shown in Fig. 3(c) satisfies the original control goals as well as possible. In terms of Fig. 2, the dotted curve should be as close as possible to the solid curve in the time interval $[0, \infty)$, minimising the difference $\boldsymbol{x}_\Delta$. The new controller may use all available elements of the control input $\boldsymbol{u}_f$ and all elements of the measured output $\boldsymbol{y}_f$, also those ignored by the nominal controller. This reconfiguration problem is equivalent to a closed-loop model-matching problem, which we will, however, not solve directly. Instead, we impose a special structure on the new controller $\Sigma_{Cr} = (\Sigma_R, \Sigma_C)$, which is split into the original nominal controller Σ_C and a reconfiguration block Σ_R.

The reconfiguration block Σ_R takes as inputs the control signal $\boldsymbol{u}_c$ from the nominal controller and the output $\boldsymbol{y}_f$ from the faulty plant. It produces as outputs the translated control signal $\boldsymbol{u}_f$ for the faulty plant and the corrected output $\boldsymbol{y}_c$ for the nominal controller. The reconfiguration block also depends on an internal initial condition $\boldsymbol{\zeta}_0$. Its inner structure will be based on a model for the difference $\boldsymbol{x}_\Delta$ between system trajectories generated by nominal dynamics and system trajectories generated by reconfigured dynamics, with detailed definitions given below in Section 5. Together with the faulty plant (6), the reconfiguration block Σ_R forms the *reconfigured plant* $\Sigma_{Pr} = (\Sigma_{Pf}, \Sigma_R)$ to which the nominal controller (4) is connected by means of the signal pair $(\boldsymbol{u}_c, \boldsymbol{y}_c)$ (see Fig. 3c).

The following goal makes sure that the original controller "sees" the fault-free plant behaviour when attached to the reconfigured plant. It enables keeping the nominal controller as a part of the overall reconfigured closed-loop system, and it will be instrumental in guaranteeing the stability of the reconfigured closed-loop system, formalised in Problem 1 below.

Definition 2 (*Weak Fault-Hiding Goal*)**.** The reconfigured plant $\Sigma_{Pr} = (\Sigma_{Pf}, \Sigma_R)$ satisfies the weak fault-hiding goal, if for zero disturbance ($\boldsymbol{d} \equiv \boldsymbol{0}$) it follows that

$$\forall \boldsymbol{x}_0, \exists \boldsymbol{\zeta}_0 \text{ such that } \forall t \in \mathbb{R}^+, \forall \boldsymbol{u}_c(t) \in \mathcal{L}_1^{\text{loc}} : \boldsymbol{y}(t) - \boldsymbol{y}_c(t) = \boldsymbol{0}.$$

In words, for every plant initial condition, there must exist a matching reconfiguration block initial condition such that the reconfigured plant behaviour equals the fault-free plant behaviour in the absence of disturbances. We speak of "weak" fault-hiding because the initial condition $\boldsymbol{\zeta}_0$ of the reconfiguration block depends on the initial condition $\boldsymbol{x}_0$ of the faulty plant. We will use this goal in Section 5.

This approach, which is called the fault-hiding approach, offers the following advantages.

- The design of the reconfiguration block Σ_R is independent of the controller and therefore usable with any nominal controller (for instance, different people taking shifts in operating a plant). The reconfiguration block can be inserted into existing control schemes without having to touch the nominal controller (acceptance for replacing working control schemes is low in many industries).
- The fault-hiding strategy opens the way for minimum-invasive alterations of the loop. If the controller is automatic and the fault affects small parts of the plant only, then large parts of the nominal controller are still valid and should be kept instead of performing a complete redesign, which may be costly and time-consuming.
- If the nominal controller is a human operator, e.g. a pilot, then the fault-hiding approach reduces the difficulty of dealing with a faulty system, because the reconfigured system behaves like the nominal system. As a consequence, it reduces training efforts for large numbers of fault scenarios and stress during fault situations.

The reconfiguration block will be designed such that the fault is not visible for (in other words, hidden from) the nominal controller. Furthermore, the following additional closed-loop objectives are added to the synthesis.

4.3. Reconfiguration objectives

From a control point of view, it is of interest to at least recover the ISS and setpoint tracking properties for the reconfigured closed-loop system $(\Sigma_{Pf}, \Sigma_R, \Sigma_C)$ as formulated in Assumption 2, where Σ_{Pf} is described by (6). Recall that $\boldsymbol{z}$ is the output of (Σ_P, Σ_C) and $\boldsymbol{z}_f$ is the output of $(\Sigma_{Pf}, \Sigma_R, \Sigma_C)$. Indeed, we wish to solve the following problem.

Problem 1 (*Stability and Setpoint Tracking Recovery*)**.** Consider the nominal controller (4), the nominal PWA system (1), and the faulty PWA system (6). Find a reconfiguration block Σ_R such that

- $\{(\Sigma_P, \Sigma_C) \text{ ISS w.r.t. } (\boldsymbol{r}, \boldsymbol{d})\} \Rightarrow \{(\Sigma_{Pf}, \Sigma_R, \Sigma_C) \text{ ISS w.r.t. } (\boldsymbol{r}, \boldsymbol{d})\}$, and
- for all $\boldsymbol{x}_0 \in \mathbb{R}^n$, $\boldsymbol{x}_{c0} \in \mathbb{R}^{n_c}$ and for $\boldsymbol{d}(t) = \bar{\boldsymbol{d}}\rho(t)$, $\bar{\boldsymbol{d}} \in \mathbb{R}^k$, and $\boldsymbol{r}(t) = \bar{\boldsymbol{r}}\rho(t)$, $\bar{\boldsymbol{r}} \in \mathbb{R}^q$, it holds that $\{\limsup_{t\to\infty} \|\boldsymbol{r}(t) - \boldsymbol{z}(t)\| \le K\} \Rightarrow \{\limsup_{t\to\infty} \|\boldsymbol{r}(t) - \boldsymbol{z}_f(t)\| \le K\}$ for any initialisation $\boldsymbol{\zeta}_0$ of Σ_R.

In other words, we wish to recover the nominal stability and steady-state setpoint tracking properties. The solution to Problem 1 is given in the next section.

5. Stability and setpoint tracking recovering reconfiguration method

5.1. Reconfiguration block

In this section, the realisation of the reconfiguration block Σ_R is described. We first address the recovery from sensor faults and

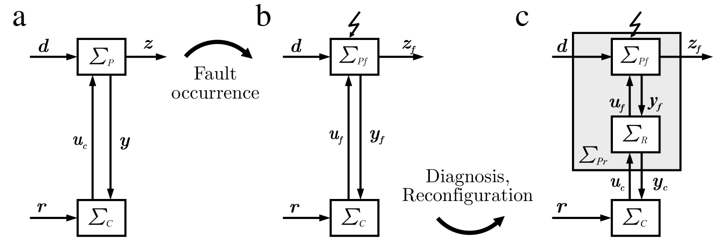

Fig. 3. (a) Nominal closed-loop system, (b) reconfigured closed-loop system with new controller, (c) reconfigured closed-loop system for fault-hiding.

assume in this section that an exogenous system

$$\dot{\boldsymbol{d}}(t) = \mathbf{0} \tag{7}$$

with unknown initial condition $\boldsymbol{d}(0) = \boldsymbol{d}_0$ generates the constant disturbance $\boldsymbol{d}(t) = \boldsymbol{d}_0\ \forall t \geq 0$. The approach consists in the estimation $\hat{\boldsymbol{d}}$ of the disturbance $\boldsymbol{d}$. The estimate $\hat{\boldsymbol{d}}$ is used as an input to the virtual sensor, and we obtain the *extended PWA virtual sensor*

$$\bar{\Sigma}_S : \begin{cases} \dot{\hat{\boldsymbol{x}}}_f(t) = (\boldsymbol{A}_i - \boldsymbol{L}\boldsymbol{C}_f)\hat{\boldsymbol{x}}_f(t) + \boldsymbol{a}_{f,i} + \boldsymbol{B}_f\boldsymbol{u}_f(t) + \boldsymbol{L}\boldsymbol{y}_f(t) \\ \quad + \boldsymbol{B}_d\hat{\boldsymbol{d}}(t) \quad \text{for } \hat{\boldsymbol{x}}_f \in \Lambda_i, i \in \{1, \ldots, p\} \\ \dot{\hat{\boldsymbol{d}}}(t) = \boldsymbol{L}_d\left(\boldsymbol{y}_f(t) - \boldsymbol{C}_f\hat{\boldsymbol{x}}_f(t)\right) \\ \hat{\boldsymbol{y}}_c(t) = \boldsymbol{P}\boldsymbol{y}_f(t) + (\boldsymbol{C} - \boldsymbol{P}\boldsymbol{C}_f)\hat{\boldsymbol{x}}_f(t) \end{cases} \tag{8}$$

(Fig. 4) with the initial conditions $\hat{\boldsymbol{x}}_f(0) = \hat{\boldsymbol{x}}_{f,0}$, $\hat{\boldsymbol{d}}(0) = \hat{\boldsymbol{d}}_0$ and the free parameters $\boldsymbol{L}$, $\boldsymbol{L}_d$, and $\boldsymbol{P}$. The extended virtual sensor is an extension of the PWA virtual sensor introduced in Richter et al. (2008), where the disturbance estimator and the throughput gain $\boldsymbol{P} \in \mathbb{R}^{(r\times r)}$ have been added. One useful choice of $\boldsymbol{P}$ is an identity matrix modified by setting every row corresponding to a faulty sensor to zero. Consequently, the faulty measurement $\boldsymbol{y}_f$ is transformed into an estimate $\hat{\boldsymbol{y}}_c$ of the fictitious true measurement $\boldsymbol{C}\boldsymbol{x}_f$ (obtained without sensor faults), where $\boldsymbol{P}$ admits the parts of $\boldsymbol{y}_f$ unaffected by faults, and the correction term $(\boldsymbol{C} - \boldsymbol{P}\boldsymbol{C}_f)\hat{\boldsymbol{x}}_f$ adds the information gained in the state estimation process. If only sensor faults but no actuator faults are present, then the extended PWA virtual sensor (8) is the complete reconfiguration block and its output $\hat{\boldsymbol{y}}_c$ is directly connected to the nominal controller.

The state estimation error $\boldsymbol{e}$ and the disturbance estimation error $\boldsymbol{e}_d$ are defined as

$$\boldsymbol{e}(t) \triangleq \hat{\boldsymbol{x}}_f(t) - \boldsymbol{x}_f(t) \tag{9}$$

$$\boldsymbol{e}_d(t) \triangleq \hat{\boldsymbol{d}}(t) - \boldsymbol{d}(t) \tag{10}$$

with unknown initial conditions $\boldsymbol{e}(0) = \hat{\boldsymbol{x}}_{f,0} - \boldsymbol{x}_0$ and $\boldsymbol{e}_d(0) = \hat{\boldsymbol{d}}_0 - \boldsymbol{d}_0$. The extended system (8) is rewritten in terms of the extended observation error $\bar{\boldsymbol{e}}$ and the extended state $\bar{\boldsymbol{x}}$

$$\bar{\boldsymbol{e}}(t) = \begin{pmatrix} \boldsymbol{e}(t) \\ \boldsymbol{e}_d(t) \end{pmatrix}, \qquad \bar{\boldsymbol{x}}(t) = \begin{pmatrix} \boldsymbol{x}_f(t) \\ \boldsymbol{d}(t) \end{pmatrix} \tag{11}$$

as the following observation error dynamics

$$\bar{\Sigma}_e : \dot{\bar{\boldsymbol{e}}}(t) = \bar{\boldsymbol{k}}_e\left(\bar{\boldsymbol{e}}(t) + \bar{\boldsymbol{x}}(t)\right) - \bar{\boldsymbol{k}}_e\left(\bar{\boldsymbol{x}}(t)\right), \tag{12}$$

where $\bar{\boldsymbol{e}}(0) = \bar{\boldsymbol{e}}_0 \triangleq (\boldsymbol{e}(0)^T\ \boldsymbol{e}_d(0)^T)^T$ and

$$\bar{\boldsymbol{k}}_e\left(\begin{pmatrix} \boldsymbol{\xi} \\ \boldsymbol{\eta} \end{pmatrix}\right) \triangleq (\bar{\boldsymbol{A}}_{e,i} - \bar{\boldsymbol{L}}\bar{\boldsymbol{C}}_f)\begin{pmatrix} \boldsymbol{\xi} \\ \boldsymbol{\eta} \end{pmatrix} + \bar{\boldsymbol{a}}_{f,i} \quad \text{for } \boldsymbol{\xi} \in \Lambda_i, \quad i \in \{1, \ldots, p\} \tag{13}$$

$$\bar{\boldsymbol{A}}_{e,i} \triangleq \begin{pmatrix} \boldsymbol{A}_i & \boldsymbol{B}_d \\ \mathbf{0} & \mathbf{0} \end{pmatrix}, \qquad \bar{\boldsymbol{a}}_{f,i} \triangleq \begin{pmatrix} \boldsymbol{a}_{f,i} \\ \mathbf{0} \end{pmatrix}, \qquad \bar{\boldsymbol{L}} \triangleq \begin{pmatrix} \boldsymbol{L} \\ \boldsymbol{L}_d \end{pmatrix},$$

$$\bar{\boldsymbol{C}}_f \triangleq \begin{pmatrix} \boldsymbol{C}_f & \mathbf{0} \end{pmatrix}. \tag{14}$$

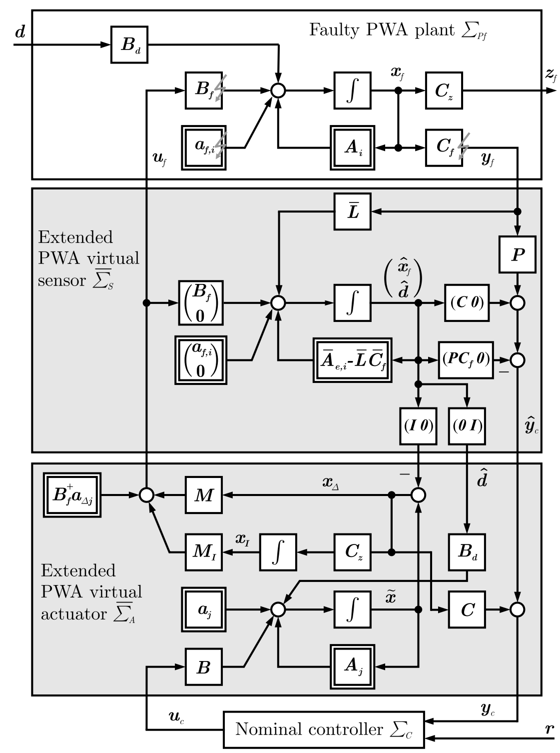

Fig. 4. Extended PWA reconfiguration block in the reconfigured closed-loop system.

The relevant free parameters $\boldsymbol{L}$ and $\boldsymbol{L}_d$ of the extended PWA virtual sensor will be designed so that the extended estimation error dynamics $\bar{\Sigma}_e$ is stabilised. Note that the disturbance $\boldsymbol{d}$ is not a genuine input to (12).

We next introduce the part of the reconfiguration block necessary for the recovery from actuator faults. The solution taken in this paper consists in extending the PWA virtual actuator (Richter et al., 2008) with added integrator states $\boldsymbol{x}_I \in \mathbb{R}^q$ representing an internal model of the reference input. The number of integrators is chosen to match the number of components of the output $\boldsymbol{z}$. This idea results in an *extended PWA virtual actuator*

$$\bar{\Sigma}_A : \begin{cases} \dot{\tilde{\boldsymbol{x}}}(t) = \boldsymbol{A}_j\tilde{\boldsymbol{x}}(t) + \boldsymbol{a}_j + \boldsymbol{B}\boldsymbol{u}_c(t) + \boldsymbol{B}_d\hat{\boldsymbol{d}}(t) \\ \quad \text{for } \tilde{\boldsymbol{x}} \in \Lambda_j, j \in \{1, \ldots, p\} \\ \dot{\boldsymbol{x}}_I(t) = \boldsymbol{C}_z\boldsymbol{x}_\Delta(t) \\ \boldsymbol{y}_c(t) = \hat{\boldsymbol{y}}_c(t) + \boldsymbol{C}\boldsymbol{x}_\Delta(t) \\ \boldsymbol{u}_f(t) = \boldsymbol{M}\boldsymbol{x}_\Delta(t) + \boldsymbol{M}_I\boldsymbol{x}_I(t) + \boldsymbol{B}_f^+\boldsymbol{a}_{\Delta,j} \\ \quad \text{for } \tilde{\boldsymbol{x}} \in \Lambda_j, j \in \{1, \ldots, p\} \end{cases} \tag{15}$$

with the initial conditions $\tilde{\boldsymbol{x}}(0) = \hat{\boldsymbol{x}}_{f,0}$, $\boldsymbol{x}_I(0) = \mathbf{0}$ as shown in Fig. 4, where

$$\boldsymbol{x}_\Delta(t) \triangleq \tilde{\boldsymbol{x}}(t) - \hat{\boldsymbol{x}}_f(t), \qquad \boldsymbol{a}_{\Delta,j} \triangleq \boldsymbol{a}_j - \boldsymbol{a}_{f,j}, \tag{16}$$

where $\boldsymbol{M}$ and $\boldsymbol{M}_I$ are free parameters, and where $\boldsymbol{B}_f^+$ is the pseudoinverse of the input matrix of the faulty plant (6). The affine input $\boldsymbol{B}_f^+\boldsymbol{a}_{\Delta,j}$ is introduced to compensate the bias caused by the difference between the affine terms of the nominal and the faulty plant, which arises, for example, from blocking actuators as discussed before. Consider now the blockage of some actuators whose column indices in the matrix $\boldsymbol{B}$ are collected in the index set J. In accordance with Eq. (5), the difference (16) between the nominal and faulty affine term

$$\boldsymbol{a}_\Delta = \sum_{k\in J} \boldsymbol{b}_k \bar{u}_k \tag{17}$$

is not mode-dependent. The desired compensation is successful if and only if the condition

$$\boldsymbol{a}_\Delta \in \operatorname{im} \boldsymbol{B}_f \tag{18}$$

is satisfied. We introduce satisfaction of this condition as an assumption, but note that the stability recovery does not depend on its satisfaction, and the methods presented here are also useful if this condition is violated (see also Remark 2 below).

Assumption 4. The faulty PWA system (6) satisfies Condition (18).

Using the function

$$\bar{\boldsymbol{k}}_\Delta\left(\begin{pmatrix}\boldsymbol{\xi}\\ \boldsymbol{\eta}\end{pmatrix}\right) \triangleq \left(\bar{\boldsymbol{A}}_{a,j} - \bar{\boldsymbol{B}}_f\bar{\boldsymbol{M}}\right)\begin{pmatrix}\boldsymbol{\xi}\\ \boldsymbol{\eta}\end{pmatrix} + \bar{\boldsymbol{a}}_{f,j} \quad \text{for } \boldsymbol{\xi} \in \Lambda_j, \quad j \in \{1,\ldots,p\}, \tag{19}$$

where

$$\bar{\boldsymbol{A}}_{a,j} \triangleq \begin{pmatrix}\boldsymbol{A}_j & \boldsymbol{0}\\ \boldsymbol{C}_z & \boldsymbol{0}\end{pmatrix}, \qquad \bar{\boldsymbol{a}}_{f,j} \triangleq \begin{pmatrix}\boldsymbol{a}_{f,j}\\ \boldsymbol{0}\end{pmatrix}, \qquad \bar{\boldsymbol{B}}_f \triangleq \begin{pmatrix}\boldsymbol{B}_f\\ \boldsymbol{0}\end{pmatrix},$$

$$\bar{\boldsymbol{M}} \triangleq \begin{pmatrix}\boldsymbol{M} & \boldsymbol{M}_I\end{pmatrix}, \tag{20}$$

and Assumption 4, it is straightforward to obtain the following combined dynamics of the extended difference system:

$$\bar{\Sigma}_\Delta : \begin{pmatrix}\dot{\boldsymbol{x}}_\Delta(t)\\ \dot{\boldsymbol{x}}_I(t)\end{pmatrix} = \bar{\boldsymbol{k}}_\Delta\left(\begin{pmatrix}\tilde{\boldsymbol{x}}(t)\\ \boldsymbol{x}_I(t)\end{pmatrix}\right) - \bar{\boldsymbol{k}}_\Delta\left(\begin{pmatrix}\tilde{\boldsymbol{x}}(t) - \boldsymbol{x}_\Delta(t)\\ \boldsymbol{0}\end{pmatrix}\right) + \begin{pmatrix}\boldsymbol{B}\boldsymbol{u}_c(t) + \boldsymbol{L}\boldsymbol{C}_f\boldsymbol{e}(t)\\ \boldsymbol{0}\end{pmatrix}. \tag{21}$$

It is first shown that the reconfiguration block (8), (15) in combination with the faulty plant (6) achieves the weak fault-hiding goal.

Theorem 1 (*Weak Fault-Hiding*)**.** *The reconfigured plant (6), (8), (15) satisfies the weak fault-hiding goal.*

Proof. Based on Assumption 4, the relevant part of the reconfigured plant model is given by the equations (index k such that $\tilde{\boldsymbol{x}}(t) \in \Lambda_k$)

$$\begin{pmatrix}\dot{\tilde{\boldsymbol{x}}}(t)\\ \dot{\bar{\boldsymbol{e}}}(t)\\ \dot{\boldsymbol{x}}_\Delta(t)\\ \dot{\boldsymbol{x}}_I(t)\end{pmatrix} = \begin{pmatrix}\boldsymbol{A}_k\tilde{\boldsymbol{x}}(t) + \boldsymbol{a}_k + \boldsymbol{B_d}\hat{\boldsymbol{d}}(t)\\ \bar{\boldsymbol{k}}_e\left(\bar{\boldsymbol{e}}(t) + \bar{\boldsymbol{x}}(t)\right) - \bar{\boldsymbol{k}}_e\left(\bar{\boldsymbol{x}}(t)\right)\\ \boldsymbol{k}_\Delta(\tilde{\boldsymbol{x}}(t)) - \boldsymbol{k}_\Delta(\tilde{\boldsymbol{x}}(t) - \boldsymbol{x}_\Delta(t)) + \boldsymbol{L}\boldsymbol{C}_f\boldsymbol{e}(t)\\ \boldsymbol{C}_z\boldsymbol{x}_\Delta(t)\end{pmatrix} + \begin{pmatrix}\boldsymbol{B}\\ \boldsymbol{0}\\ \boldsymbol{B}\\ \boldsymbol{0}\end{pmatrix}\boldsymbol{u}_c(t), \tag{22a}$$

$$\boldsymbol{y}_c(t) = \begin{pmatrix}\boldsymbol{C} & -\boldsymbol{P}\bar{\boldsymbol{C}}_f & \boldsymbol{0} & \boldsymbol{0}\end{pmatrix}\begin{pmatrix}\tilde{\boldsymbol{x}}(t)\\ \bar{\boldsymbol{e}}(t)\\ \boldsymbol{x}_\Delta(t)\\ \boldsymbol{x}_I(t)\end{pmatrix},$$

$$\begin{pmatrix}\tilde{\boldsymbol{x}}(0)\\ \bar{\boldsymbol{e}}(0)\\ \boldsymbol{x}_\Delta(0)\\ \boldsymbol{x}_I(0)\end{pmatrix} = \begin{pmatrix}\hat{\boldsymbol{x}}_{f,0}\\ \bar{\boldsymbol{e}}_0\\ \boldsymbol{0}\\ \boldsymbol{0}\end{pmatrix}, \tag{22b}$$

where $\boldsymbol{k}_\Delta(\boldsymbol{\xi}) \triangleq (\boldsymbol{A}_j - \boldsymbol{B}_f\boldsymbol{M})\boldsymbol{\xi} + \boldsymbol{a}_{f,j}$ for $\boldsymbol{\xi} \in \Lambda_j, j \in \{1,\ldots,p\}$. Weak fault-hiding (Definition 2) is achieved by the matching initialisation $\hat{\boldsymbol{x}}_{f,0} = \boldsymbol{x}_0$, $\hat{\boldsymbol{d}}_0 = \boldsymbol{0}$ which implies that $\bar{\boldsymbol{e}}_0 = \boldsymbol{0}$ for $\boldsymbol{d}(t) = \boldsymbol{0}\ \forall t \in \mathbb{R}_+$, and $\hat{\boldsymbol{d}}(t) \equiv \boldsymbol{0}$. The nominal controller is attached to the reference system

$$\Sigma_{\tilde{P}} : \begin{cases}\dot{\tilde{\boldsymbol{x}}}(t) = \boldsymbol{A}_j\tilde{\boldsymbol{x}}(t) + \boldsymbol{a}_j + \boldsymbol{B}\boldsymbol{u}_c(t) + \boldsymbol{B_d}\hat{\boldsymbol{d}}(t)\\ \quad \text{for } \tilde{\boldsymbol{x}} \in \Lambda_j, j \in \{1,\ldots,p\}\end{cases}$$

governed by nominal dynamics. The reference state $\tilde{\boldsymbol{x}}$ is decoupled from the observation error $\bar{\boldsymbol{e}}$ and the difference state $\boldsymbol{x}_\Delta$ for $\hat{\boldsymbol{d}}_0 = \boldsymbol{0}$ since $\boldsymbol{d}_0 = \boldsymbol{0}$. The output $\boldsymbol{y}_c$ depends on $\tilde{\boldsymbol{x}}$ and $\bar{\boldsymbol{e}}$, where the observation error $\bar{\boldsymbol{e}}$ is autonomous and from $\boldsymbol{d} \equiv \boldsymbol{0}$ and $\hat{\boldsymbol{d}}(0) = \boldsymbol{0}$ it follows that $\bar{\boldsymbol{e}} \equiv \boldsymbol{0}$. □

The latter matching initialisation $\hat{\boldsymbol{x}}_{f,0} = \boldsymbol{x}_0$ is in general practically not achievable, because $\boldsymbol{x}_0$ is not completely measured. Furthermore, the disturbance does not appear in the output $\boldsymbol{y}_c$. However, stability recovery as described in the next section is achieved for *arbitrary* initialisation and considerable mismatch in disturbance behaviour.

5.2. Problem reformulation

In this section, we characterise the tracking part of Problem 1 in alternative form. In words, we address the question how to ensure that the reconfigured closed-loop system $(\Sigma_{Pf}, \bar{\Sigma}_S, \bar{\Sigma}_A, \Sigma_C)$ tracks reference trajectories $\boldsymbol{r}$ with stable dynamics based on Assumption 2.

The relevant output $\boldsymbol{z}$ is defined in Eq. (1). We study under which conditions on the free gains $\boldsymbol{L}$, $\boldsymbol{L}_d$, $\boldsymbol{P}$, $\boldsymbol{M}$, and $\boldsymbol{M}_I$, we can conclude that the corresponding output $\boldsymbol{z}_f$ of the faulty system defined in Eq. (6) asymptotically tracks the reference input to the same precision $K \geq 0$ as in the nominal case:

$$\limsup_{t\to\infty} \|\boldsymbol{e_z}(t)\| \triangleq \limsup_{t\to\infty} \|\boldsymbol{r}(t) - \boldsymbol{z}_f(t)\| = \limsup_{t\to\infty} \|\boldsymbol{r}(t) - \boldsymbol{C_z}\boldsymbol{x}_f(t)\| \leq K.$$

From the definition (9) and (16) of the observation error $\boldsymbol{e}$ and the difference system state $\boldsymbol{x}_\Delta$ respectively, one obtains the equivalent goal

$$\limsup_{t\to\infty} \|\boldsymbol{e_z}(t)\| = \limsup_{t\to\infty} \|\boldsymbol{r}(t) - \boldsymbol{C_z}\tilde{\boldsymbol{x}}(t) + \boldsymbol{C_z}(\boldsymbol{x}_\Delta(t) + \boldsymbol{e}(t))\| \leq K, \tag{23}$$

where it is known from Assumption 2 that $\limsup_{t\to\infty} \|\boldsymbol{r}(t) - \boldsymbol{C_z}\tilde{\boldsymbol{x}}(t)\| \leq K$ if $\lim_{t\to\infty} \bar{\boldsymbol{e}}(t) = \boldsymbol{0}$. It is thus desired that $\limsup_{t\to\infty} \boldsymbol{C_z}(\boldsymbol{x}_\Delta(t) + \boldsymbol{e}(t)) = \boldsymbol{0}$ holds. Observing that $\boldsymbol{e}$ and $\boldsymbol{x}_\Delta$ are driven by different inputs $\boldsymbol{d}$ and $\boldsymbol{u}_c$, achieving the special case $\boldsymbol{C_z}\boldsymbol{e}(\infty) = -\boldsymbol{C_z}\boldsymbol{x}_\Delta(\infty)$, is unrealistic in most cases. We thus focus on separately decoupling $\boldsymbol{C_z}\boldsymbol{e}$ from $\boldsymbol{d}$, which is implied by the previous requirement that $\lim_{t\to\infty} \bar{\boldsymbol{e}}(t) = \boldsymbol{0}$, as well as decoupling $\boldsymbol{C_z}\boldsymbol{x}_\Delta$ from $\boldsymbol{u}_c$ and obtain the following sufficient conditions for solving Problem 1:

$$\lim_{t\to\infty} \bar{\boldsymbol{e}}(t) = \boldsymbol{0} \tag{24}$$

$$\lim_{t\to\infty} \boldsymbol{C_z}\boldsymbol{x}_\Delta(t) = \boldsymbol{0}. \tag{25}$$

We will focus on decoupling the extended observation error from the disturbance by means of disturbance estimation for the case where the constant disturbance is generated by an appropriate exo-system (Section 5.3) to achieve (24), and the output-relevant difference system is decoupled from the input $\boldsymbol{u}_c$ to achieve (25). It remains to justify that the state $\tilde{\boldsymbol{x}}$ is indeed governed by completely nominal dynamics, and that the observation error and difference system introduced by the reconfiguration block preserve the claimed properties. The formal justification of this intuitively sketched solution approach is given in the proof of Theorem 4 below.

5.3. Main results on stability and tracking recovery

The following theorem provides the recovery from sensor faults by solving the problem of stable and convergent state and disturbance estimation for constant disturbance.

Theorem 2 (*Disturbance-decoupled observation*). *Consider the faulty PWA system (6), suppose that Assumptions 1 and 3 hold, and suppose that the disturbance is constant ($\dot{\boldsymbol{d}}(t) = \boldsymbol{0}$). If there exist matrices $\bar{\boldsymbol{X}}_s \in \mathbb{R}^{(n+k)\times(n+k)}$ and $\bar{\boldsymbol{Y}}_s \in \mathbb{R}^{(n+k)\times(r+k)}$ that satisfy the LMIs*

$$\bar{\boldsymbol{X}}_s = \bar{\boldsymbol{X}}_s^T \succ 0 \tag{26a}$$

$$\bar{\boldsymbol{X}}_s\bar{\boldsymbol{A}}_{e,i} + \bar{\boldsymbol{A}}_{e,i}^T\bar{\boldsymbol{X}}_s - \bar{\boldsymbol{Y}}_s\bar{\boldsymbol{C}}_f - \bar{\boldsymbol{C}}_f^T\bar{\boldsymbol{Y}}_s^T \prec 0, \quad i = 1,\ldots,p \tag{26b}$$

then the system (8) with $\bar{\boldsymbol{L}} \triangleq \bar{\boldsymbol{X}}_s^{-1}\bar{\boldsymbol{Y}}_s$ is a state and disturbance observer for the faulty system (6). The extended observation error $\bar{\boldsymbol{e}}$ defined in Eq. (11) satisfies the relation

$$\|\bar{\boldsymbol{e}}(t)\| \le c\mathrm{e}^{-at}\|\bar{\boldsymbol{e}}(0)\|, \quad t \in [0,\infty), \tag{27}$$

where the real numbers $c > 0$ and $a > 0$ depend only on $\bar{\boldsymbol{X}}_s$ and $\bar{\boldsymbol{Y}}_s$.

Proof. See Appendix. □

The conditions (26) ensure that the extended PWA virtual sensor (8) estimates both the constant disturbance and the system state in spite of the unknown discrete system mode. The gain $\boldsymbol{P}$ does not affect stability, but may be used, for example, to throughput the non-faulty measurements ($\boldsymbol{P} = \boldsymbol{I}$).

Remark 1 (*Occasional setpoint changes*). In practice, this virtual sensor scheme will still work appropriately for piecewise constant disturbance with infrequent discontinuous changes. Disturbance jumps may then be interpreted as changes of the observation error initial condition. "Infrequent" means that the disturbance should remain constant for several integer multiples of $1/a$, where a is defined in Eq. (27). The robustness against stronger variations of the disturbance is discussed in Section 6 below.

The following theorem provides the recovery from actuator faults by the stable output regulation problem for the output-relevant difference system.

Theorem 3 (*Extended difference system ISS*). *Consider the faulty PWA system (6) and suppose that Assumptions 1, 3 and 4 are satisfied. If there exist matrices $\bar{\boldsymbol{X}}_a \in \mathbb{R}^{(n+q)\times(n+q)}$ and $\bar{\boldsymbol{Y}}_a \in \mathbb{R}^{(m\times(n+q))}$ that satisfy the linear matrix inequalities*

$$\bar{\boldsymbol{X}}_a = \bar{\boldsymbol{X}}_a^T \succ 0 \tag{28a}$$

$$\bar{\boldsymbol{A}}_{a,j}\bar{\boldsymbol{X}}_a + \bar{\boldsymbol{X}}_a\bar{\boldsymbol{A}}_{a,j}^T - \bar{\boldsymbol{B}}_f\bar{\boldsymbol{Y}}_a - \bar{\boldsymbol{Y}}_a^T\bar{\boldsymbol{B}}_f^T \prec 0, \quad j = 1,\ldots,p, \tag{28b}$$

then the extended difference system (21) of the extended virtual actuator (15) with $\bar{\boldsymbol{M}} \triangleq \bar{\boldsymbol{Y}}_a\bar{\boldsymbol{X}}_a^{-1}$ is 0-GES for $\boldsymbol{u}_c, \boldsymbol{e} \equiv \boldsymbol{0}$. Moreover, any solution of the unforced difference system (21) (i.e. with $\boldsymbol{u}_c, \boldsymbol{e} \equiv \boldsymbol{0}$ but arbitrary $\tilde{\boldsymbol{x}}$) satisfies the relation

$$\|\boldsymbol{x}_\Delta(t)\| + \|\boldsymbol{x}_I(t)\| \le c\mathrm{e}^{-at}\left(\|\boldsymbol{x}_\Delta(0)\| + \|\boldsymbol{x}_I(0)\|\right), \tag{29}$$

where the real numbers $c > 0$ and $a > 0$ depend only on $\bar{\boldsymbol{X}}_a$ and $\bar{\boldsymbol{Y}}_a$. In other words, the difference state $\boldsymbol{x}_\Delta$ asymptotically converges to the origin: $\lim_{t\to\infty}\boldsymbol{x}_\Delta(t) = \boldsymbol{0}$ for zero inputs. Furthermore, the extended difference system is ISS w.r.t. the input $(\boldsymbol{u}_c, \boldsymbol{e})$. If the steady-state control input $\boldsymbol{u}_c$ is constant and $\lim_{t\to\infty}\boldsymbol{e}(t) = \boldsymbol{0}$, then $\lim_{t\to\infty}\boldsymbol{C_z}\boldsymbol{x}_\Delta(t) = \boldsymbol{0}$.

Proof. See Appendix. □

Note that formally, $\tilde{\boldsymbol{x}}$ is also an external input to the extended difference system, but the obtained properties are valid independently of $\tilde{\boldsymbol{x}}$. Combining the results of Theorems 1–3, we obtain the following main result on recovery of stability and tracking for the reconfigured closed-loop system, which provides the solution to Problem 1.

Theorem 4 (*Reconfigured closed-loop stability and tracking recovery*). *Suppose that Assumptions 1–4 as well as the LMIs (26) and (28) are satisfied. Then, the reconfigured closed-loop system $(\Sigma_{Pf}, \bar{\Sigma}_S, \bar{\Sigma}_A, \Sigma_C)$ consisting of the controller (4), the faulty PWA system (6), the extended PWA virtual sensor (8), and the extended PWA virtual actuator (15) is globally ISS w.r.t. the input $(\boldsymbol{r}, \boldsymbol{d})$. Moreover, the output $\boldsymbol{z}_f$ asymptotically tracks any constant reference $\boldsymbol{r}(t) = \bar{\boldsymbol{r}}\rho(t)$ for any constant disturbance $\boldsymbol{d}(t) = \bar{\boldsymbol{d}}\rho(t)$ to nominal precision K in the sense that $\limsup_{t\to\infty}\|\boldsymbol{r}(t) - \boldsymbol{z}_f(t)\| \le K$ for all initial conditions $\boldsymbol{x}_0$, $\hat{\boldsymbol{x}}_{f,0}$, and $\boldsymbol{x}_{c,0}$.*

Proof. See Appendix. □

Both extensions that together provide stability and tracking are based on the internal model principle. Namely, models of exosystems creating the admissible disturbance and reference inputs have been embedded in the reconfiguration block (8), (15).

Remark 2 (*Actuator Blockage*). The violation of Assumption 4 is not problematic. If the actuator blockage cannot be statically compensated, the extended difference system (21) is augmented by an additional term:

$$\bar{\Sigma}_\Delta : \begin{pmatrix}\dot{\boldsymbol{x}}_\Delta(t)\\ \dot{\boldsymbol{x}}_I(t)\end{pmatrix} = \bar{\boldsymbol{k}}_\Delta\left(\begin{pmatrix}\tilde{\boldsymbol{x}}(t)\\ \boldsymbol{x}_I(t)\end{pmatrix}\right) - \bar{\boldsymbol{k}}_\Delta\left(\begin{pmatrix}\tilde{\boldsymbol{x}}(t) - \boldsymbol{x}_\Delta(t)\\ \boldsymbol{0}\end{pmatrix}\right) + \begin{pmatrix}\boldsymbol{B}\boldsymbol{u}_c(t) + \boldsymbol{L}\boldsymbol{C}_f\boldsymbol{e}(t)\\ \boldsymbol{0}\end{pmatrix} + \begin{pmatrix}(\boldsymbol{I} - \boldsymbol{B}_f\boldsymbol{B}_f^+)\boldsymbol{a}_\Delta g(t)\\ \boldsymbol{0}\end{pmatrix},$$

where $g(t) = 1$, which acts like a constant additive input on the extended difference system. As asserted in Theorem 4, the output $\boldsymbol{C_z}$ converges to zero for constant steady-state control input. The added term is constant and may be considered as an addition to the constant steady-state control input $\boldsymbol{u}_c$. In other words, its effect on the output is compensated by the remaining control inputs due to the extension by integrators. The compensation is only successful if in every mode, the steady-state gain allows fault-compensation at the output, and if the actuation range is sufficiently large.

5.4. Control reconfiguration algorithm

The design procedure for the reconfiguration block for recovering stability and tracking is summarised in Algorithm 1. The steps 1–4 describe the nominal closed-loop operation before any faults occur. Once faults are detected and isolated in step 5, the virtual sensor and virtual actuator design activates in steps 6–11, where the gains $\boldsymbol{L}, \boldsymbol{L_d}, \boldsymbol{M}$, and $\boldsymbol{M}_I$ are designed (the gain $\boldsymbol{P}$ is arbitrary). After completed gain calculations, the reconfigured closed-loop system is executed in step 12.

If the relevant LMIs are infeasible, then a stabilising virtual sensor and virtual actuator scheme might exist, but it cannot be found using the sufficient stability conditions presented in this paper. This problem appears to be fundamentally unavoidable, since the problem of deciding whether all trajectories of a given PWA system are bounded is undecidable (Blondel & Tsitsiklis, 2000). In practice, objective reconfiguration may complement control reconfiguration, for example by removing rows from the output matrix $\boldsymbol{C_z}$ according to a priority list until feasible solutions are found.

At first glance, it might seem that a reduction of the conservatism might be achievable by seeking continuous piecewise

Algorithm 1 Tracking PWA virtual actuator and sensor synthesis

Require: PWA model $\boldsymbol{A}_i$, $\boldsymbol{a}_i$, $\boldsymbol{B}$, $\boldsymbol{B_d}$, $\boldsymbol{C}$, $\boldsymbol{C_z}$ for $i \in \{1, \dots, p\}$, initial time $t_0 < 0$, guessed initial condition $\hat{\boldsymbol{x}}_{f,0}$

1: Initialise the nominal closed-loop system (1), (4), (8), (15), with $\boldsymbol{C}_f = \boldsymbol{C}, \boldsymbol{B}_f = \boldsymbol{B}, \boldsymbol{a}_{f,i} = \boldsymbol{a}_i, \boldsymbol{L} = \boldsymbol{0}, \boldsymbol{L}_d = \boldsymbol{0}, \boldsymbol{P} = \boldsymbol{I}, \boldsymbol{M} = \boldsymbol{0}, \boldsymbol{M}_l = \boldsymbol{0}, \boldsymbol{x}(t_0) = \boldsymbol{x}_0$, $\boldsymbol{x}_c(t_0) = \boldsymbol{x}_{c0}, \hat{\boldsymbol{x}}_f(t_0) = \hat{\boldsymbol{x}}_{f,0}, \hat{\boldsymbol{d}}(t_0) = \boldsymbol{0}, \tilde{\boldsymbol{x}}(t_0) = \hat{\boldsymbol{x}}_{f,0}, \boldsymbol{x}_l(t_0) = \boldsymbol{0}$.
2: Solve LMI (26) with $\boldsymbol{C}_f = \boldsymbol{C}$ and compute a stabilising virtual sensor gain $\bar{\boldsymbol{L}} \triangleq \bar{\boldsymbol{X}}_s^{-1}\bar{\boldsymbol{Y}}_s$, update extended PWA virtual sensor (8)
3: **repeat**
4: Run nominal closed-loop system
5: **until** actuator or sensor fault f isolated
6: Construct fault model $\boldsymbol{a}_{f,i}, \boldsymbol{B}_f, \boldsymbol{C}_f$ and update the extended PWA virtual sensor (8) and the extended virtual actuator (15)
7: Solve LMIs (26) and (28) for $\bar{\boldsymbol{X}}_s, \bar{\boldsymbol{Y}}_s, \bar{\boldsymbol{X}}_a, \bar{\boldsymbol{Y}}_a$
8: Compute $\bar{\boldsymbol{L}} \triangleq \bar{\boldsymbol{X}}_s^{-1}\bar{\boldsymbol{Y}}_s$ and $\bar{\boldsymbol{M}} \triangleq \bar{\boldsymbol{Y}}_a\bar{\boldsymbol{X}}_a^{-1}$
9: Update extended PWA virtual sensor (8) with $\bar{\boldsymbol{L}}$, and arbitrary $\boldsymbol{P}$
10: Wait for virtual sensor to converge for specified time interval
11: Update PWA virtual actuator (15) with $\bar{\boldsymbol{M}}$ and initialise $\tilde{\boldsymbol{x}}(t_r) = \hat{\boldsymbol{x}}(t_r)$
12: Run reconfigured closed-loop system (4), (6), (8), (15)

Result: Globally ISS reconfigured closed-loop system that tracks constant reference inputs in spite of constant disturbances.

quadratic Lyapunov functions instead of common quadratic Lyapunov functions as in Johansson and Rantzer (1998) and Lin and Antsaklis (2009). However, we note that in those works, sufficient conditions for stability (and stabilisation) in terms of the existence of piecewise quadratic Lyapunov functions have been obtained for *equilibria* of PWA systems. In the approach presented in this paper, we require the stability of time-varying solutions of certain PWA systems (such as the observation error and difference systems), and we study such stability properties using the concepts of convergence/incremental stability. To our knowledge, no such characterisation of incremental stability or convergence for PWA systems in terms of piecewise quadratic Lyapunov functions exists to date. The derivation of such a theory seems to be far from trivial.

6. Robustness analysis

In the previous sections, it has been tacitly assumed that the nominal and faulty plants are accurately modeled as PWA systems, and moreover, that the disturbance is constant. This section relaxes these assumptions and studies the robustness of the reconfiguration scheme against model approximation errors and time-varying disturbance. Both issues are serious in practice. For the following analysis, we assume that the faulty nonlinear system is an input-affine system of the form

$$\Sigma_{Pf,NL} : \left\{ \dot{\boldsymbol{x}}_f(t) = \boldsymbol{f}(\boldsymbol{x}_f(t)) + \boldsymbol{B}_f\boldsymbol{u}_f(t) + \boldsymbol{B_d}\boldsymbol{d}(t) \right. \qquad (30)$$

with $\boldsymbol{f}$ continuous, whereas the PWA virtual sensor (8) is based on the PWA model (6). The difference between the input-affine model (30) and the PWA model (6),

$$\boldsymbol{\varepsilon}(\boldsymbol{x}_f) = \boldsymbol{f}(\boldsymbol{x}_f) - \boldsymbol{A}_i\boldsymbol{x}_f - \boldsymbol{a}_{f,i} \quad \text{for } \boldsymbol{x}_f \in \Lambda_i, i \in \{1, \dots, p\}, \qquad (31)$$

represents the unknown nonlinear approximation error. The disturbance is now assumed to be time-varying and generated by the exo-system

$$\Sigma_d : \left\{ \dot{\boldsymbol{d}}(t) = \boldsymbol{\varrho}(t), \ \boldsymbol{d}(0) = \boldsymbol{d}_0, \right. \qquad (32)$$

where the disturbance variation rate is modelled through $\boldsymbol{\varrho}$. Using the model approximation error (31), the input-affine system (30) is re-written as a perturbed PWA system:

$$\Sigma_{Pf,NL} : \begin{cases} \dot{\boldsymbol{x}}_f(t) = \boldsymbol{A}_i\boldsymbol{x}_f(t) + \boldsymbol{a}_{f,i} + \boldsymbol{B}_f\boldsymbol{u}_f(t) + \boldsymbol{B_d}\boldsymbol{d}(t) + \boldsymbol{\varepsilon}(\boldsymbol{x}_f(t)) \\ \quad \text{for } \boldsymbol{x}_f(t) \in \Lambda_i, \ i \in \{1, \dots, p\}. \end{cases}$$

It is assumed that the model approximation error $\boldsymbol{\varepsilon}$ is uniformly bounded, which is always achievable on a compact subset $\mathcal{X}$ of state space by sufficient refinement of the state-space partition that underlies the PWA system model, and it is likewise assumed that the disturbance variation rate $\boldsymbol{\varrho}$ is globally bounded:

$$\exists E \text{ such that } \forall \boldsymbol{x}_f \in \mathcal{X} \subset \mathbb{R}^n : \ \|\boldsymbol{\varepsilon}(\boldsymbol{x}_f)\| \le E \qquad (33)$$

$$\exists F \text{ such that } \forall t \in \mathbb{R} : \|\boldsymbol{\varrho}(t)\| \le F. \qquad (34)$$

In order to obtain robustness for the reconfigured closed-loop system, Assumption 2 is replaced by the following assumption about robustness of the nominal control scheme.

Assumption 5 (*Robust stabilising and tracking nominal control*)**.** The feedback interconnection (Σ_P, Σ_C) of the nominal PWA system (1) with bounded measurement noise $\boldsymbol{n_y}$ $(\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{n_y}(t))$ and the nominal controller (4) is ISS w.r.t. the input $(\boldsymbol{r}, \boldsymbol{d}, \boldsymbol{n_y})$ and IOS w.r.t. the input $(\boldsymbol{r}, \boldsymbol{d}, \boldsymbol{n_y})$ and the output $(\boldsymbol{x}, \boldsymbol{u}_c)$. Furthermore, constant reference commands $\boldsymbol{r}(t) = \bar{\boldsymbol{r}}\rho(t)$, $\bar{\boldsymbol{r}} \in \mathbb{R}^q$, are asymptotically tracked to precision $K' \ge 0$ in the presence of time-varying disturbances $\boldsymbol{d}(t)$ and measurement noise $\boldsymbol{n_y}(t)$ $(\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{n_y}(t)$ with the property $\lim_{t\to\infty} \boldsymbol{n_y}(t) \neq \boldsymbol{0})$ with constant steady-state control input $\bar{\boldsymbol{u}}_c \in \mathbb{R}^m$ in the sense that for all $\boldsymbol{x}_0, \boldsymbol{x}_{c0}$

$$\{\boldsymbol{d}(t) \text{according to (32)}, \ \boldsymbol{r}(t) = \bar{\boldsymbol{r}}\rho(t)\}$$
$$\Rightarrow \begin{cases} \limsup_{t\to\infty} \|\boldsymbol{r}(t) - \boldsymbol{z}(t)\| \le K' \\ \lim_{t\to\infty} \boldsymbol{u}_c(t) = \bar{\boldsymbol{u}}_c. \end{cases}$$

Note that due to the time-varying disturbance and the persistent measurement noise, the tracking precision K' is typically larger than the nominal tracking precision of Assumption 2. The magnitude of K' will typically depend on the variation bound E on $\|\boldsymbol{\varrho}\|$ as well as on a bound on measurement noise $\|\boldsymbol{n_y}\|$.

The inclusion of the model approximation error and the time-varying disturbance leads to the following new dynamics for the extended observation error:

$$\bar{\Sigma}_e : \left\{ \dot{\bar{\boldsymbol{e}}}(t) = \bar{\boldsymbol{k}}_e(\bar{\boldsymbol{x}}(t) + \bar{\boldsymbol{e}}(t)) - \bar{\boldsymbol{k}}_e(\bar{\boldsymbol{x}}(t)) - \bar{\boldsymbol{\varepsilon}}(\boldsymbol{x}_f(t)) - \bar{\boldsymbol{\varrho}}(t), \right. \qquad (35)$$

where

$$\bar{\boldsymbol{\varepsilon}}(\boldsymbol{x}_f(t)) \triangleq \begin{pmatrix} \boldsymbol{\varepsilon}(\boldsymbol{x}_f(t)) \\ \boldsymbol{0} \end{pmatrix}, \qquad \bar{\boldsymbol{\varrho}}(t) = \begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{\varrho}(t) \end{pmatrix},$$

and the extended observation error $\bar{\boldsymbol{e}}$, the extended state $\bar{\boldsymbol{x}}$, and the function $\bar{\boldsymbol{k}}_e(\cdot)$ as in Eqs. (11) and (13).

Theorem 5 (*Robustness against model approximation error and time-varying disturbance*)**.** *Consider the faulty nonlinear system (30) reconfigured by means of the extended PWA virtual sensor (8) and the extended PWA virtual actuator (15), and suppose that Assumptions 1, 3 and 5 as well as the LMIs (26) and (28) are satisfied. The reconfigured closed-loop system $(\Sigma_{Pf,NL}, \bar{\Sigma}_S, \bar{\Sigma}_A, \Sigma_C)$ is ISpS w.r.t. the input $(\boldsymbol{r}, \boldsymbol{\varrho})$. Moreover, if the reference input and the steady-state control input are constant, and if the nominal closed-loop system tracks the reference to precision K', then the reconfigured closed-loop system tracks the reference input to degraded precision $K' + c \cdot E + d \cdot F$, where $c, d > 0$.*

Proof. See Appendix. □

This result shows that the reconfigured closed-loop stability and tracking recovery properties are not suddenly lost if assumptions regarding model knowledge and constant disturbance inputs are violated. Rather, the tracking accuracy degrades gradually as the model error and the disturbance variation increase. Due to the visibility of the observation error at the output $\boldsymbol{y}_c$, the controller may reject the disturbance induced by the model approximation error, as the example in Section 7 below will demonstrate.

In addition, it can be shown that the reconfigured closed-loop system is small-gain robust against fault diagnosis uncertainty (Richter, Heemels, van de Wouw, & Lunze, 2010). The analysis is omitted due to lack of space.

7. Example application

A successful application of Algorithm 1 to the model of a two-tank system is presented in this section. The plant consists of tanks T_1 and T_2 of cross-sectional area 0.0177 m^2 with levels h_1 and h_2, respectively in m. The tanks are interconnected by a lower valve u_L and an upper valve u_U, where T_1 is filled via pump u_P and disturbed by unmodelled outflow d in ml/s. With the state $\boldsymbol{x} = (h_1, h_2)^T$ and the input vector $\boldsymbol{u} = (u_P, u_L, u_U)^T$, the plant is approximately described by the model (1) with the parameters

$$\boldsymbol{B} = 10^{-3}\begin{pmatrix} 8.1 & -2.9 & -3.4 \\ 0 & 2.9 & 3.4 \end{pmatrix}, \qquad \boldsymbol{B_d} = 10^{-5}\begin{pmatrix} 5.64 \\ 0 \end{pmatrix},$$

$$\boldsymbol{B}_f = 10^{-3}\begin{pmatrix} 8.1 & 0 & -0.68 \\ 0 & 0 & 0.68 \end{pmatrix}, \qquad \boldsymbol{C} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$\boldsymbol{C}_f = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},$$

where the mode-dependent model parameters ($\boldsymbol{A}_i$, $\boldsymbol{a}_i$, $i = 1, \dots, 22$) are available from Richter et al. (2010) and the gains include suitable unit conversions where applicable. The fault-free tanks system is controlled by two linear decentralised proportional-integral feedback controllers and a constant input:

$$\begin{pmatrix} u_P(t) \\ u_L(t) \\ u_U(t) \end{pmatrix} = \begin{pmatrix} 50 \cdot (r_1(t) - y_1(t)) + 4 \cdot \int_0^t (r_1(\tau) - y_1(\tau))\mathrm{d}\tau \\ 50 \cdot (r_2(t) - y_2(t)) + 4 \cdot \int_0^t (r_2(\tau) - y_2(\tau))\mathrm{d}\tau \\ 0.8 \end{pmatrix}.$$

The controlled quantities are the levels h_1, h_2, for which the control aims are firstly stability, and secondly regulation to a given setpoint. The considered faults are an abrupt and non-transient blockage of the level sensor for h_1 ($f_1 : y_{f,1}(t) = 0.3$ for $t > t_{f1}$) at time $t_{f1} = 60$ s, an abrupt and non-transient failure of the lower valve, and gain reduction for the upper valve ($f_2 : u_{f,L}(t) = 0$ for $t > t_{f2}$, $u_{f,U}(t) = 0.2u_U(t)$ for $t > t_{f2}$) at fault time $t_{f2} = 80$ s. The plant is excited by reference steps $r_1(t) = 0.15$ m for $t \leq 30$ s and $r_1(t) = 0.45$ m for $t > 30$ s for the level h_1 as well as $r_2(t) = 0.05$ m for $t \leq 100$ s and $r_2(t) = 0.08$ m for $t > 100$ s for the level h_2. The steps drive the process through a large operating range, and thus realistically describe a startup procedure. A non-modelled outflow from tank T_1 starting at 65 s is represented as a disturbance d, where $d(t) = 0$ ml/s for $0\ \text{s} \leq t < 65$ s, and $d(t) = -20$ ml/s for $65\ \text{s} \leq t \leq 300$ s. Note that the fault breaks the loop at several points and the reconfiguration method must change the control loop structure to meet the control objectives.

Fig. 5 shows the behaviour of the reconfigured closed-loop system with a periodic reference input r_2 with peak-to-peak amplitude 0.015 m and period $T = 40$ s. The plant is represented by a detailed nonlinear model instead of the PWA model that is used in the reconfiguration blocks, so as to demonstrate the robustness of our method with respect to model uncertainties and time-varying disturbance. Times $t \in [0, t_{f1}]$ correspond to the steps 1–4 of Algorithm 1. The application of steps 5–11 of Algorithm 1 result in $\boldsymbol{L}, \boldsymbol{L}_d$ at $t = 60$ s $\boldsymbol{M}, \boldsymbol{M}_l$ at $t = 80$ s as follows (where $\boldsymbol{P} = \boldsymbol{0}$):

$$\boldsymbol{L} = \begin{pmatrix} 0 & 334.7 \\ 0 & 34.4 \end{pmatrix}, \qquad \boldsymbol{L}_d = \begin{pmatrix} 0 & 6.9 \end{pmatrix},$$

$$\boldsymbol{M} = \begin{pmatrix} 114.6 & 116.4 \\ 0 & 0 \\ -6.6 & 1350 \end{pmatrix}, \qquad \boldsymbol{M}_l = \begin{pmatrix} 167.1 & 168 \\ 0 & 0 \\ 1.4 & 1977 \end{pmatrix}.$$

Each gain computation phase took about 6 s using MATLAB 7 on a Pentium D 2.8 GHz with 1GB RAM using YALMIP (Löfberg, 2004) and Sedumi 1.05 (Sturm, 1999).

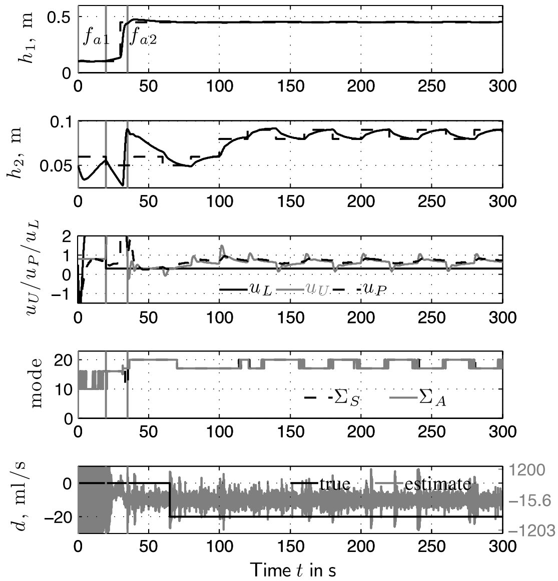

Fig. 5. Robust behaviour of the disturbed reconfigured closed-loop system with periodic excitation (faulty plant represented by detailed nonlinear model).

The actuator blockage of u_L is compensated by u_U, as the figure clearly shows. It is clearly visible that the tank levels and the control inputs are periodic with the same period. The observation error $\boldsymbol{e}$ (not shown) tends to zero, while the difference system $\boldsymbol{x}_\Delta$ (not shown) does not stay at the origin, but is periodically perturbed by the control input $\boldsymbol{u}_c$. Nevertheless, the difference is small, less than or equal to one millimeter. Consequently, practical tracking is achieved also for periodic reference inputs in this case, and in spite of considerable modelling error: the detailed nonlinear model is of the form $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u}_c$ (Blanke et al., 2006), whereas the state-dependent input gain $\boldsymbol{g}$ must be approximated by a constant gain in the class of PWA models considered in this paper. The disturbance estimate is affected by the model uncertainties, but its mean (−15.6 ml/s) is reasonably close to the true disturbance (−20 ml/s). Additional experiments not presented here have shown that linear reconfiguration blocks are not capable to achieve stabilising reconfiguration during this transient startup operation. This observation highlights the value of PWA model-based control reconfiguration.

In summary, this example has demonstrated the value of our method for complex PWA systems, and its usefulness in the presence of considerable model uncertainty and time-varying disturbances. Due to space limitations, reports on an example with larger state-space dimension have to be deferred to a future publication.

8. Conclusions

A novel approach to the reconfigurable control of piecewise affine systems was presented, which works by placing a reconfiguration block between the faulty plant and the nominal controller. This idea is a generalisation of the fault-hiding framework from linear systems towards piecewise affine systems. The main problems that had to be overcome in this generalisation arose from the fact that in piecewise affine systems, the superposition principle is lost and the separation principle does not hold in the same way as in the linear case. Therefore, the proposed solutions required a completely new design perspective for reconfiguration block. The gains of the reconfiguration block are designed based on feasible solutions to a set of linear matrix inequalities, which are efficiently solvable (Algorithm 1). The feasibility of these LMIs implies a reconfigured closed-loop system that recovers input-to-state stability and setpoint tracking properties (Theorem 4). The robustness of the approach with respect to model uncertainties and non-constant disturbances in the sense of input-to-state stability (Theorem 5) was shown. Finally, we demonstrated the strength and robustness of the reconfiguration solution in the startup procedure of an example.

Experimental trials of the approach presented in this paper on a large-scale thermofluid process that has been used previously to evaluate linear fault-hiding approaches (Richter et al., 2007) are currently underway. As an outlook, the explicit consideration of actuator saturations and state constraints, which are frequently present in technical systems, is a relevant topic for future extension. A reduction of the conservatism resulting from the use of common quadratic Lyapunov functions would be desirable. However, the problem is extremely challenging as it requires a general characterisation of incremental stability properties in terms of piecewise quadratic Lyapunov functions, which is at present an open problem.

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Appendix. Proofs

Proof of Theorem 2. Noting that the function (13) is continuous, we construct a Lyapunov function $V(\bar{\boldsymbol{e}}) = \frac{1}{2}\bar{\boldsymbol{e}}^T\bar{\boldsymbol{X}}_s\bar{\boldsymbol{e}}$. Proposition 1 implies that

$$\dot{V}(\bar{\boldsymbol{e}}) = \bar{\boldsymbol{e}}^T\bar{\boldsymbol{X}}_s\dot{\bar{\boldsymbol{e}}} = \bar{\boldsymbol{e}}^T\bar{\boldsymbol{X}}_s(\bar{\boldsymbol{k}}_e(\bar{\boldsymbol{x}}+\bar{\boldsymbol{e}}) - \bar{\boldsymbol{k}}_e(\bar{\boldsymbol{x}})) \le -a\bar{\boldsymbol{e}}^T\bar{\boldsymbol{X}}_s\bar{\boldsymbol{e}}, \quad a > 0, \tag{A.1}$$

which immediately implies the inequality (27). The inequality (A.1) follows from Proposition 1 if the extended system satisfies

$$\bar{\boldsymbol{X}}_s(\bar{\boldsymbol{A}}_{e,i} - \bar{\boldsymbol{L}}\bar{\boldsymbol{C}}_f) + (\bar{\boldsymbol{A}}_{e,i} - \bar{\boldsymbol{L}}\bar{\boldsymbol{C}}_f)^T\bar{\boldsymbol{X}}_s \prec 0, \quad i = 1,\ldots,p,$$

which is equivalent to the LMIs (26) after introduction of the new variable $\bar{\boldsymbol{Y}}_s = \bar{\boldsymbol{X}}_s\bar{\boldsymbol{L}}$. □

Proof of Theorem 3. Noting that the function (19) is continuous, we construct a Lyapunov function $V((\boldsymbol{x}_\Delta^T, \boldsymbol{x}_I^T)^T) = \frac{1}{2}\begin{pmatrix}\boldsymbol{x}_\Delta^T & \boldsymbol{x}_I^T\end{pmatrix}\bar{\boldsymbol{X}}_a^{-1}\begin{pmatrix}\boldsymbol{x}_\Delta^T & \boldsymbol{x}_I^T\end{pmatrix}^T$, and use Proposition 1 to obtain the following properties of its derivative

$$\begin{aligned}\dot{V}((\boldsymbol{x}_\Delta^T, \boldsymbol{x}_I^T)^T) &= \begin{pmatrix}\boldsymbol{x}_\Delta^T & \boldsymbol{x}_I^T\end{pmatrix}\bar{\boldsymbol{X}}_a^{-1}\left(\bar{\boldsymbol{k}}_\Delta(\tilde{\boldsymbol{x}}, \boldsymbol{x}_I) - \bar{\boldsymbol{k}}_\Delta(\tilde{\boldsymbol{x}} - \boldsymbol{x}_\Delta, \boldsymbol{0})\right.\\ &\quad \left. + \begin{pmatrix}\boldsymbol{B}\boldsymbol{u}_c + \boldsymbol{L}\boldsymbol{C}_f\boldsymbol{e}\\ \boldsymbol{0}\end{pmatrix}\right)\\ &\le -a\begin{pmatrix}\boldsymbol{x}_\Delta\\ \boldsymbol{x}_I\end{pmatrix}^T\bar{\boldsymbol{X}}_a^{-1}\begin{pmatrix}\boldsymbol{x}_\Delta\\ \boldsymbol{x}_I\end{pmatrix}\\ &\quad + \begin{pmatrix}\boldsymbol{x}_\Delta\\ \boldsymbol{x}_I\end{pmatrix}^T\bar{\boldsymbol{X}}_a^{-1}\begin{pmatrix}\boldsymbol{B}\boldsymbol{u}_c + \boldsymbol{L}\boldsymbol{C}_f\boldsymbol{e}\\ \boldsymbol{0}\end{pmatrix}\end{aligned}$$

for some $a > 0$, which is readily transformed into a Lyapunov characterisation of ISS with respect to the input $(\boldsymbol{u}_c, \boldsymbol{e})$. The latter inequality follows from Proposition 1 if the extended difference system satisfies the inequality

$$\bar{\boldsymbol{X}}_a^{-1}(\bar{\boldsymbol{A}}_{a,j} - \bar{\boldsymbol{B}}_f\bar{\boldsymbol{M}}) + (\bar{\boldsymbol{A}}_{a,j} - \bar{\boldsymbol{B}}_f\bar{\boldsymbol{M}})^T\bar{\boldsymbol{X}}_a^{-1} \prec 0, \quad j = 1,\ldots,p,$$

which is equivalent to the LMIs (28) after multiplication of the LMI with $\bar{\boldsymbol{X}}_a$ from left and right and introduction of the new variable $\bar{\boldsymbol{Y}}_a = \bar{\boldsymbol{M}}\bar{\boldsymbol{X}}_a$. We have thus proven that the extended difference system (21) is ISS w.r.t. the input $(\boldsymbol{u}_c, \boldsymbol{e})$ if the given LMIs are satisfied. It remains to be proven that $\lim_{t\to\infty}\boldsymbol{C_z}\boldsymbol{x}_\Delta(t) = \boldsymbol{0}$ as $\boldsymbol{u}_c$ becomes constant in steady state. This property is proven by showing that the extended difference system (21) is exponentially convergent, and thus a constant steady-state input implies a constant steady-state solution for the extended difference system. Consider a candidate Lyapunov function V for exponential convergence:

$$V = \left((\boldsymbol{x}_{\Delta,2} - \boldsymbol{x}_{\Delta,1})^T \quad (\boldsymbol{x}_{I,2} - \boldsymbol{x}_{I,1})^T\right)\boldsymbol{P}\begin{pmatrix}\boldsymbol{x}_{\Delta,2} - \boldsymbol{x}_{\Delta,1}\\ \boldsymbol{x}_{I,2} - \boldsymbol{x}_{I,1}\end{pmatrix}.$$

Along solutions of (21) the time derivative of V satisfies

$$\begin{aligned}\dot{V} &= \left((\boldsymbol{x}_{\Delta,2} - \boldsymbol{x}_{\Delta,1})^T \quad (\boldsymbol{x}_{I,2} - \boldsymbol{x}_{I,1})^T\right)\boldsymbol{P}\cdot\left(\bar{\boldsymbol{k}}_\Delta\left(\begin{pmatrix}\tilde{\boldsymbol{x}}\\ \boldsymbol{x}_{I,2}\end{pmatrix}\right)\right.\\ &\quad \left. - \bar{\boldsymbol{k}}_\Delta\left(\begin{pmatrix}\tilde{\boldsymbol{x}}\\ \boldsymbol{x}_{I,1}\end{pmatrix}\right) - \bar{\boldsymbol{k}}_\Delta\left(\begin{pmatrix}\tilde{\boldsymbol{x}} - \boldsymbol{x}_{\Delta,2}\\ \boldsymbol{0}\end{pmatrix}\right) + \bar{\boldsymbol{k}}_\Delta\left(\begin{pmatrix}\tilde{\boldsymbol{x}} - \boldsymbol{x}_{\Delta,1}\\ \boldsymbol{0}\end{pmatrix}\right)\right)\end{aligned}$$

Using twice the fact that the function $\bar{\boldsymbol{k}}_\Delta$ satisfies the inequality (3), one obtains that there exists an $a > 0$ such that

$$\begin{aligned}\dot{V} &\le -a\left(\boldsymbol{0}^T(\boldsymbol{x}_{I,2} - \boldsymbol{x}_{I,1})^T\right)\boldsymbol{P}\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{x}_{I,2} - \boldsymbol{x}_{I,1}\end{pmatrix}\\ &\quad - a\left((\boldsymbol{x}_{\Delta,2} - \boldsymbol{x}_{\Delta,1})^T\boldsymbol{0}^T\right)\boldsymbol{P}\begin{pmatrix}\boldsymbol{x}_{\Delta,2} - \boldsymbol{x}_{\Delta,1}\\ \boldsymbol{0}\end{pmatrix}.\end{aligned} \tag{A.2}$$

Since $\boldsymbol{P} = \boldsymbol{P}^T \succ 0$, it follows that $\boldsymbol{P}$ has the structure $\bar{\boldsymbol{P}} = \begin{pmatrix}\boldsymbol{P}_{11} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{P}_{22}\end{pmatrix} \succ 0$ from the partition $\boldsymbol{P} = \begin{pmatrix}\boldsymbol{P}_{11} & \boldsymbol{P}_{12}\\ \boldsymbol{P}_{12} & \boldsymbol{P}_{22}\end{pmatrix}$. Hence, Eq. (A.2) gives

$$\dot{V} \le -a\left((\boldsymbol{x}_{\Delta,2} - \boldsymbol{x}_{\Delta,1})^T(\boldsymbol{x}_{I,2} - \boldsymbol{x}_{I,1})^T\right)\boldsymbol{P}\begin{pmatrix}\boldsymbol{x}_{\Delta,2} - \boldsymbol{x}_{\Delta,1}\\ \boldsymbol{x}_{I,2} - \boldsymbol{x}_{I,1}\end{pmatrix}.$$

Therefore, the extended difference system (21) is uniformly exponentially convergent and its solutions converge to a unique steady-state solution, which is constant if the control input is constant (Pavlov et al., 2006, Property 2.23). From Theorem 2, $\lim_{t\to\infty}\bar{\boldsymbol{e}} = \boldsymbol{0}$ for $\dot{\boldsymbol{d}} = \boldsymbol{0}$. Due to Assumption 2, $\lim_{t\to\infty}\boldsymbol{u}_c(t) = \bar{\boldsymbol{u}}_c$ is true, so $\boldsymbol{u}_c$ becomes constant in the limit and $(\boldsymbol{x}_\Delta, \boldsymbol{x}_I)$ converges to a constant steady-state solution due to Pavlov et al. (2006, Property 2.25). According to (15), $\boldsymbol{x}_I$ constant and $\boldsymbol{x}_\Delta$ constant together imply that $\boldsymbol{C_z}\boldsymbol{x}_\Delta = \boldsymbol{0}$, and it follows that $\lim_{t\to\infty}\boldsymbol{C_z}\boldsymbol{x}_\Delta(t) = \boldsymbol{0}$, as claimed. The boundedness of solutions of the extended difference system in the case of non-constant inputs is guaranteed by its ISS property. □

Proof of Theorem 4. The interconnection $(\bar{\Sigma}_e, \bar{\Sigma}_\Delta)$ is proven to be ISS with respect to the input $(\boldsymbol{u}_c, \tilde{\boldsymbol{x}}, \boldsymbol{d})$ using Theorems 2 and 3 as follows. It was shown in Theorem 2 that $\|\bar{\boldsymbol{e}}(t)\| \le ce^{-at}\|\bar{\boldsymbol{e}}(0)\|$ for $t \ge 0$. In other words, the ISS gain of the system $\bar{\Sigma}_e$ from $(\boldsymbol{x}_\Delta, \tilde{\boldsymbol{x}})$ to $\bar{\boldsymbol{e}}$ is zero. The system $\bar{\Sigma}_\Delta$ has finite ISS gain from its inputs $\boldsymbol{u}_c, \tilde{\boldsymbol{x}}$ and $\boldsymbol{e}$ to $(\boldsymbol{x}_\Delta, \boldsymbol{x}_I)$. Moreover, the IOS gain (see Jiang et al., 1994) from $\tilde{\boldsymbol{x}}$ to $\boldsymbol{x}_\Delta$ is zero. From the ISS small-gain theorem (Khalil, 2002, Theorem 5.6) and the IOS small gain theorem (Jiang et al., 1994, Theorem 2.1), it follows that the feedback interconnection $(\bar{\Sigma}_e, \bar{\Sigma}_\Delta)$ is ISS w.r.t. the input $(\boldsymbol{u}_c, \tilde{\boldsymbol{x}}, \boldsymbol{d})$, hence also IOS w.r.t. the outputs $(\boldsymbol{e}, \boldsymbol{x}_\Delta)$. An explicit proof based on elementary manipulations of comparison functions is straightforward to obtain.

Next, the ISS property for the reconfigured extended closed-loop system $(\Sigma_{Pf}, \bar{\Sigma}_S, \bar{\Sigma}_A, \Sigma_C)$ must be verified, which is graphically shown in Fig. 6. In particular, the feedback signal $\bar{\boldsymbol{e}} = (\boldsymbol{e}^T, \boldsymbol{e}_{\boldsymbol{d}}^T)^T$ exponentially converges to zero by Theorem 2. The state variable $\boldsymbol{x}_I$ in $\bar{\Sigma}_\Delta$ is not part of a feedback interconnection. However, the state observation error $\boldsymbol{e}$ and the disturbance observation error $\boldsymbol{e}_{\boldsymbol{d}}$ are in feedback interconnection with $\Sigma_{\tilde{P}}$. We note that by Theorem 2, the signal $\boldsymbol{e}_{\boldsymbol{d}}$ exponentially converges to zero for arbitrary inputs $\boldsymbol{u}_c, \tilde{\boldsymbol{x}}$. Therefore, the IOS gain of the system $(\bar{\Sigma}_e, \bar{\Sigma}_\Delta)$ from the input $(\boldsymbol{u}_c, \tilde{\boldsymbol{x}})$ to the output $(\boldsymbol{e}, \boldsymbol{e}_{\boldsymbol{d}})$ is zero and it follows from the

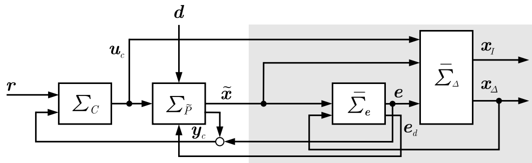

Fig. 6. Transformed extended closed-loop system (4), (6), (8), (15).

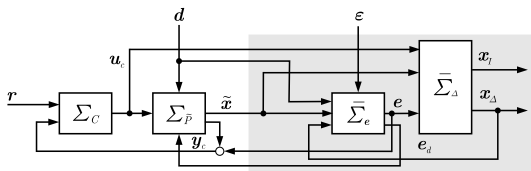

Fig. 7. Transformed extended reconfigured closed-loop system (4), (6), (8), (15) with model approximation error.

IOS small-gain theorem (Jiang et al., 1994, Theorem 2.1) and from Assumption 2 that the reconfigured closed-loop system is ISS. Note that Assumption 2 is applicable to $(\Sigma_{\tilde{P}}, \Sigma_C)$ since $\lim_{t\to\infty} \bar{\boldsymbol{e}}(t) = \boldsymbol{0}$ holds and $(\Sigma_{\tilde{P}}, \Sigma_C)$ is assumed to be ISS w.r.t. $\bar{\boldsymbol{e}}$. Therefore, it has been shown that Problem 1 is solved with respect to stability recovery.

It remains to be verified that Problem 1 is also solved with respect to setpoint tracking recovery. In Section 5.2, it has been shown that the reconfigured closed-loop system tracks constant setpoints to precision K provided that the nominal closed-loop system tracks them to this precision, and provided that the extended observation error $\bar{\boldsymbol{e}}$ vanishes and difference systems state $\boldsymbol{x}_\Delta$ seen through the output matrix $\boldsymbol{C_z}$ vanishes. According to Fig. 6, the system $(\Sigma_{\tilde{P}}, \Sigma_C)$ is governed by nominal dynamics except for the observation errors $\boldsymbol{e}$ and $\boldsymbol{e}_d$ that perturb the nominal closed-loop system in the form of intermittent measurement noise, since $\lim_{t\to\infty} \boldsymbol{e}(t) = \boldsymbol{0}$ and $\lim_{t\to\infty} \boldsymbol{e}_d(t) = \boldsymbol{0}$. Consequently, Assumption 2 also applies to the system $(\Sigma_{\tilde{P}}, \Sigma_C)$ and the complete solution to Problem 1 is provided. □

Proof of Theorem 5. Noting that the function $\bar{\boldsymbol{k}}_e$ in (13) is continuous, a Lyapunov function $V(\bar{\boldsymbol{e}}) = \frac{1}{2}\bar{\boldsymbol{e}}^T\boldsymbol{X}\bar{\boldsymbol{e}}$ is constructed for the system (35). The satisfaction of LMI (26) implies according to Proposition 1 that there exists $b > 0$ and $\theta \in (0, 1)$ such that

$$\begin{aligned}\dot{V}(\bar{\boldsymbol{e}}) &= \bar{\boldsymbol{e}}^T\boldsymbol{X}\dot{\bar{\boldsymbol{e}}} = \bar{\boldsymbol{e}}^T\boldsymbol{X}\left(\bar{\boldsymbol{k}}_e(\bar{\boldsymbol{x}}+\bar{\boldsymbol{e}}) - \bar{\boldsymbol{k}}_e(\bar{\boldsymbol{x}}) - \bar{\boldsymbol{\varepsilon}}(\boldsymbol{x}_f) - \bar{\boldsymbol{\varrho}}\right)\\ &\le -(1-\theta)b\|\bar{\boldsymbol{e}}\|^2 - \theta b\|\bar{\boldsymbol{e}}\|^2 + \|\bar{\boldsymbol{e}}\|\cdot\|\boldsymbol{X}\|E\\ &\quad + \|\bar{\boldsymbol{e}}\|\cdot\|\boldsymbol{X}\|\cdot\|\boldsymbol{\varrho}\|, \quad b > 0,\ \theta\in(0,1)\\ &\le -(1-\theta)b\|\bar{\boldsymbol{e}}\|^2 \quad \text{if } \|\bar{\boldsymbol{e}}\| > \frac{\|\boldsymbol{X}\|}{\theta b}(E+F)\end{aligned}$$

which is a Lyapunov characterisation of the ISpS property (Jiang et al., 1994). In the presence of disturbance variation, the extended observation error converges to a ball proportional in size to the bound on the disturbance variation $\left(\frac{\|\boldsymbol{X}\|}{\theta b}(E+F)\frac{\lambda_{\max}(\boldsymbol{X})}{\lambda_{\min}(\boldsymbol{X})}\right)$.

With this result for the ISpS of the observation error, the remaining proof of closed-loop ISpS follows closely the reasoning of the proof of Theorem 4, which is not repeated here. The proof is based on the observation that the model error $\boldsymbol{\varepsilon}$ only affects the observation error, but neither the difference system, nor the ISS small-gain properties of the interconnection $(\Sigma_e, \Sigma_\Delta)$ (Fig. 7).

The reduced tracking precision follows from the consideration that the observation error is bounded by a constant proportional to the model error bound E, that the bounded observation error induces a bounded difference system state whose bound is also proportional to the model error bound E, and the fact that the steady-state tracking error satisfies the relation $\limsup_{t\to\infty}\|\boldsymbol{e_z}(t)\| = \limsup_{t\to\infty}\|\boldsymbol{r}(t) - \boldsymbol{C_z}\tilde{\boldsymbol{x}}(t) + \boldsymbol{C_z}(\boldsymbol{x}_\Delta(t) + \boldsymbol{e}(t))\|$, where $\limsup_{t\to\infty}\|\boldsymbol{r}(t) - \boldsymbol{C_z}\tilde{\boldsymbol{x}}(t)\| \le K'$ from Assumption 5, $\lim_{t\to\infty}\|\boldsymbol{C_z e}(t)\| \le c(E+F)$ for $c = \|\boldsymbol{X}\|/(\theta b)$, and $\lim_{t\to\infty}\|\boldsymbol{C_z x}_\Delta(t)\| \le c\cdot d\cdot(E+F)$ where d is the ultimate gain of $\bar{\Sigma}_\Delta$ w.r.t. the input $\boldsymbol{e}$ and the output $\boldsymbol{x}_\Delta$, and therefore $\limsup_{t\to\infty}\|\boldsymbol{e_z}(t)\| \le K' + c\cdot d\cdot(E+F)$. Note that $\boldsymbol{e}$ and $\boldsymbol{e_d}$ act as persistent measurement disturbances on the system $\Sigma_{\tilde{P}}$. □

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