51st IEEE Conference on Decision and Control
December 10-13, 2012. Maui, Hawaii, USA

String Stability of Interconnected Vehicles Under Communication Constraints

Sinan Öncü, Nathan van de Wouw, W. P. Maurice H. Heemels and Henk Nijmeijer

***Abstract*— In this paper, we present a novel modelling and string stability analysis method for an interconnected vehicle string in which information exchange takes place via wireless communication. The usage of wireless communication introduces time-varying sampling intervals, delays, and communication constraints of which the impact on string stability requires a careful analysis. In particular, we study a Cooperative Adaptive Cruise Control (CACC) system which regulates inter-vehicle distances in a vehicle string and utilizes information exchange between vehicles through wireless communication in addition to local sensor measurements. The propagation of disturbances through the interconnected vehicle string is inspected by using the notion of so-called *string stability* which is formulated here in terms of an $\mathscr{L}_2$-gain requirement from disturbance inputs to controlled outputs. This paper provides conditions on the uncertain sampling intervals and delays under which string stability can still be guaranteed. These results support the design of CACC systems that are robust to uncertainties introduced by wireless communication.**

I. Introduction

The ever increasing demand for mobility in today's life brings additional burden on the existing ground transportation infrastructure for which a feasible solution in the near future lies in the more efficient use of currently available means of transportation. For this purpose, the development of Intelligent Transportation Systems (ITS) technologies that contribute to improved traffic flow stability, throughput and safety are needed. In particular, Cooperative Adaptive Cruise Control (CACC), which is an extension of the currently available Adaptive Cruise Control (ACC) technology with the addition of information exchange between vehicles through wireless Vehicle-to-Vehicle (V2V) and Vehicle-to-Infrastructure (V2I) communication, seems to be a promising solution. Wireless information exchange between vehicles provides means of overcoming sensory limitations of human or ACC operated vehicles and, therefore, can contribute significantly to improving the traffic flow, especially on highways [1].

In this paper, we approach the problem of regulating inter-vehicle distances in a CACC system from a Networked Control Systems (NCS) perspective [3], [4]. In the scope of CACC, control over a wireless communication network is the enabling technology that makes CACC realizable; however, very few studies of CACC consider the imperfections that are introduced by the network [2], [5], [6], [17]. This is mainly due to the fact that systematic tools for the modelling and analysis of NCS arose relatively recently. In [2], a continuous-time transfer function analysis of constant time delays was carried out which considers a time slotted token-passing type network where each vehicle is assumed to transmit within evenly separated and fixed time intervals. Additionaly, they assumed that all shared information is implemented at the same time. Here, we allow for uncertain and time-varying sampling/transmission intervals and communication delays and obtain bounds on maximum allowable transmission intervals (MATI) and maximum allowable delays (MAD) while string stability is still guaranteed.

Most works on NCS focus on the stability analysis of closed-loop NCS consisting of a plant and controller interconnected by a wireless network [4], [7], [8], [9], [10], [11], [12]. Relatively few works consider sensitivity of NCS to perturbations, see e.g. [13], [15]. In the context of CACC, this is highly relevant as firstly, wireless communications take place between controlled vehicles and, secondly, string stability relates to the attenuation of disturbances along the string. Therefore, we employ $\mathscr{L}_p$-stability results for NCS developed in [13] using an emulation-based approach and cast the interconnected vehicle string dynamics in a form amendable for such analysis. As such, the main contributions of this paper are the development of a novel modelling method encompassing all these important issues, and the analysis of string stability for vehicle platooning while taking into account the effect of time-varying transmission intervals and delays. The framework is set up in a general manner such that the inclusion of scheduling constraints induced by the wireless communication between vehicles can be envisioned as well.

II. Interconnected Vehicle String Model

The general objective of a Cooperative Adaptive Cruise Control (CACC) system is to pack the driving vehicles together as tightly as possible in order to increase traffic flow while preventing amplification of disturbances throughout the string, which is known as string instability [16], [19], [18]. Some other equally important requirements related to safety, comfort, fuel consumption, etc., are outside the scope of the present work.

A. Vehicle Following Objective

To model a wirelessly interconnected vehicle platoon, we have to realize that the vehicles forming the platoon are 'interconnected' through the vehicle following objective, as implemented through CACC. Each vehicle is controlled to follow its predecessor while maintaining a desired but not necessarily constant distance. Here, we consider a constant time headway spacing policy where the desired spacing

This work was supported by HTAS Connect & Drive Project, the Innovational Research Incentives Scheme under the VICI grant Wireless control systems: A new frontier in automation (No. 11382) awarded by NWO (The Netherlands Organization for Scientific Research) and STW (Dutch Science Foundation), and the European 7th Framework Network of Excellence: Highly-complex and networked control systems (HYCON2) (grant agreement no. 257462).

S. Öncü, N. van de Wouw, W.P.M.H. Heemels and H. Nijmeijer are with the Department of Mechanical Engineering, Eindhoven University of Technology, 5600 MB Eindhoven, The Netherlands. s.oncu@tue.nl; n.v.d.wouw@tue.nl; m.heemels@tue.nl; h.nijmeijer@tue.nl

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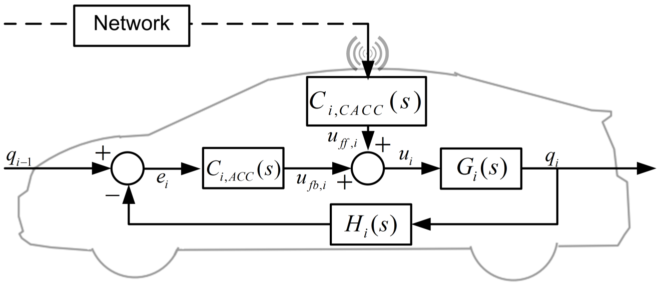


Fig. 1. Control structure block diagram of a single CACC equipped vehicle.

($d_{r,i}$) between the front bumper of the i-th vehicle to its predecessor's (vehicle $i-1$) rear bumper is given by

$$d_{r,i} = r_i + h_{d,i} v_i, \tag{1}$$

where i is the vehicle index and r_i is a constant term that forms the desired gap between consecutive vehicles at standstill, $h_{d,i}$ is the headway-time constant representing the time that it will take the i-th vehicle to arrive at the same position as its predecessor when $r_i = 0$ and v_i is the vehicle velocity. Without loss of generality, $r_i = 0$ is adopted in this paper since it does not affect the dynamics of the platoon in the scope of this work. For similar reasons, the car length (L_i) will also be taken as zero. The actual distance (d_i) between two consecutive vehicles i and $i-1$ is then given by

$$d_i = q_{i-1} - (q_i + L_i) = q_{i-1} - q_i, \tag{2}$$

where q_i is the absolute position of i-th vehicle in global coordinates. The local vehicle following control objective can now be defined as regulating the error

$$\begin{aligned} e_i &= d_i - d_{r,i} \\ &= q_{i-1} - q_i - h_{d,i} v_i \end{aligned} \tag{3}$$

to zero.

B. CACC Control Structure

For the i-th vehicle in the string, the longitudinal dynamics is given as

$$\dot{a}_i = -\eta_i^{-1} a_i + \eta_i^{-1} u_i, \tag{4}$$

where a_i is the longitudinal acceleration, η_i represents the internal actuator dynamics and u_i is the desired acceleration for the i-th vehicle. The control structure for a single CACC equipped vehicle (vehicle i) is as shown in Fig. 1. CACC operation is introduced in a feedforward fashion as an addition to the underlying ACC. The total control command for the i-th vehicle, $u_i = u_{fb,i} + u_{ff,i}$, consists of feedback ($u_{fb,i}$) and feedforward ($u_{ff,i}$) components. The signal conditioning block, $H_i(s) = 1 + h_{d,i}s$, is used to implement the spacing strategy given in (1). The feedback controller, $C_{i,ACC}(s)$ that constitutes the ACC part is a PD-type controller that acts on locally sensed data (e.g. using radar) to achieve the vehicle following objective and is given as

$$\begin{aligned} u_{fb,i} &= k_{p,i} e_i + k_{d,i} \dot{e}_i, \\ &= k_{p,i} e_i + k_{d,i}(v_{i-1} - v_i - h_{d,i} a_i), \end{aligned} \tag{5}$$

where $k_{p,i}$ and $k_{d,i}$ are respectively the proportional and derivative gains of the ACC controller. The desired acceleration of the directly preceding vehicle (u_{i-1}) is used in a feedforward fashion to improve tracking performance and forms the CACC part of the controller ($C_{i,CACC}(s)$ in Fig. 1). Additional dynamics is introduced in the controller due to the velocity-dependent spacing policy in (1), which gives the following additional differential equation for the CACC feedforward filter

$$\dot{u}_{ff,i} = -h_{d,i}^{-1} u_{ff,i} + h_{d,i}^{-1} u_{i-1}, \tag{6}$$

to be used in the state-space representation of the CACC vehicle model presented next. For more details on the control structure, we refer the interested reader to [17], [18], [19].

C. Closed-Loop CACC Model

The general form of the closed-loop CACC vehicle model is then obtained by combining the longitudinal dynamics in (4) with the distance error equation in (3), the feedback control law (5), and the feedforward control law (6) with u_{i-1} replaced by $\hat{u}_{i-1}$, where the notation $\hat{u}_{i-1}$ is used to denote that u_{i-1} is transmitted over the network. Note that $\hat{u}_{i-1}$ typically differs from u_{i-1} due to network-introduced effects (sample-and-hold, delays, communication constraints, etc.). By choosing the state variables as $x_i^T = [e_i \; v_i \; a_i \; u_{ff,i}] \in \mathbb{R}^{n_x}$, the i-th CACC equipped vehicle dynamics in an n-vehicle string is described by

$$\begin{aligned} \dot{x}_i &= A_{i,i} x_i + A_{i,i-1} x_{i-1} + \underbrace{B_{s,i} u_i}_{ACCpart} + \underbrace{B_{c,i} \hat{u}_{i-1}}_{CACCpart}, \\ A_{i,i} &= \begin{bmatrix} 0 & -1 & -h_{d,i} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\eta_i^{-1} & 0 \\ 0 & 0 & 0 & -h_{d,i}^{-1} \end{bmatrix}, B_{s,i} = \begin{bmatrix} 0 \\ 0 \\ \eta_i^{-1} \\ 0 \end{bmatrix}, \\ A_{i,i-1} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B_{c,i} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ h_{d,i}^{-1} \end{bmatrix}, \end{aligned} \tag{7}$$

for $1 \le i \le n$ and where $B_{s,i}$ is the input vector corresponding to the input u_i which is generated by using locally available (sensed) data and $B_{c,i}$ is the input vector for the additional CACC input $\hat{u}_{i-1}$. A time-domain representation of the total feedback/feedforward control input with the given spacing policy is given by

$$\begin{aligned} u_i &= u_{fb,i} + u_{ff,i}, \qquad 1 \le i \le n, \\ &= K_{i,i-1} x_{i-1} + K_{i,i} x_i, \end{aligned} \tag{8}$$

$$K_{i,i-1} = \begin{bmatrix} 0 \\ k_{d,i} \\ 0 \\ 0 \end{bmatrix}^T, K_{i,i} = \begin{bmatrix} k_{p,i} \\ -k_{d,i} \\ -k_{d,i} h_{d,i} \\ 1 \end{bmatrix}^T.$$

A reference vehicle (denoted by index $i = 0$ and with state x_0) is introduced, which may either represent the rest of the traffic as seen by the lead vehicle (with index $i = 1$) in the string or a trajectory generator in the lead vehicle in case there are no preceding vehicles and is described by

$$\begin{aligned} \dot{x}_0 &= A_0 x_0 + B_{s,0} u_r, \\ A_0 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\eta_0^{-1} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \; B_0 = \begin{bmatrix} 0 \\ 0 \\ \eta_0^{-1} \\ 0 \end{bmatrix}, \end{aligned} \tag{9}$$

where $x_0^T = [e_0^T, \, v_0^T, \, a_0^T, \, u_{ff,0}^T]^T$, and u_r is the reference acceleration profile. In (9), state variables are chosen in

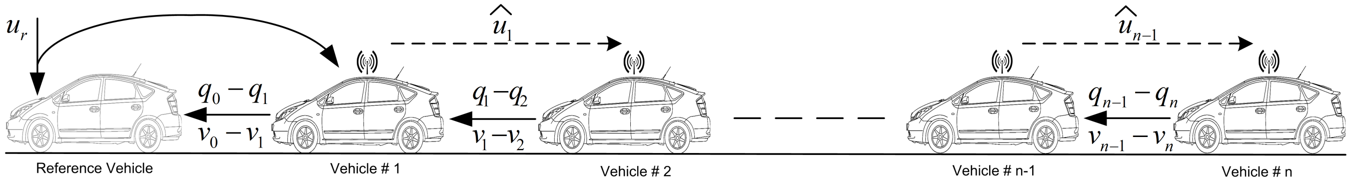


Fig. 2. Schematic representation of the n-vehicle string.

accordance with the real vehicles in the string and, therefore, redundant states exist. However, this choice results in a uniform representation for the upcoming vehicle string model. Also, the lead vehicle (with state x_1) in the string requires special consideration. The CACC input is locally available to this vehicle without any network-induced imperfection since it is generated locally by this vehicle and, therefore, $\hat{u}_0 = u_r$.

By considering these two special cases for the reference and the lead vehicles and using the CACC vehicle model in (7) for each operational CACC subsystem, an n-vehicle string as in Fig. 2 is modeled by collecting n subsystems together with the reference model to form the new state vector $\bar{x}_n = [x_0^T \; x_1^T \; x_2^T \; \cdots \; x_n^T]^T$ and use the input of the reference vehicle model (u_r) as the exogenous input to the cascaded system which can now be represented as

$$\dot{\bar{x}}_n = \bar{A}_n \bar{x}_n + \bar{B}_{s,n} \bar{u}_n + \bar{B}_{c,n} \hat{\bar{u}}_n + B_r u_r \tag{10}$$

with

$$\bar{A}_n = \begin{bmatrix} A_0 & 0 & 0 & \cdots & 0 \\ A_{1,0} & A_{1,1} & 0 & \cdots & 0 \\ 0 & A_{2,1} & A_{2,2} & \cdots & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & A_{n,n-1} & A_{n,n} \end{bmatrix},$$

$$\bar{B}_{s,n} = \begin{bmatrix} 0 & \cdots & \cdots & 0 \\ B_{s,1} & 0 & \cdots & 0 \\ 0 & B_{s,2} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & B_{s,n} \end{bmatrix}, B_r = \begin{bmatrix} B_{s,0} \\ B_{c,1} \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\bar{B}_{c,n} = \begin{bmatrix} 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & 0 \\ B_{c,2} & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & B_{c,n} & 0 \end{bmatrix},$$

where $\bar{u}_n = [u_1 \; u_2 \; \ldots \; u_n]^T$ and $\hat{\bar{u}}_n = [\hat{u}_1 \; \hat{u}_2 \; \ldots \; \hat{u}_n]^T$. The local ACC control defining $\bar{u}_n$ can be incorporated into (10) by using (8), i.e. $u_i = [K_{i,i-1} \; K_{i,i}][x_{i-1}^T \; x_i^T]^T$, for each sub-system $i \in \{1, 2, \ldots, n\}$, to obtain

$$\bar{u}_n = \begin{bmatrix} u_1^T & u_2^T & \cdots & u_n^T \end{bmatrix}^T = \bar{K}_n \bar{x}_n, \tag{11}$$

with

$$\bar{K}_n = \begin{bmatrix} K_{1,0} & K_{1,1} & 0 & \cdots & 0 \\ 0 & K_{2,1} & K_{2,2} & \cdots & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ 0 & \cdots & & K_{n,n-1} & K_{n,n} \end{bmatrix}. \tag{12}$$

In this interconnected vehicle string model, the wirelessly communicated control commands in $\hat{\bar{u}}_n$ are kept separate for future analysis.

As a special case, the Network-free(NF)-CACC model is obtained by assuming the CACC control inputs are perfectly transmitted (i.e. no network effects) and therefore, $\hat{\bar{u}}_n = \bar{u}_n = \bar{K}_n \bar{x}_n$ can also be incorporated in (10) to yield

$$\begin{aligned} \dot{\bar{x}}_n &= (\bar{A}_n + (\bar{B}_{s,n} + \bar{B}_{c,n})\bar{K}_n)\bar{x}_n + B_r u_r, \\ &= A_{\bar{x}_n}^{NF} \bar{x}_n + B_r u_r. \end{aligned} \tag{13}$$

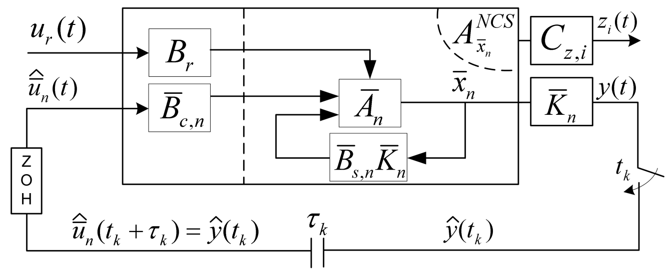


Fig. 3. NCS model.

The NF-CACC model will be used to perform analyses which will serve as a reference for the evaluation of the networked system performance.

III. CACC NCS Model

In the previous section, the interconnected vehicle string was formulated such that the control inputs (namely, $\bar{u}_n$ for ACC, and $\hat{\bar{u}}_n$ for CACC) are kept separate according to their way of being acquired by the host vehicle (i.e. through direct measurement or through wireless communication). Also, the model permits to express the CACC control commands that are actually feedforward signals as state feedback control laws. Now, by adopting the realistic assumption that a much higher sampling rate is employed for the locally sensed data that is used for the ACC functionality, we can consider the ACC vehicle following controller as inherently continuous-time dynamic coupling between vehicles. Continuous-time plant and controller equations for the NCS setup depicted in Fig. 3 can be obtained by substituting (11) into (10):

$$\begin{aligned} \dot{\bar{x}}_n &= A_{\bar{x}_n}^{NCS} \bar{x}_n + \bar{B}_{c,n} \hat{\bar{u}}_n + B_r u_r, \\ y &= \bar{u}_n = \bar{K}_n \bar{x}_n, \\ \hat{\bar{u}}_n &= \hat{y}, \end{aligned} \tag{14}$$

where $A_{\bar{x}_n}^{NCS} = \bar{A}_n + \bar{B}_{s,n} \bar{K}_n$, $y \in \mathbb{R}^{n_y}$ is the output of the plant, and $u_r \in \mathbb{R}^{n_r}$ is the exogenous input.

At each transmission instant $t_k, k \in \mathbb{N}$, CACC control commands are generated by using the sampled measurement data which are subsequently sent over the network. They arrive at the controller after a transmission delay of τ_k. Therefore, the controller updates occur at $t_k + \tau_k$ and the control input is implemented through a zero-order-hold (ZOH). The difference between the implemented piecewise continuous control command ($\hat{\bar{u}}_n$) and the actual CACC control command ($\bar{u}_n$) is captured as the error introduced by the network and is defined as

$$e_u := \hat{\bar{u}}_n - \bar{u}_n. \tag{15}$$

In between the control command updates, the network operates in a zero-order-hold (ZOH) fashion and, therefore,

$$\dot{\hat{\bar{u}}}_n = 0. \tag{16}$$

Now, the NCS model dynamics in between the control command updates can be written in terms of plant and the error states by using (15) and (16) in (14) as follows:

$$\begin{aligned}\dot{\bar{x}}_n &= f(\bar{x}_n, e_u, u_r) := A_{11}\bar{x}_n + A_{12}e_u + A_{13}u_r, &\text{(17a)}\\ \dot{e}_u &= g(\bar{x}_n, e_u, u_r) := A_{21}\bar{x}_n + A_{22}e_u + A_{23}u_r, &\text{(17b)}\end{aligned}$$

with

$$\begin{aligned}&A_{11} = A^{NCS}_{\bar{x}_n} + \bar{B}_{c,n}\bar{K}_n, && A_{12} = \bar{B}_{c,n},\\ &A_{13} = B_r, && A_{21} = -\bar{K}_n(A^{NCS}_{\bar{x}_n} + \bar{B}_{c,n}\bar{K}_n),\\ &A_{22} = -\bar{K}_n\bar{B}_{c,n}, && A_{23} = -\bar{K}_n B_r.\end{aligned} \tag{18}$$

At update instants $t_k + \tau_k$, the error is reset according to

$$\begin{aligned}e_u((t_k+\tau_k)^+) &= \hat{\bar{u}}_n((t_k+\tau_k)^+) - \bar{u}_n((t_k+\tau_k)),\\ &= \bar{u}_n(t_k) + h(k, e_u(t_k)) - \bar{u}_n(t_k+\tau_k),\\ &= h(k, e_u(t_k)) - e_u(t_k) + e_u(t_k+\tau_k),\end{aligned} \tag{19}$$

where $h(k, e_u(t_k))$ is related to the protocol that is employed and determines which node gets access to the network at each transmission instant, see [13], [14] for more details. In this work, we consider the sampled-data (SD) protocol with $h(k, e_u(t_k)) = 0$ in (19), although recently we have shown that the presented framework can be used also to study Round Robin (RR) type of protocols for CACC applications in a similar way. The NCS model is transformed as in [13] into the hybrid system framework as developed in [20] for the upcoming stability and performance analysis. For this purpose, we introduce the auxiliary variables $s \in \mathbb{R}^n$, $\kappa \in \mathbb{N}$, $\tau \in \mathbb{R}_{\geq 0}$ and $\ell \in \{0, 1\}$ to reformulate the model in terms of so-called flow equations and reset equations. The variable s is used to store the error value (e_u) according to (19) at the last transmission instant to be used at the next control command update instant, κ is the transmission counter, τ is a timer, and l is a boolean logic operator that determines whether the next event in the hybrid system will be a transmission or an update reset. The hybrid system $\mathcal{H}_{NCS}$ is now given by the flow equations

$$\begin{aligned}\dot{\bar{x}}_n &= f(\bar{x}_n, e_u, u_r),\\ \dot{e}_u &= g(\bar{x}_n, e_u, u_r),\\ \dot{s} &= 0, \quad \dot{\tau} = 1,\\ \dot{\kappa} &= 0, \quad \dot{\ell} = 0,\end{aligned} \tag{20}$$

when ($\ell = 0 \wedge \tau \in [0, \tau_{mati}]$) or ($\ell = 1 \wedge \tau \in [0, \tau_{mad}]$) where $\tau_{mati} \geq t_{k+1} - t_k, k \in \mathbb{N}$, is the maximum allowable transmission interval (MATI) and $\tau_{mad} \leq \tau_{mati}$ is the maximum allowable delay (MAD), hence $\tau_k \leq \min(\tau_{mad}, t_{k+1} - t_k), k \in \mathbb{N}$. This condition implies that an update will occur before the next transmission instant. Transmission ($\ell = 0$) and update ($\ell = 1$) reset equations are given, respectively, as

$$\begin{aligned}(\bar{x}^+_n, e^+_u, s^+, \tau^+, \kappa^+, l^+) &= (\bar{x}_n, e_u, h(k, e_u) - e_u, 0, \kappa+1, 1),\\ (\bar{x}^+_n, e^+_u, s^+, \tau^+, \kappa^+, l^+) &= (\bar{x}_n, s + e_u, -s - e_u, \tau, \kappa, 0).\end{aligned} \tag{21}$$

For more details on this NCS hybrid system formulation, see [13].

IV. Problem Formulation and Analysis Approach

An important requirement in a CACC system is to avoid amplification of disturbances throughout the string as the vehicle index increases. For the evaluation of string stability, one considers the amplification of the distance error, the velocity, the acceleration or the control effort along the vehicle string [16], [19], [18].

A. Problem Formulation

The CACC NCS model allows us to inspect the influence of the exogenous input u_r on a particular controlled output

$$z_i = q_i(\bar{x}_n), \tag{22}$$

in terms of an induced $\mathscr{L}_p$-gain. The hybrid model $\mathcal{H}_{NCS}$ expanded with the output (22) is denoted by $\mathcal{H}^z_{NCS}$.

Definition 1: [13] Consider $p \in \mathbb{N}$ with $p \geq 1$ and let $\theta \geq 0$ be given. The hybrid system $\mathcal{H}^z_{NCS}$ is said to be $\mathscr{L}_p$-stable with gain smaller than or equal to θ, if there is a $\mathscr{K}_\infty$-function S such that for any $0 < \delta \leq \tau_{mati}$, any exogenous input $u_r \in \mathscr{L}_p$, and any initial condition, each corresponding solution to $\mathcal{H}^z_{NCS}$ satisfies

$$\|z_i\|_p \leq S(|\xi(0)|) + \theta \|u_r\|_p, \tag{23}$$

where $\xi = (\bar{x}^T_n, e^T_u, s^T, \kappa, \tau, \ell)^T$ denotes the state of the hybrid system (20),(21).

Problem 1: Given the plant and controller in (14) which was designed without the consideration of the network effects (i.e. $\hat{\bar{u}}_n = \bar{u}_n$), determine values of τ_{mati} and τ_{mad} so that the CACC NCS model $\mathcal{H}^z_{NCS}$ still has a guaranteed $\mathscr{L}_p$-gain (i.e. θ in (23)).

In this paper, we consider the propagation of the control effort, (i.e. $u_i, i \in \{2, \cdots, n\}$) as the particular output of interest and require $\mathscr{L}_2$-gain from u_r to u_i to be less than or equal to one ($\theta \leq 1$) to guarantee string stability. Control commands of individual vehicles can be selected by using $z_i = q_i(\bar{x}_n) = C_{z,i}\bar{x}_n$ accordingly in (22).

B. Stability and Performance Analysis

An $\mathscr{L}_2$-gain analysis of a hybrid system requires conditions on the flow (20) and jumps (21) during resets.

1) Conditions on Resets: We assume that there exists a Lyapunov function $W : \mathbb{N} \times \mathbb{R}^{n_e} \to \mathbb{R}_{\geq 0}$ which satisfies

$$\begin{aligned}\underline{\alpha}_W |e_u| \leq W(\kappa, e_u) &\leq \overline{\alpha}_W |e_u|, &\text{(24a)}\\ W(\kappa+1, h(\kappa, e_u)) &\leq \lambda W(\kappa, e_u), &\text{(24b)}\end{aligned}$$

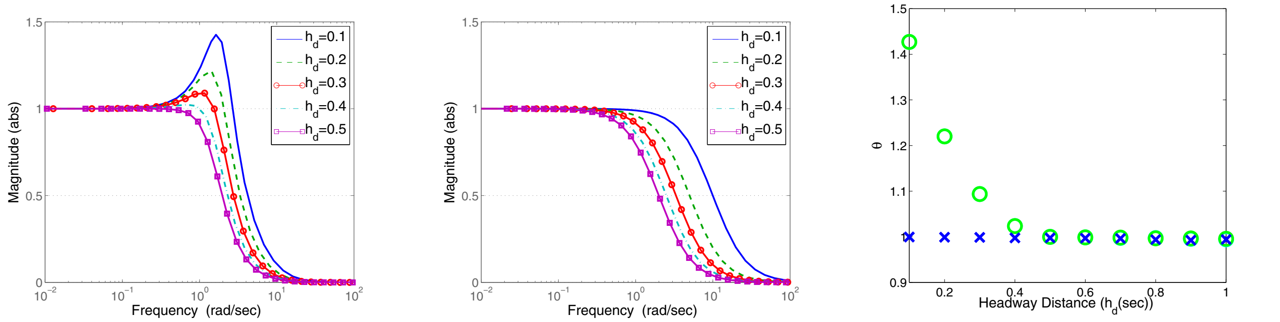
for constants $0 < \underline{\alpha}_W \leq \overline{\alpha}_W$ and $0 < \lambda < 1$. Additionally, it is assumed that

$$W(\kappa+1, e_u) \leq \lambda_W W(\kappa, e_u), \tag{25}$$

for some constant $\lambda_W \geq 1$ for allmost all $e_u \in \mathbb{R}^{n_e}$ and all $\kappa \in \mathbb{N}$. Moreover, it is assumed that

$$\left|\frac{\partial W}{\partial e_u}(\kappa, e_u)\right| \leq M_1, \tag{26}$$

for some constant $M_1 > 0$. For the SD protocol considered here there is a $W : \mathbb{N} \times \mathbb{R}^{n_e} \to \mathbb{R}_{\geq 0}$ that is locally Lipschitz in its second argument and satisfies (24a), (25), (26) with $\underline{\alpha}_W = \overline{\alpha}_W = \lambda_W = M_1 = 1$ and (24b) for any $\lambda \in (0, 1)$.



(a) $\frac{u_2(s)}{u_r(s)}$ for an ACC string.

(b) $\frac{u_2(s)}{u_r(s)}$ for a CACC string.

(c) $\mathscr{L}_2$-gain (θ) from u_r to u_2 for ACC (o) and CACC (x).

Fig. 4. String stability analysis of the Network-Free (C)-ACC vehicle string for different headway-times (h_d).

2) Conditions on Flow: The following growth condition on the flow of the NCS model (17) is used:

$$|g(\bar{x}_n, e_u, u_r)| \leq m_{\bar{x}_n}(\bar{x}_n, u_r) + M_{e_u}|e_u|, \tag{27}$$

where $m_{\bar{x}_n} : \mathbb{R}^{n_x} \times \mathbb{R}^{n_r} \to \mathbb{R}_{\geq 0}$ and $M_{e_u} \geq 0$ is a constant, and, additionally, it is assumed that a storage function $V : \mathbb{R}^{n_x} \to \mathbb{R}_{\geq 0}$ exists which satisfies the condition

$$\begin{aligned}\langle \nabla V(\bar{x}_n), f(\bar{x}_n, e_u, u_r)\rangle \leq -m^2_{\bar{x}_n}(\bar{x}_n, u_r) + \gamma^2 W^2(\kappa, e_u)\\ + \mu(\theta^2|u_r|^2 - |z_i|^2),\end{aligned} \tag{28}$$

for all $i \in \{1, 2, \ldots, n\}$, where $f(\bar{x}_n, e_u, u_r)$ is as in (17a), with $\mu > 0, \gamma > 0$ as constants, and the bounds

$$\underline{\alpha}_V(|\bar{x}_n|) \leq V(\bar{x}_n) \leq \overline{\alpha}_V(|\bar{x}_n|), \tag{29}$$

for some $\mathscr{K}_\infty$-functions $\underline{\alpha}_V$ and $\overline{\alpha}_V$. Essentially, the condition above is a (slightly extended version of) dissipativity-based formulation for the system $\dot{\bar{x}}_n = f(\bar{x}_n, e_u, u_r)$ to have an $\mathscr{L}_2$-gain smaller than or equal to θ between the exogenous input u_r and the output z_i as in (22). Consider now the differential equations

$$\dot{\phi}_0 = -2L_0\phi_0 - \gamma_0(\phi_0^2 + 1), \tag{30a}$$

$$\dot{\phi}_1 = -2L_1\phi_1 - \gamma_0(\phi_1^2 + \frac{\gamma_1^2}{\gamma_0^2}), \tag{30b}$$

where $L_\ell \geq 0$ and $\gamma_\ell > 0$, $\ell = 0, 1$, are the real constants

$$L_0 = \frac{M_1 M_e}{\underline{\alpha}_W};\ L_1 = \frac{M_1 M_e \lambda_W}{\lambda \underline{\alpha}_W};\ \gamma_0 = M_1\gamma;\ \gamma_1 = \frac{M_1\gamma\lambda_W}{\lambda}. \tag{31}$$

Based on these conditions, we can formulate the following theorem guaranteeing upper bounds on $\mathscr{L}_2$-gain of $\mathcal{H}^z_{NCS}$.

Theorem 4.1: [13] Consider the system $\mathcal{H}^z_{NCS}$ that satisfies the aforementioned conditions. Suppose $\tau_{mati} \geq \tau_{mad} \geq 0$ satisfy

$$\phi_0(\tau) \geq \lambda^2\phi_1(0) \text{ for all } 0 \leq \tau \leq \tau_{mati} \tag{32a}$$

$$\phi_1(\tau) \geq \phi_0(\tau) \text{ for all } 0 \leq \tau \leq \tau_{mad} \tag{32b}$$

for solutions ϕ_0 and ϕ_1 of (30) corresponding to certain chosen initial conditions $\phi_\ell(0) > 0$, $\ell = 0, 1$, with $\phi_1(0) \geq \phi_0(0) \geq \lambda^2\phi_1(0) \geq 0$, $\phi_0(\tau_{mati}) > 0$ and λ as in (24b). Then, the system $\mathcal{H}^z_{NCS}$ is $\mathscr{L}_2$-stable with gain θ.

By using a numerical search algorithm, quantitative numbers for τ_{mati} and τ_{mad} can be obtained with the help of the above theorem by constructing the solutions to (30) for various initial conditions. Computing the τ value of the intersection of ϕ_0 and the constant line $\lambda^2\phi_1(0)$ provides τ_{mati} according to (32a), while the intersection of ϕ_0 and ϕ_1 gives a value for τ_{mad} due to (32b). Different values of the initial conditions $\phi_0(0)$ and $\phi_1(0)$ lead to different solutions of the differential equations in (30), and thus, to different storage functions in (29). In this way, tradeoff curves between τ_{mati} and τ_{mad} can be obtained that indicate when $\mathscr{L}_2$-stability of the NCS is still guaranteed with gain θ.

V. String Stability Analysis

A. Network-Free CACC String Stability Analysis

The NF-CACC model in (13) will be used to perform analyses which will serve as a reference for the evaluation of string stability of the networked system.

The control system (13) has an $\mathscr{L}_2$-gain from u_r to output $z_i = C_{z,i}\bar{x}_n$ less than or equal to θ if there exists a positive definite and proper storage function V such that the dissipation inequality

$$\langle \nabla V(\bar{x}_n), f(\bar{x}_n, 0, u_r)\rangle \leq \theta^2 \left\|u_r\right\|^2 - \left\|C_{z,i}\bar{x}_n\right\|^2, \tag{33}$$

is satisfied, and the string stability condition requires $\theta \leq 1$. Note that for the linear system (13), the $\mathscr{L}_2$-gain of the system according to (33) equals the $\mathcal{H}_\infty$-norm $\left\|H_{C_{z,i}}\right\|_\infty = \sup_{\omega\in\mathbb{R}}\left\|H_{C_{z,i}}(j\omega)\right\|$ of the transfer function

$$H_{C_{z,i}}(s) = C_{z,i}(sI - A^{NF}_{\bar{x}_n})^{-1}B_r. \tag{34}$$

Therefore, the string stability requirement can also be interpreted as a condition on the maximal amplification of the corresponding LTI system (13) to a sinusoidal input, i.e.,

$$|H_{C_{z,i}}(jw)| \leq 1, \forall\omega,\ \ i \geq 1. \tag{35}$$

To verify (33) a quadratic storage function $V(\bar{x}_n) = \bar{x}_n^T P \bar{x}_n$ was chosen to compute the $\mathscr{L}_2$-gain from u_r to u_i (e.g. $C_{z,2} = [0_{1\times4}\ K_{2,1}\ K_{2,2}\ 0_{1\times4} \ldots 0_{1\times4}]_{1\times n_x(n+1)}$ for $i = 2$) and we minimize θ subject to the LMIs

$$\begin{pmatrix} (A^{NF}_{\bar{x}_n})^T P + P A^{NF}_{\bar{x}_n} + C^T_{z,i}C_{z,i} & PB_r \\ B_r^T P & -\theta^2 I \end{pmatrix} \preceq 0,\ \ P \succ 0. \tag{36}$$

This analysis has been performed for various headway-time values which yield the results presented in Fig. 4c for $n = 2$. The analysis shows that the response of the ACC string satisfies the condition (36) with $\theta = 1$ only for relatively large headway-time values (larger than 0.5 sec), whereas the NF(ideal)-CACC system is string stable according to (35) for all headway-time values that were considered. Also, the $\mathscr{L}_2$-gains for corresponding headway-time values equal the peak amplitude values of corresponding Bode plots in

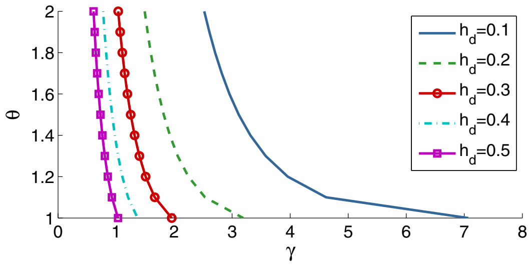


Fig. 5. Tradeoff curves for θ and γ with different headway-times (h_d).

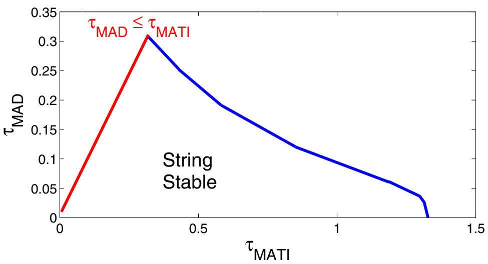


Fig. 6. String stable solution region for a fixed headway-time ($h_d = 0.5$).

Fig. 4a-b due to the relation between the $\mathscr{L}_2$-gain and the $\mathcal{H}_\infty$-norm. This analysis sets the basis for the evaluation of the CACC NCS model $\mathcal{H}^z_{NCS}$ by providing the ideally achievable (no network) $\mathscr{L}_2$-gain in (28) and shows that θ is lower bounded by $\theta^* = 1$. This lower bound of 1 on θ is a direct consequence of the vehicle following objective (achieved by the proposed (C)ACC controller).

B. MATI-MAD Analysis for Sampled-Data Setting

In the networked CACC setting considered here, we assume that a single node samples and transmits all vehicle data synchronously in the network (i.e. the protocol function $h = 0$ in (19)) corresponding to the SD protocol. In practice, this implies a synchronised sampling and transmission of the wirelessly transmitted measurements, which could be implemented using GPS-based clock-synchronisation. We note that the framework set up here can also be applied to more general Round-Robin protocols, for which conditions (24-26) can also be shown to hold [13].

The flow conditions in (28) are checked for the system H^z_{NCS} for the controlled output $z_i = u_i = C_{z,i}\bar{x}_n$ by using a quadratic storage function $V(\bar{x}_n) = \bar{x}_n^T P \bar{x}_n$, and taking $m(\bar{x}_n, u_r) = |A_{21}\bar{x}_n + A_{23}u_r|$, and $W(\kappa, e_u) = |e_u|$. This leads to the following LMIs:

$$\begin{pmatrix} \Omega_i & PA_{12} & A_{21}^T A_{23} + PA_{13} \\ A_{12}^T P & -\gamma^2 I & 0 \\ A_{13}^T P + A_{23}^T A_{21} & 0 & A_{23}^T A_{23} - \mu\theta^2 I \end{pmatrix} \preceq 0, P \succ 0, \tag{37}$$

where $\Omega_i = A_{11}^T P + PA_{11} + A_{21}^T A_{21} + \mu C_{z,i}^T C_{z,i}$. These LMIs are solved for $n = 2$ to obtain tradeoff curves between the $\mathscr{L}_2$-gain θ and γ in (28) with the controlled output $z_2 = u_2 = C_{z,2}\bar{x}_n$ for different headway-time constants (h_d) as presented in Fig. 5. Now, by selecting the string stable pairs $(\theta, \gamma) = (1, \gamma^*)$ derived from the results in Fig. 5, we can obtain the constants given in (31) that are used to solve the differential equations (30). Finally, from Theorem 4.1, quantitative numbers for τ_{mati} and τ_{mad} are obtained which result in the confined region shown in Fig. 6, where $\mathcal{H}^z_{NCS}$ is $\mathscr{L}_2$-stable with gain $\theta^* = 1$.

VI. Discussion and Conclusions

In this paper, we presented a novel modeling and analysis framework for string stability of interconnected vehicle strings in the face of communication effects induced by the wireless network in between the vehicles. $\mathscr{L}_p$-stability results for Networked Control Systems based on hybrid system models were used to perform the string stability analyses for the resulting Cooperative Adaptive Cruise Controller (CACC) strategy. These analyses provided bounds on tolerable transmission intervals and delays in face of scheduling constraints requiring network protocols. Even though the framework laid down in this paper can accommodate broader classes of protocols, for illustrative purposes we focused on the case of the sampled-data (SD) protocol. For more general classes including the Round Robin (RR) protocol, initial promising results are recently obtained based on the provided framework, showing its potential in this context.

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