Analyzing the non-smooth dynamics induced by a split-path nonlinear integral controller

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Summary. In this paper, we introduce a novel non-smooth integral controller, which aims at achieving a better transient response in terms of overshoot of a feedback controlled dynamical system. The resulting closed-loop system can be represented as a non-smooth system with different continuous dynamics being active in dedicated regions of the state-space. The dynamical behavior of the hybrid system will be studied through simulations.

Introduction

The vast majority of linear motion systems in industry, such as e.g. wafer scanners, pick-and-place machines, are still controlled using linear (PID-type) controllers [1]. It is well known, however, that linear controllers suffer from inherent performance trade-offs/limitations [4]. In this paper, we will focus on the trade-off induced by including integral action in a controller: integral action removes steady-state errors, however, in a step-response, the transient performance will degrade in terms of increased overshoot. In order to reduce the effect of degrading transient performance in terms of increased overshoot, we propose a non-smooth type of integral action which will be introduced in the next section.

Controller design and non-smooth system formulation

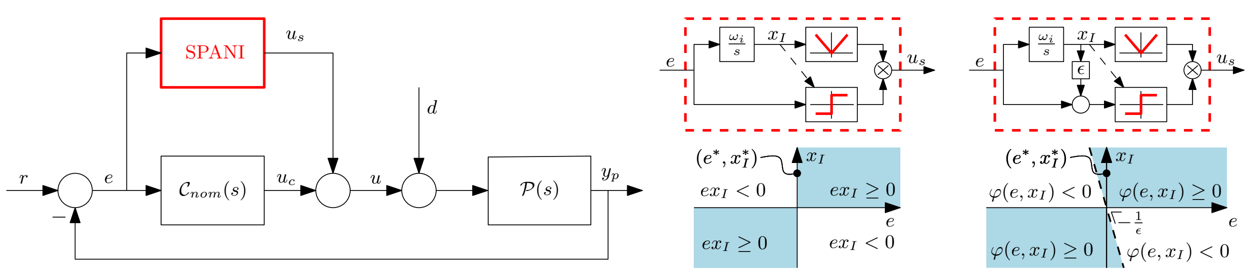
Consider the control scheme depicted in Figure 1.(a), comprising a linear plant with transfer function $\mathcal{P}(s)$, $s \in \mathbb{C}$, linear controller $\mathcal{C}_{nom}(s)$ without integral action, reference r, disturbance d, and tracking error $e := r - y_p$, with y_p the output of the plant $\mathcal{P}(s)$. The total control effort $u := u_c + u_s$ consists of the control effort u_c of $\mathcal{C}_{nom}(s)$ and the control effort u_s of the split-path nonlinear integrator (SPANI), which is based on a split-path nonlinear filter proposed in [2].
The rationale behind the design of this SPANI filter can be best understood by considering a step response on a system containing integral control. In order to achieve a zero steady-state error, the integrator integrates the error e over time resulting in build-up of integral buffer x_I. If the error e becomes zero, i.e., $e = 0$, the integrator still has the integrated error stored in its state x_I. Due to the phase lag introduced by the integrator, it takes some time to empty this buffer, causing the error to overshoot. In contrast to a linear integrator, i.e., $u_s = x_I$, the nonlinear SPANI enforces, due to the absolute value and sign element, see Figure 1.(b), the integral action to take the same sign as the error signal. This results in non-smooth behavior as soon as $e = 0$, i.e., at that specific time instant an instantaneous switch of the sign of the integral action u_s takes place, inducing a reduction in overshoot. In order to obtain a closed-loop model of the system in Figure 1, let (A_p, B_p, C_p) be a state-space realization of the strictly proper plant $\mathcal{P}(s)$, (A_c, B_c, C_c, D_c) be a state-space realization of the linear controller $\mathcal{C}_{nom}(s)$, and let the integrator dynamics be described by $\dot{x}_I = \omega_i e$, with gain ω_i. The SPANI configuration, see Figure 1.(b), enforces $u_s = +x_I$ if $ex_I \geq 0$ and $u_s = -x_I$ if $ex_I < 0$. This results in a switched dynamical model of the closed-loop system with default SPANI filter, see Figure 1.(a) and 1.(b), given by

$$\dot{x} = \begin{cases} \bar{A}_1 x + \bar{B}_r r + \bar{B}_d d & \text{if } ex_I \geq 0 \\ \bar{A}_2 x + \bar{B}_r r + \bar{B}_d d & \text{if } ex_I < 0, \end{cases} \tag{1a}$$

$$y_p = \bar{C}x, \tag{1b}$$

with $x = [x_p^T, x_c^T, x_I^T]^T$ containing the state of the plant, controller, and integrator, respectively, and where

$$\bar{A}_1 = \begin{bmatrix} A_p - B_p D_c C_p & B_p C_c & +B_p \\ -B_c C_p & A_c & 0 \\ -\omega_i C_p & 0 & 0 \end{bmatrix}, \bar{A}_2 = \begin{bmatrix} A_p - B_p D_c C_p & B_p C_c & -B_p \\ -B_c C_p & A_c & 0 \\ -\omega_i C_p & 0 & 0 \end{bmatrix}, \bar{B}_r = \begin{bmatrix} B_p D_c \\ B_c \\ \omega_i \end{bmatrix}, \bar{B}_d = \begin{bmatrix} B_p \\ 0 \\ 0 \end{bmatrix}, \bar{C} = \begin{bmatrix} C_p^T \\ 0 \\ 0 \end{bmatrix}^T.$$



(a) Control scheme with SPANI filter (b) Default SPANI (c) Modified SPANI

Figure 1: Closed-loop control scheme with the nonlinear SPANI filter (a). The default SPANI (b) and modified SPANI (c) configurations are shown, together with the regions in the (e, x_I)-plane in which the dynamics $\bar{A}_1$ and $\bar{A}_2$ are active, see (1).

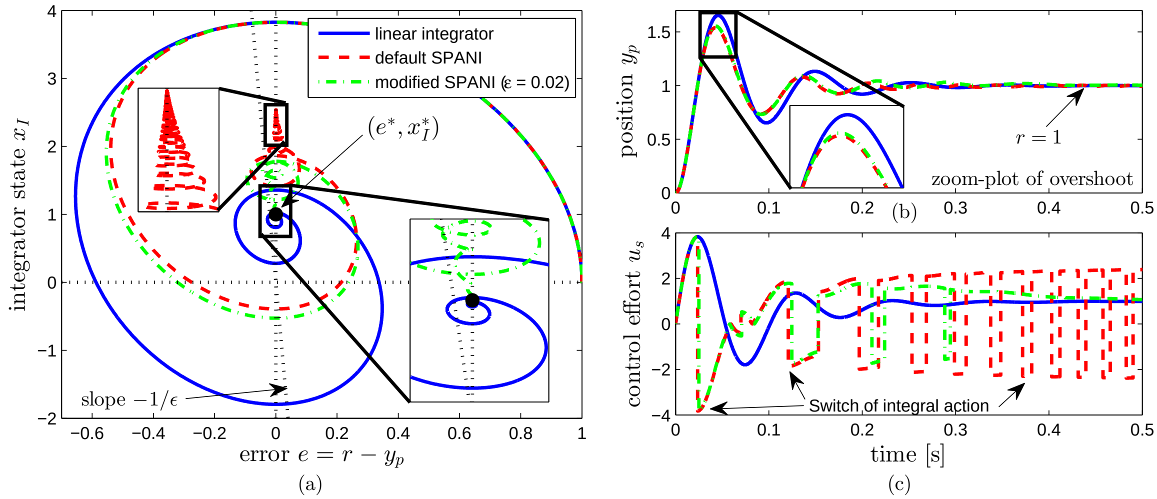


Figure 2: Simulation result of the controlled mass-spring-damper system for the linear integrator, default SPANI and modified SPANI. Presented are the (e, x_I) phase-portrait in (a), the step-response with a zoom-plot of the overshoot in (b), and the integrator control effort u_s in (c).

Nonlinear dynamical behavior

Let us consider the example of a controlled mass-spring-damper system, which is taken from [3]. The default SPANI filter reduces the amount of overshoot in a step response, compared to default integral action, see Figure 2.(b). Up to the moment that the position y_p crosses $r = 1$, i.e., up to the moment that the error $e = 0$, the response of the linear integrator and the default SPANI are identical. However, when crossing $e = 0$, the SPANI sets the integral action to $u_s = -x_I$, and hence, changes sign, see Figures 1.(b) and 2.(c). This reduces the amount of overshoot compared to the linear integrator, however, from Figure 2.(a) we also conclude that the default SPANI does not converge to the equilibrium point (e^*, x_I^*) of the closed-loop system. Moreover, from Figure 2 we observe unwanted oscillations in the steady-state behavior of the closed-loop system.

The problem of the unwanted steady-state oscillations stems from the fact that the equilibrium point (e^*, x_I^*), lies exactly on the switching plane $e = 0$, since $e^* = 0$ is enforced by the integral action $\dot{x}_I = \omega_i e$, see Figure 1.(b) or 2.(a). As a solution to this problem, we therefore propose to use a modified version of the SPANI filter, see Figure 1.(c) with 'tilted quadrants', i.e. change the switching condition slightly, such that

$$\dot{x} \quad = \quad \begin{cases} \bar{A}_1 x + \bar{B}_r r + \bar{B}_d d & \text{if } \varphi(e, x_I) = x_I(\epsilon x_I + e) \geq 0 \\ \bar{A}_2 x + \bar{B}_r r + \bar{B}_d d & \text{if } \varphi(e, x_I) = x_I(\epsilon x_I + e) < 0, \end{cases} \tag{2}$$

and $\epsilon > 0$ denoting the amount of tilting, see Figure 1.(c). From Figure 2 we observe that this resolves the unwanted oscillatory steady-state behavior, and that the system converges to the equilibrium point (e^*, x_I^*), while still warranting a reduction in overshoot compared to the linear integrator. For the modified SPANI, see Figure 1.(c) and (2), formal Lyapunov-based stability conditions in terms of linear matrix inequalities can be derived in order to guarantee global exponential stability of the equilibrium point (even in case sliding modes would be present).

Conclusions

In this paper, we introduced a split-path nonlinear integral controller in order to improve the transient response of a feedback controlled system in terms of overshoot. The closed-loop system can be written as a non-smooth system, with different continuous-time dynamics active in dedicated regions of the state-space. The default SPANI suffers from stability problems which can be resolved by slightly modifying the switching planes of the non-smooth SPANI controller.

Acknowledgements

This research is financially supported by the Dutch Technology Foundation STW.

References

[1] K.H. Ang, G. Chong, and Y. Li. PID control system analysis, design, and technology. *IEEE Trans Control Syst Technol*, 13(4):559–576, 2005.

[2] W. Foster, D. Gieseking, and W. Waymeyer. A nonlinear filter for independent gain and phase (with application). *Trans. ASME J. Basic Eng.*, 78:457–462, 1966.

[3] B.G.B. Hunnekens, N. van de Wouw, and H. Nijmeijer. Variable gain motion control for transient performance improvement. In *Proc. American Control Conference, Montréal, Canada*, pages 2467–2472, 2012.

[4] M.M. Seron, J.H. Braslavsky, and G.C. Goodwin. *Fundamental Limitations in Filtering and Control*. Berlin: Springer, 1997.