Suboptimal Event-triggered Control over Unreliable Communication Links with Experimental Validation

B. Asadi Khashooei, B. van Eekelen, D. J. Antunes and W. P. M. H. Heemels**

Abstract—We propose an event-triggered control policy for a discrete-time linear system with unreliable actuators’ and sensors’ links, captured by Bernoulli packet dropout models. The proposed policy is a threshold-based policy by which transmissions occur if a weighted norm of an error state vector exceeds a threshold. The threshold and the weights of the norm depend on the underlying characteristics of the packet dropout model. Such a policy is shown to guarantee a given performance, defined in terms of a quadratic cost, while reducing transmissions. The proposed event-triggered control policy is experimentally validated in the context of remotely steering an omni-directional ground robot along a predefined trajectory over an unreliable wireless network, while keeping transmissions to a minimum.

I. INTRODUCTION

The increasing availability of cost-effective and reliable sensors and actuators with wireless/wired networking capabilities, opens new possibilities for the manipulation of the physical world through so-called cyber devices resulting in new cyber-physical systems. In particular, there has been a growing interest in networked control systems (NCSs), for which sensors, controllers and actuators communicate via (shared) networks to obtain desired behavior for a given process.

One of the main challenges in NCS is the design of tools for efficient resource management in terms of energy, communication, and computation resources. Periodic time-triggered execution of sampling, communication and control tasks has been the main paradigm for decades. However, periodic strategies, although simple, lack the flexibility one needs to efficiently manage these resources. A recent alternative is event-triggered control (ETC, see, e.g., [1]–[15]). The main idea behind ETC is to include state or output information for the decision making of the data-transmission times in a control loop. Therefore, in a networked control setting, one can consider the ETC unit as an agent trying to create a balance between the performance of the system and the use of feedback.

The main design approaches for ETC can be classified based on the considered system’s model. Some consider nonlinear systems and follow emulation-based ETC approaches (later also using hybrid system approaches, see, e.g., [4], [5], [8], [16], [17]) and some focus on linear systems with stochastic disturbances [6], [7], [9], [12], [13]). The latter works often study co-design of triggering and control policies by considering a cost function, typically a quadratic function, penalizing both communication resources and control actions. We call the latter, optimization-based approaches. Optimal ETC design addresses the problem of characterizing the control and scheduling policies that minimize the considered performance criteria. However, computing optimal ETC policies is typically intractable. For example, many approaches cast the problem in a dynamic programming formulation that is limited by the curse of dimensionality. Suboptimal ETC design, on the other hand, focuses on finding simple and easy to implement control and scheduling policies that have guaranteed performance and therefore try to overcome the bottlenecks that arise with the optimal ETC design.

In the design of ETC, it is important to incorporate network artifacts such as packet dropouts, packet delays, and quantization errors, as these are present in almost all practical applications. This asks for the adaptation of the body of work in ETC to take into account these network induced artifacts. The work [18] considers the optimal control policy design for scalar systems with limited control actions under unreliable actuator links. It is shown that the optimal policy is an ETC policy in which the new control input is transmitted to the actuator if the norm of the state estimate is greater than a threshold. This threshold is obtained through a look-up table that depends on the knowledge of whether or not a dropout has happened and is determined by enumerating all possible scenarios. In [19], the triggering policy consists of a deterministic threshold-based policy and a probabilistic network access protocol determining which agent gets access to the shared network. There it was shown that if the triggering policy is aware of whether or not a packet has been dropped then the proposed policy is robust in the sense that the quadratic measure of the aggregate error remains bounded in expectation. In [20], the control of scalar Brownian motion under delta sampling, a special class of level-triggered sampling, is investigated. Furthermore, it is shown that the distortion of the deterministic sampler is always higher than the distortion due to the delta sampling. In [21], a combination of an event-based predictive control and a network compensator is shown to provide closed-loop stability of nonlinear continuous-time systems in the presence of packet dropouts. In [22], by introducing more realistic models for the communication channels with packet dropouts...
and time-delay, it has been shown that certainty equivalence is still optimal if an instantaneous error-free acknowledgment channel exists. In [23], utilizing a performance index in terms of the second moment of a scalar stochastic linear system, an ETC mechanism is designed such that, in the presence of packet dropouts, the second moment of the state converges exponentially to a desired set in finite time. It is further shown that the proposed policy in [23] under mild conditions provides guaranteed bounds over the transmission rate. In [24] the design of an output-based dynamic event-triggered mechanism for nonlinear systems subject to packet losses both with and without acknowledgment is studied. It was shown that the proposed mechanism can admit a maximum allowable number of successive packet dropouts, while still maintaining the desired stability and performance properties.

In this paper, we focus on the development of an ETC policy that provides guaranteed closed-loop performance in the presence of packet losses. To this effect, we assume that the network links connecting the sensors to the controller and the controller to actuators are subject to failures (see Figure 1). In this context, we provide a policy that takes into account UDP-like information structures at the controller side, i.e., there is no acknowledgment of actuation signal packets available to the controller [25]. We show that, taking into account a quadratic performance index, we can design a threshold event-triggered mechanism based on a weighted norm of the state estimate such that its performance is within a predefined neighborhood of the all-time transmission policy proposed in [25]. Furthermore, the applicability of the proposed policy is validated through experimental results. The experimental set-up consists of an omni-directional robot moving, and a controller on a host PC. Images of the robot on the world frame. Then, the remote controller steers the robot via a LAN network. These images are processed to obtain control actions to the robot via a wireless network with a UDP-like protocol.

The structure of the remainder of the paper is as follows. In Section II we formulate the problem, in Section III we explain the proposed policy. Section IV provides simulation results and in Section V the experimental setup is introduced and the results of the experiments are provided. Section VI concludes the paper. For the sake of brevity the proofs are omitted.

II. CONTROL STRUCTURE

A schematic of the control setup is depicted in Figure 1. The plant is a linear discrete-time dynamical system

\[ x_{k+1} = Ax_k + Bu_k + v_k, \quad k \in \mathbb{S} \]  

where \( \mathbb{S} = \{0, 1, \ldots, N - 1\} \), \( N \in \mathbb{N} \) and \( x_k \in \mathbb{R}^{n_x} \), \( u_k \in \mathbb{R}^{n_u} \) are the state and the actuation signal, respectively. Moreover, \( (v_k)_{k=0}^{N-1} \) is a sequence of Gaussian i.i.d. random vectors with zero mean and covariance \( \Phi_v \). We consider a network between the sensors and the controller \( N_1 \), and a network between the controller and the actuators \( N_2 \).

In a similar setting as in [25], [26], we assume that the networks are lossy with drop out probabilities \( \alpha \) and \( \beta \), respectively. The control unit not only computes the control action \( u_k \) but also decides whether or not to transmit the computed control action based on the available information. This scheduling is modeled via a decision variable \( \sigma_k \in \{0, 1\} \), \( k \in \mathbb{S} \), where \( \sigma_k = 1 \) denotes the occurrence of a transmission and \( \sigma_k = 0 \) a non-transmission event. At the actuator side, we consider a control-input-generator (CIG) that sets the actuation signal to zero when no control input is received from the controller. Let \( (\alpha_k)_{k \in \mathbb{S}} \) and \( (\beta_k)_{k \in \mathbb{S}} \) be i.i.d. Bernoulli processes which model the unreliable nature of the link from the controller to the actuators and sensors to controller, respectively. Then,

\[ \hat{u}_k = \alpha_k \sigma_k u_k, \quad k \in \mathbb{S} \]  

and the system (1) can be modeled as

\[ x_{k+1} = Ax_k + \alpha_k \sigma_k Bu_k + v_k \]

\[ y_k = \begin{cases} x_k, & \beta_k = 1, \\ \emptyset, & \text{otherwise}. \end{cases} \]

The packet dropouts have the following probability distribution

\[ P[\alpha_k = 1] = 1 - \alpha = \bar{\alpha} \]

\[ P[\alpha_k = 0] = \alpha \]  

and

\[ P[\beta_k = 1] = 1 - \beta = \bar{\beta} \]

\[ P[\beta_k = 0] = \beta. \]

We assume that \( (\alpha_k)_{k \in \mathbb{S}} \) and \( (\beta_k)_{k \in \mathbb{S}} \) are independent processes, also independent of the disturbance process \( (v_k)_{k \in \mathbb{S}} \), and the initial state \( x_0 \). Moreover, note that \( y_k \in \mathbb{R}^{n_y} \) denotes the accessible state of the plant to the controller and the symbol \( y_k = \emptyset \) is used to denote the absence of information when a packet drop occurs.

To design and analyze the controller, we consider the following finite-horizon quadratic performance criterion

\[ J = \mathbb{E}[x_N^T Q x_N + \sum_{k=0}^{N-1} x_k^T Q x_k + \hat{u}_k^T R \hat{u}_k] \]

where \( Q \) and \( R \) are positive definite weighting matrices of proper dimensions. To penalize transmissions, we follow a
We denote the average triggering rate as applied. The information vector can be represented as like protocol in which there is no acknowledgment signal is considered. We assume that both actuator and sensor network links are prone to failure, i.e., \( \beta > 0 \), \( \alpha > 0 \), and there is no acknowledgment of successful transmission in the actuator network. Then the available information at the controller is characterized by (10). In this setting, if transmissions are prone to failure, i.e., \( \beta > \alpha \), the state estimate whose dynamics are governed by the optimal estimator (see, [25, eq. 17]),

\[
\hat{x}_k = \begin{cases} 
A\hat{x}_{k-1} + \bar{\alpha}Bu_{k-1} & \text{if } \beta_k = 0 \\
x_k & \text{if } \beta_k = 1.
\end{cases}
\]  

Lemma 1: The control policy (12), (13) for all-time transmission with a UDP-like network protocol results in the overall performance of

\[
V_0^{\pi_{\text{all}}}(I_0^{UDP}) = J_0^{\pi_{\text{all}}}(I_0^{UDP}) + N\lambda
\]

where

\[
J_0^{\pi_{\text{all}}}(I_0^{UDP}) = E[x_0^T K_0 x_0 | I_0^{UDP}] + E[e_0^T P_0 e_0 | I_0^{UDP}] + \sum_{i=0}^{N-1} E[v_i^T (K_{i+1} + \beta P_{i+1}) v_i].
\]

In the following theorem, the proposed ETC policy under the information structure (10) is introduced. The goal of the ETC design is to find a policy that provides a guaranteed performance under unreliable communication links, while reducing the number of transmissions in the actuator network.

Theorem 1: Consider the system (3) with control and scheduling policy \( \pi \) defined as

\[
(u_k, \sigma_k) = \begin{cases} 
(L_k\hat{x}_k, 1), & \text{if } \hat{x}_k^T \Gamma_k \hat{x}_k > \lambda \\
(\emptyset, 0), & \text{otherwise}
\end{cases}
\]

\[
\Gamma_k = \bar{\alpha} A^T K_{k+1} B (R + B^T (K_{k+1} + \alpha \beta P_{k+1}) B)^{-1} 
\times B^T K_{k+1} A
\]

Then the cost of the policy \( \pi \), denoted by \( J_0^\pi(I_0^{UDP}) \), satisfies

\[
J_0^\pi(I_0^{UDP}) \leq J_0^{\pi_{\text{all}}}(I_0^{UDP}) + N\lambda.
\]  

IV. Simulation results

We consider a discretized model of a double integrator controlled over two networks as depicted in Figure 1. The discretized system with the sampling time of 0.1 (sec) and the cost parameters are given by

\[
A = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0.005 \\ 0.1 \end{bmatrix},
\]

\[
\Phi_v = 0.01, \quad N = 100, \quad Q_N = Q = I, \quad R = 1.
\]

In this example, we consider the drop-out probabilities of \( \beta = 0.6 \) and \( \alpha = 0.3 \) for the sensor and the actuator networks, respectively. Figure 2 shows the simulation results for the proposed policy with \( \lambda \in [0, 1] \) based on Monte Carlo simulation of 600 realizations. As can be seen the proposed policy led to a system's operation within the guaranteed analytical performance bounds.
time-invariant system

\[
\begin{bmatrix}
x_{k+1} \\
y_{k+1} \\
\psi_{k+1}
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_k \\
y_k \\
\psi_k
\end{bmatrix}
+ T_s \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\hat{u}_x, k \\
\hat{u}_y, k \\
\hat{u}_z, k
\end{bmatrix}
+ v_k
\]

\[y_k = \begin{cases} X_k, & \beta_k = 1, \\
\emptyset, & \text{otherwise,}
\end{cases}
\]

where \(T_s\) is the sampling time, which is set to 0.1 second. The control input \(\hat{u}_k\) consists of a feedforward term \(\hat{u}_k^f\) and a feedback term \(\hat{u}_k^r\). The feedforward term is computed from the desired model-based trajectory \(X_k^r\), \(k \in \mathbb{S}\), and obtained through

\[X_{k+1}^r = AX_k^r + BU_k^r.\]  \hspace{1cm} (20)

The feedback term, on the other hand, regulates the robot towards the desired trajectory in the presence of disturbances. Since the system is linear we can use the proposed ETC policy to design the feedback control input based on the error between the position of the robot \(X_k\) and the reference, \(X_k^r\) which has the dynamics

\[z_{k+1} = A z_k + B \hat{u}_k^r + v_k\]

where \(z_k = X_k - X_k^r\). We assume that both the remote controller and the robot have access to the feedforward control inputs and the controller only sends the feedback input to the robot. Therefore, the control input is obtained by

\[\hat{u}_k = \hat{u}_k^f + \hat{u}_k^r\]  \hspace{1cm} (21)

where

\[\hat{u}_k^r = \sigma_k \alpha_k u_k^r.\]  \hspace{1cm} (22)

Therefore, if no control input is received by the robot either because of packet dropout \(\alpha_k = 0\) or because the triggering condition is not active, i.e., \(\sigma_k = 0\), then the feedforward term \(\hat{u}_k^f\) is still applied to the actuators, i.e., only the feedback term \(\hat{u}_k^r\) of the input \(\hat{u}_k\) is set to zero.

V. EXPERIMENTAL VALIDATION FOR REMOTE CONTROL OF A GROUND ROBOT

In this section, we use the developed algorithm for model-based tracking and remote regulation of a ground robot towards a predefined trajectory. The experimental set-up consists of an omni-directional robot, see Figure 3, a top camera above a soccer field, where the robot is moving and a controller on a host PC. Images of the robot on the soccer field are sent from the top camera to the controller, these images are processed to obtain the position and orientation of the robot with respect to the world frame. Then based on the calculated position control actions are computed as will be discussed in the sequel.

Figure 4 shows a schematic representation of the control structure. The plant represents the omni-directional robot, whose dynamics can be described by a linear model. The robot model is given by the following discrete-time linear
were introduced to acquire a drop-out probability of \( \alpha \). Power of the proposed policy, randomly induced packet drops in the network is approximately 1\%.

The actual percentage of packets that was lost in the actuator network, \( \hat{\alpha} \), is always below the guaranteed bounds, \( J_{\text{UDP}} \), which validates the proposed scheme in the experimental set-up.

![Fig. 5](image1.png)

**Fig. 5.** The tracking in the \( x, y \)-plane for the proposed event-triggered mechanism (16) with threshold \( \lambda = 0.01 \), only the tracking for the first perambulation is shown. The instants at which a triggering occurs are depicted with red dots.

![Fig. 6](image2.png)

**Fig. 6.** Performance of the proposed ETC in the considered experimental setup where \( J = \mathbb{E}[z_N^T Q_N z_N + \sum_{k=0}^{N-1} z_k^T Q z_k + \hat{\alpha}_k \hat{\alpha}_k^T R \hat{\alpha}_k] \).

**Experimental results**

In the experiments, we consider the performance index (6) with

\[
Q = I_3, \quad Q_N = I_3, \quad R = I_3, \quad N = 250.
\]

The actual percentage of packets that was lost in the actuator network is approximately 1\%. Therefore, to illustrate the power of the proposed policy, randomly induced packet drops were introduced to acquire a drop-out probability of \( \alpha = 0.1 \) (accounts for 10\% drop-out rate). In the sensor network the packet drop-outs were mainly due to missing the computation deadline. This leads to a dropout probability corresponding to \( \beta = 0.25 \). For this experiment we only consider the UDP-like protocol thus no acknowledgment is available in the actuator network.

Since the computation of a (new) feedback control value can not be done instantaneously at sampling times, we introduce a unit delay for applying controller actions and, therefore, we update the state estimation \( \hat{\dot{z}}_k \) by prediction i.e.,

\[
u_k = L_k (\hat{\dot{z}}_{k-1} + \hat{\alpha} B u_{k-1}).
\]

The reference trajectory consists of two tangent circles with radius of 1 meter as depicted by blue dashes in Figure 5. Moreover, the real trajectory and transmission instants of the remote robot are also depicted in Figure 5.

Figure 6 shows the experimental results for four different values of \( \lambda \in \{0, 0.001, 0.002, 0.01\} \). Each experiment was carried out for 5 rounds and the obtained performance is averaged over these experiments. Interestingly, the performance of the proposed ETC policy, \( J^{\text{UDP}} \), is always below the guaranteed bounds, \( J_{\text{UDP}} \), which validates the proposed scheme in the experimental set-up.

**VI. Conclusions**

A suboptimal ETC policy with guaranteed performance under unreliable actuator and sensor links was provided in this paper. The proposed policy belongs to the class of threshold-based policies, whose parameters are influenced by the underlying characteristics of the packet dropout behaviour in the communication network and the cost of communication in the considered quadratic performance index. An experimental setup for validation of the proposed algorithm has been developed. In the experiment a remote controller is used to steer an omni-directional ground robot, along a predefined trajectory. The conducted experiments validated the theoretical results and the applicability of the proposed ETC policy.

**References**


