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Rollout Strategies for Output-Based Event-Triggered Control

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Abstract— **Recent research advocates that replacing the periodic communication paradigm by an event-triggered paradigm can have significant benefits for control systems. While in many event-triggered control solutions transmission decisions are based on full state information, in most applications only partial state information is available for feedback (output measurements). Here we propose an optimization-based output-feedback event-triggered solution for linear discrete-time systems which guarantees a performance bound with respect to periodic control, while reducing the communication load. Performance is measured by a quadratic cost. The usefulness of the results is illustrated through a numerical example.**

I. Introduction

Periodic sampling and control is the long-standing paradigm in digital feedback applications. It is simple to implement and quite efficient in the context of embedded systems where the sampling period, restricted only by hardware constraints, can typically be made much smaller than the time-constants of the controlled physical system. The advent of networked control applications in which several control systems share the same communication network calls for a new paradigm to cope with bandwidth limitations. Event-triggered control (ETC) has the potential to play this role. The fundamental idea behind ETC is that transmissions should be triggered by events inferred from the state or the output, as opposed to being triggered periodically. This in general leads to an improvement of the trade-off between transmissions and control performance when compared to periodic control, see, e.g. [1]–[4].

Several event-triggered control solutions are available in the literature. Some focus on designing only the triggering rule for transmissions assuming a given controller (emulation approach), while others design both the controller and the triggering rule (co-design). One of the research lines within ETC (see, e.g., [3], [5]–[9]) addresses the problem of reducing the transmission rate while preserving stability and other desired properties such as guaranteed $\mathcal{L}_2$-gain to a certain extent. In another research line the emphasis is placed on optimizing a performance criterion taking into account the closed-loop performance and the networked usage (see, e.g. [1], [2], [4], [10]–[14]). Often, performance is defined through a quadratic cost, as in the celebrated LQR problem. The most common way of introducing a penalty on the network usage, is to add a constant additive communication cost irrespective of the state, output or input knowledge (see [10]–[12]). An alternative way, proposed in [15], considers a multiplicative penalty leading to a Lyapunov-based rule similar to [6]. The work [15] also establishes that the performance of such a policy is within a constant factor of the quadratic cost of periodic control while reducing transmissions. Moreover, [15] proposes a different strategy by which performance is better than that of periodic control for the same transmission rate [2]. Recent work proposes an alternative scheme capable of outperforming rollout strategies [14], when the disturbances are sporadic.

Although some of the above mentioned works consider the case where only partial information about the state is known (via output measurements), see e.g. [3], [7], [9], [11], [16], there does not seem to be too many output-based ETC schemes including performance guarantees by design. For state-feedback controller such results are available, see e.g. [1], [2], [4], [11], [13], [14], [17].

Given that in many practical situation the complete state information is not available for feedback, in the present work we propose a new output-feedback event-triggered controller with guaranteed performance bounds. We follow the approach in [15] considering discounted and average quadratic costs which penalize transmissions in a multiplicative way. Considering this cost and inspired by rollout ideas in the context of dynamic programming [18], our method consists of the selection of optimal control inputs and transmission decisions over a receding horizon while assuming that a base policy is used after the horizon. By choosing the base policy as that of an optimal periodic control strategy, performance guarantees can be obtained. We also address the co-design problem in the sense that we simultaneously design both the control and communication policies. As such, our contribution can be perceived as the extension of [15] to the output-feedback case, while still providing bounds for the performance both in discounted and average cost problem. Moreover, we also provide a stability proof of the proposed scheme in the average cost case. To illustrate the usefulness of our results, we apply the new scheme in the control of a mass-spring system using a communication network. The numerical results show that approximately 40% communication reduction is achieved while guaranteeing a performance within 1.1 of the standard optimal periodic policy communicating all the time.

The remainder of the paper is organized as follows. In section II we formulate the output-feedback ETC problem and we explain the proposed ETC method. Section III provides the guarantees on the performance and stability

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results for the proposed ETC methods. Section IV presents the numerical example and Section V provides concluding remarks and directions for future work.

II. Problem formulation

We consider the linear discrete-time system given by

$$\begin{aligned} x_{k+1} &= Ax_k + B\hat{u}_k + s_k \\ y_k &= Gx_k + v_k, \end{aligned} \tag{1}$$

where $x_k \in \mathbb{R}^{n_x}$, $\hat{u}_k \in \mathbb{R}^{n_u}$ and $y_k \in \mathbb{R}^{n_y}$ denote the state, the input, and the output, respectively, and s_k and v_k denote the state disturbance and the measurement noise, respectively, at time $k \in \mathbb{N}_0 := \{0, 1, 2, \dots\}$. We assume that the disturbance and the noise are Gaussian zero-mean, independent random vectors with covariance Φ_s and Φ_v, respectively. The initial state is assumed to be either a Gaussian random variable with mean $\hat{x}_0$ and covariance Θ_0 or known in which case it equals $\hat{x}_0$ and $\Theta_0 = 0$.

Furthermore, we consider the following cost to be minimized

$$\mathbb{E}[\sum_{k=0}^{\infty} \alpha^k (x_k^T Q x_k + \hat{u}_k^T R \hat{u}_k + 2x_k^T M \hat{u}_k)], \tag{2}$$

where $0 < \alpha < 1$ is the discount factor and Q, R, M are such that

$$\begin{bmatrix} Q & M \\ M^T & R \end{bmatrix} > 0.$$

This cost is introduced for convenience as we are mostly interested in the minimization of the average cost per stage defined as

$$\lim_{N\to\infty} \frac{1}{N} \mathbb{E}[\sum_{k=0}^{N-1} (x_k^T Q x_k + \hat{u}_k^T R \hat{u}_k + 2x_k^T M \hat{u}_k)]. \tag{3}$$

As we shall see, considering the minimization of (2) will lead to a method for the minimization of (3). Note that the cost incurred in the early stages does not contribute to the average cost (3) and therefore it does not depend on the initial condition of (1) (for a formal proof see [18]).

We assume that a controller, collocated with the sensors, sends the control values to the actuators over a communication network. This controller should not only compute the control inputs, but it should also decide at which times $k \in \mathbb{N}_0$ new control inputs are sent to the actuators. In this paper, the controller will be of an event-triggered nature. The overall scheme is depicted in Fig. 1.(a).

To model the occurrence of transmissions in the network, we introduce $\sigma_k \in \{0, 1\}$, $k \in \mathbb{N}_0$, as a decision variable indicating if a transmission occurs at time k, in which case $\sigma_k = 1$, or not ($\sigma_k = 0$). Let u_k denote the received value by the actuators at time $k \in \mathbb{N}_0$ when a transmission occurs ($\sigma_k = 1$) and have any arbitrary value otherwise. We use $u_k = \emptyset$ to denote the latter case i.e. when $\sigma_k = 0$. We assume that at the actuator side a standard zero-order hold device holds the previous value of the control action if no new control input is received at time k. Considering now an extended state $\xi_k := (x_k, \hat{u}_{k-1})$, we obtain the model

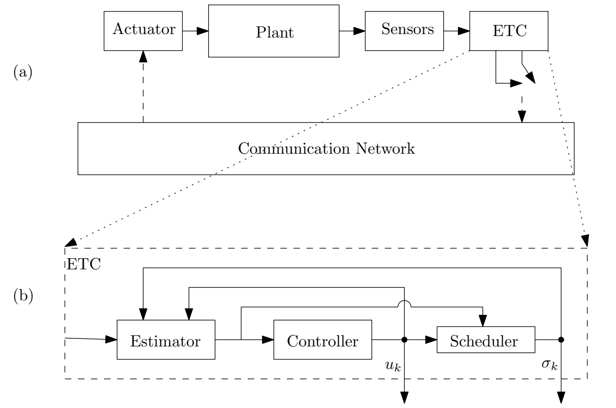

Fig. 1. The schematic of the considered structure: (a) The overall loop structure: The sensors connected to the ETC block, which decides the transmission times and computes the control input at the transmission instant. (b) Inside the ETC: A scheduler decides based on the available information whether or not the computed control action will be transmitted.

$$\begin{aligned} \xi_{k+1} &= A_{\sigma_k}\xi_k + B_{\sigma_k}u_k + \omega_k, \quad k \in \mathbb{N}_0 \\ y_k &= C\xi_k + v_k, \end{aligned} \tag{4}$$

where $\omega_k := (s_k^T, 0_{n_u}^T)^T$ and

$$\begin{aligned} A_j &:= \begin{bmatrix} A & (1-j)B \\ 0 & (1-j)I \end{bmatrix}, \quad B_j := \begin{bmatrix} jB \\ jI \end{bmatrix}, \\ C &:= \begin{bmatrix} G & 0 \end{bmatrix}, \quad j \in \{0, 1\}, \end{aligned}$$

and we arbitrate that $\hat{u}_{-1} = 0$. Moreover, the performance functions (2)-(3) can be written as

$$\mathbb{E}[\sum_{k=0}^{\infty} \alpha^k g(\xi_k, u_k, \sigma_k)] \tag{5}$$

$$\lim_{N\to\infty} \frac{1}{N} \mathbb{E}[\sum_{k=0}^{N-1} g(\xi_k, u_k, \sigma_k)], \tag{6}$$

where $g(\xi, u, j) = \xi^T Q_j \xi + u^T R_j u + 2\xi^T M_j u$ with

$$\begin{aligned} Q_j &:= \begin{bmatrix} Q & (1-j)M \\ (1-j)M^T & (1-j)R \end{bmatrix}, \quad M_j := \begin{bmatrix} jM \\ 0 \end{bmatrix} \\ R_j &:= jR, \quad j \in \{0, 1\}. \end{aligned}$$

To penalize transmission, we consider a multiplicative factor $(1 + \theta)$ in the stage cost associated with time k if a transmission occurs at time k (i.e. $\sigma_k = 1$). Therefore, we consider auxiliary cost functions described by

$$\mathbb{E}[\sum_{k=0}^{\infty} \alpha^k (1 + \theta\sigma_k) g(\xi_k, u_k, \sigma_k)], \tag{7}$$

$$\lim_{N\to\infty} \frac{1}{N} \mathbb{E}[\sum_{k=0}^{N-1} (1 + \theta\sigma_k) g(\xi_k, u_k, \sigma_k)]. \tag{8}$$

Let I_k denote the information available to the controller and scheduler at time $k \in \mathbb{N}_0$, i.e.,

$$I_k := (y_0, \dots, y_k, u_0, \dots, u_{k-1}, \sigma_0, \dots, \sigma_{k-1}, \hat{x}_0, \Theta_0)$$

for $k \in \mathbb{N}_{\geq 1}$ and $I_0 := (y_0, \hat{x}_0, \Theta_0)$. A policy $\pi := (\mu_0, \mu_1, \dots)$ is defined as a sequence of multivariate functions $\mu_k := (\mu_k^\sigma, \mu_k^u)$ that map the available information vector I_k into control actions u_k and scheduling choices σ_k, $k \in \mathbb{N}_0$.

We denote by $V_\pi^d(I_0)$ and V_π^a the costs (5) and (6), respectively, when

$$(u_k, \sigma_k) = \mu_k(I_k), \quad k \in \mathbb{N}_0, \tag{9}$$

and by $J_\pi^d(I_0)$ and J_π^a the costs (7) and (8), respectively when (9) is used.

We are interested in minimizing the costs J_π^d, J_π^a and by doing so we shall provide bounds on the costs (without penalizing transmission) V_π^d, V_π^a. The stated problem is an infinite horizon mixed integer programming problem which is computationally intractable. As such, we propose a suboptimal approach based on limited lookahead policies and in particular on the rollout algorithm (see [18, pp. 304-307]) which can outperform periodic (all-time) control. In fact, we shall prove that for our proposed policies the costs are within a constant factor of the corresponding costs using periodic control while a reduction on the average transmission rate is achieved. The average transmission rate is defined as

$$R_{trans} = \lim_{N \to \infty} \frac{1}{N} \sum_{k=0}^{N-1} \sigma_k.$$

III. Proposed method and main results

In this section, we introduce first some preliminaries and then the main result of the paper, presented in Theorem 1. Finally, we present the ideas behind the main result and the procedure to obtain to the proposed scheme.

A. All-time transmission policy

Before introducing our proposed method, we first recall the optimal control input policies that minimize (7) and (8) when $\mu_k^\sigma(I_k) = 1$ for every $k \in \mathbb{N}_0$, which is equivalent to an all-time transmission policy. Note that such policy corresponds to periodic transmission with average transmission rate $R_{trans} = 1$. The control policy which minimizes (7) can be found in standard optimal control books, (see e.g. [18]) and is described by

$$\begin{aligned} \mu_k^{u,per}(I_k) =& -(R + \alpha B^T K B)^{-1} (\alpha B^T K A + M^T) \, \mathbb{E}[x_k | I_k] \\ =& L \, \mathbb{E}[x_k | I_k], \end{aligned} \tag{10}$$

with

$$\begin{aligned} K =& Q + \alpha A^T K A - P \\ P =& (\alpha A^T K B + M)(R + \alpha B^T K B)^{-1} \\ & (\alpha B^T K A + M^T). \end{aligned} \tag{11}$$

and $\mathbb{E}[x_k|I_k]$ denotes the conditional expectation of x_k given the information vector I_k. The control policy that minimizes (8) is obtained by making $\alpha = 1$ in (10) and (11).

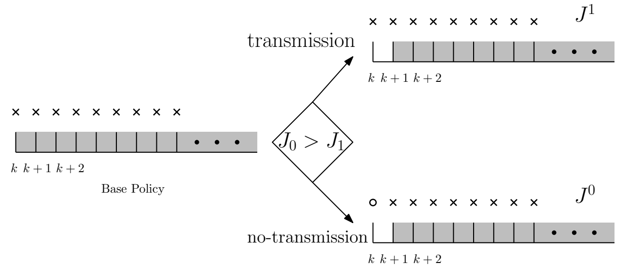

Fig. 2. Schematic of the proposed ETC

The cost (7) with policy (10) is given by

$$\begin{aligned} J_{\pi_{per}}^d(I_0) = (1+\theta)\Bigg(& \mathbb{E}[x_0^T K x_0 | I_0] + \frac{\alpha}{1-\alpha} \operatorname{Tr}(K \Phi_s) \\ & + \sum_{s=0}^{\infty} \alpha^s \operatorname{Tr}(P \Xi_s^0) \Bigg) \end{aligned} \tag{12}$$

and the cost (8) for the policy (10) is then given by

$$J_{\pi_{per}}^a := (1+\theta)\big(\operatorname{Tr}(K \Phi_s) + \operatorname{Tr}(P \bar{\Xi}) \big) \tag{13}$$

with

$$\begin{aligned} \Xi_s^l &= \mathbb{E}\left[(x_s - \mathbb{E}[x_k|I_k])^T (x_s - \mathbb{E}[x_k|I_k]) \big| I_l \right] \\ \bar{\Xi} &= \lim_{s \to \infty} \Xi_s^0, \end{aligned} \tag{14}$$

Note that the optimal control for the case of periodic scheduling resulted in a stationary policy $\pi_{per} = \big((1, \mu^{u,per}), (1, \mu^{u,per}), \dots\big)$, which is easy to implement.

B. Main result and main policy

The main result of the paper is presented next. We say that the closed loop system (4) and (9) is mean square stable if $\sup_{k \in \mathbb{N}_0} \mathbb{E}[\|x_k\|^2] < \infty$ for every $k \in \mathbb{N}_0$ (see [17]).

Theorem 1: Consider system (4) with scheduling and control policy, π_{ro}, defined as

$$\mu_k^{ro}(I_k) = \begin{cases} \big(1, L \, \mathbb{E}[x_k|I_k]\big) & \hat{\xi}_k^T \Gamma \hat{\xi}_k - \gamma_k > 0 \\ \big(0, \emptyset\big) & \text{Otherwise,} \end{cases} \tag{15}$$

where

$$\begin{aligned} \Gamma &= \begin{bmatrix} (1+\theta)P - \theta Q & \bar{M} \\ \bar{M}^T & \bar{R} \end{bmatrix} \\ \gamma_k &= \theta \operatorname{Tr}(Q \Xi_k^k) \\ \hat{\xi}_k &= \begin{bmatrix} \mathbb{E}[x_k|I_k]^T & \hat{u}_{k-1}^T \end{bmatrix}^T \end{aligned} \tag{16}$$

with

$$\begin{aligned} \bar{R} &= R + \alpha(1+\theta) B^T K B \\ \bar{M} &= M + \alpha(1+\theta) A^T K B. \end{aligned} \tag{17}$$

Then for $0 \leq \alpha < 1$

$$V_{\pi_{ro}}^d(I_0) \leq J_{\pi_{ro}}^d(I_0) \leq J_{\pi_{per}}^d(I_0) = (1+\theta) V_{\pi_{per}}^d(I_0), \tag{18}$$

for every I_0. Furthermore, for $\alpha = 1$, the system (4), (9) is mean square stable for policy π_{ro} and the associated average stage cost satisfies

$$V_{\pi_{ro}}^a \leq J_{\pi_{ro}}^a \leq J_{\pi_{per}}^a = (1+\theta) V_{\pi_{per}}^a. \tag{19}$$

Fig. 1.b shows the overall structure of the proposed event-triggered solution. The ETC block consists of three blocks, namely the estimator, which computes estimates $\mathbb{E}[x_k|I_k]$ of the state based on the available information vector, the controller, which computes control actions, and the scheduler, which makes the transmission decisions also based on information vector.

Next we provide the idea behind our method which leads to the policy defined through (15)-(16).

C. Ideas behind the main policy

We start with the discounted cost problem. At each time step the scheduler decides whether or not to transmit based on the value of the cost function of the two possible scheduling sequences as schematically depicted in Fig. 2. In both cases, after the first step (corresponding to either $\sigma_k = 0$ or $\sigma_k = 1$), all-time updates are being used. For each of these two fixed possible scheduling sequences, i.e. $\sigma = (1, 1, 1, 1, \dots)$ or $\sigma = (0, 1, 1, 1, \dots)$, one can compute the value of the cost function (7) associated with transmitting, denoted here by $J^{d,1}$, and the cost function (7) associated with not transmitting, denoted here by $J^{d,0}$, based on the available information at each iteration. If $J^{d,0} \leq J^{d,1}$ no transmission takes place and the actuator holds its current value. If $J^{d,1} < J^{d,0}$ the new optimal control value computed for all-time transmission case is sent over the network. The procedure is then repeated at the next iteration.

One can see this method as a rollout procedure [18] in the context of dynamic programming with the base policy π_{per} (10) and the cost-to-go function $J^d_{\pi_{per}}$ as in (12). In fact, $J^d_{\pi_{per}}$ is employed as the approximation of the cost-to-go function defined as in (7) for a given information vector I_k, $k \in \mathbb{N}_0$, in a one-step lookahead optimization i.e.

$$J^d_{\pi_{stp}}(I_k) = \min_{u_k,\sigma_k} \mathbb{E}[\bar{g}(\xi_k, u_k, \sigma_k) + \alpha J^d_{\pi_{per}}(I_{k+1})|I_k, u_k, \sigma_k] \quad (20)$$

with the policies

$$\mu_k^{ro,u}(I_k) = \arg\min_{u_k,\sigma_k} \mathbb{E}[\bar{g}(\xi_k, u_k, \sigma_k) + \alpha J^d_{\pi_{per}}(I_{k+1})|I_k, u_k, \sigma_k], \quad (21)$$

where $\bar{g}(\xi_k, u_k, \sigma_k) := (1+\sigma_k\theta)g(\xi_k, u_k, \sigma_k)$ and $J^d_{\pi_{stp}}(I_k)$ is the one-step lookahead cost for the information vector I_k. This cost can be computed easily by considering the possible scenarios for the scheduling variable at each step k

1) $\sigma_k = 1$, i.e. transmission at stage k followed by the periodic scheduling policy which results in the control policy (10) with the cost-to-go function

$$J^{d,1}_{\pi_{stp}}(I_k) = (1+\theta)\left(\mathbb{E}[x_k^T K x_k|I_k] + \frac{\alpha}{1-\alpha} \mathrm{Tr}(K\Phi_s) + \sum_{s=k}^{\infty} \alpha^{s-k} \mathrm{Tr}(P\Xi_s^k) \right), \quad (22)$$

which is the cost-to-go function related to all-time transmissions.

2) $\sigma_k = 0$, i.e. no transmission at time k followed by the all-time transmissions policy. Hence, no new control action would be sent to the actuators at time k denoted by $\emptyset$ in (15), then we have

$$J^{d,0}_{\pi_{stp}}(I_k) = \mathbb{E}[\xi_k^T \Omega \xi_k|I_k] + (1+\theta)\frac{\alpha}{1-\alpha} \mathrm{Tr}(K\Phi_s) + (1+\theta) \sum_{s=k+1}^{\infty} \alpha^{s-k} \mathrm{Tr}(P\Xi_s^k), \quad (23)$$

where

$$\Omega = \begin{bmatrix} \bar{Q} & \bar{M} \\ \bar{M}^T & \bar{R} \end{bmatrix},$$

with $\bar{Q} = Q + \alpha(1+\theta)A^T K A$ and Ξ_s^k, $\bar{M}$ and $\bar{R}$ defined in (16) and (17).

After computing the two possible costs with corresponding control policies, we get

$$J^d_{\pi_{stp}}(I_k) = \min_{\sigma_k \in \{0,1\}} \{J^{d,\sigma_k}_{\pi_{stp}}(I_k)\} \quad (24)$$

or in other words the scheduling variable σ_k attains the minimum

$$\mu_k^{\sigma,ro}(I_k) = \arg\min_{\sigma_k} \{J^{d,\sigma_k}_{\pi_{stp}}(I_k)\}.$$

Thus,

$$\mu_k^{ro}(I_k) = \begin{cases} \left(1, L\,\mathbb{E}[x_k|I_k]\right) & if\, J^{d,0}_{\pi_{stp}}(I_k) > J^{d,1}_{\pi_{stp}}(I_k) \\ \left(0, \emptyset\right) & if\, J^{d,0}_{\pi_{stp}}(I_k) \leq J^{d,1}_{\pi_{stp}}(I_k) \end{cases} \quad (25)$$

and substituting (22) and (23) in (25) results in (15).

Until now, we have only addressed the main concept behind the proposed policy for the discounted cost problem, while, we are also interested in the average cost problem. The policy for the average cost problem is obtained by making $\alpha = 1$ in the policy for the discounted cost problem. Building upon this connection, we will still be able to provide similar type of bounds, as in (19), also in the setting of the average cost problem. This justifies our earlier statement on the convenience of considering the discounted cost problem. Due to lack of space, the proof of the main result is omitted.

IV. Illustrative Example

We consider an output-feedback version of the numerical example considered in [2] for the average cost case. Consider two unitary masses on a frictionless surface connected by an ideal spring with spring constant k_m and moving along a one-dimensional axis, see Fig. 3. The control input is a force acting on the first mass and the outputs are the position of both masses i.e., x_1 and x_2. The state vector is $x = [x_1, x_2, v_1, v_2]^T$, where v_i are the velocities of the masses, $i \in \{1, 2\}$. The discretized plant model (1) with $k_m = 2\pi^2$ is described by

$$A = \begin{bmatrix} 0.9045 & 0.0955 & 0.0968 & 0.0032 \\ 0.0955 & 0.9045 & 0.0032 & 0.0968 \\ -1.8466 & 1.8466 & 0.9045 & 0.0955 \\ 1.8 & -1.8 & 0.0955 & 0.9045 \end{bmatrix}, B = \begin{bmatrix} 0.0049 \\ 0.0001 \\ 0.0968 \\ 0.0032 \end{bmatrix}$$

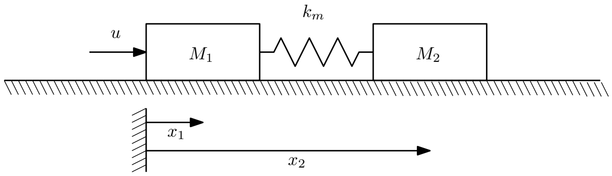

Fig. 3. Control of two masses connected with an spring via a shared network

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

We consider an average cost problem with the matrices

$$Q = \begin{bmatrix} 0.0470 & 0.0030 & 0.0023 & 0.0002 \\ 0.0030 & 0.0470 & 0.0002 & 0.0023 \\ 0.0023 & 0.0002 & 0.0002 & 0.0000 \\ 0.0002 & 0.0023 & 0.0000 & 0.0002 \end{bmatrix},$$

$$M = 10^{-4} \begin{bmatrix} 0.7781 \\ 0.0552 \\ 0.0605 \\ 0.0020 \end{bmatrix}, \; R = 0.05.$$

Considering the cost taken from the example in [2], we compare two cases. In the first case, we take $\theta = 0$, i.e., no penalty for transmission and thus obtaining all-time transmission in (7) and (8). In the second case, we apply a penalty on using the network with $\theta = 0.1$. In both cases, a Kalman filter is used to provide a state estimation based on the available information at each iteration (i.e. the error covariance of the current state and the current state estimation), i.e. we use

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + \Sigma_{k|k} C^T \Phi_v^{-1} (y_k - C\hat{x}_{k|k-1})$$
$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} + (1-\sigma_k)B\hat{u}_{k-1} + \sigma_k B u_k,$$

with initial condition $\hat{x}_{0|-1} = \hat{x}_0$ and

$$\Sigma_{k|k} = \Sigma_{k|k-1} - \Sigma_{k|k-1} C^T (C\Sigma_{k|k-1} C^T + \Phi_v)^{-1} C \Sigma_{k|k-1}$$
$$\Sigma_{k+1|k} = A\Sigma_{k|k} A^T + \Phi_s$$

Now we set in (15)-(16) $\mathbb{E}[x_k|I_k]$ as $\hat{x}_{k|k}$ and Ξ_k^k as $\Sigma_{k|k}$. Fig. 4 shows the average state norm based on Monte Carlo simulations in both cases. Interestingly, introducing the penalty for network usage not only reduces the network usage to 61% with respect to the no-penalty case, but also preserves the stability of the closed-loop system while guaranteeing a performance within 1.1 of the all-time transmission case. Fig. 5 shows the input signal in one realization of both cases with the same noise. As can be seen, for the case with penalty over the usage of network the actuator holds its previous values more often to decrease network usage.

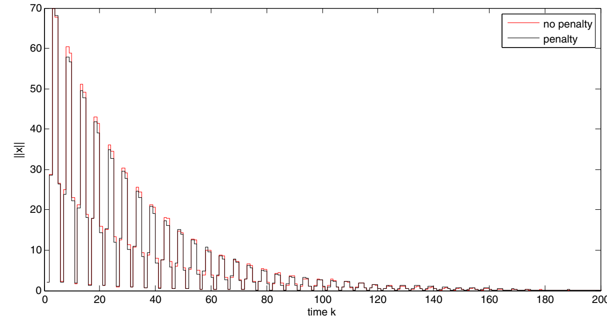

Fig. 4. Average state norm $||x||$ in two cases with $\theta = 0$ and $\theta = 0.1$

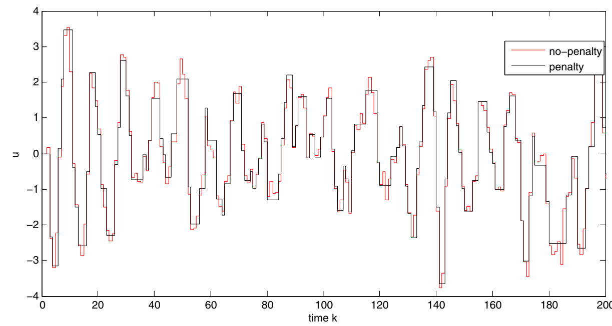

Fig. 5. Variation in control input u for two cases with $\theta = 0$ and $\theta = 0.1$

V. Conclusions

In this paper, we proposed an optimization-based output-feedback event-triggered solution for linear discrete-time systems which guarantees a performance bound with respect to periodic all-time control, while reducing the communication load. The performance is measured by a classical quadratic cost. The easy-to-implement method is obtained using a rollout strategy based on the quadratic costs appended with a multiplicative penalty for communication. The usefulness of the results was illustrated through a numerical example. For future work, we will further exploit these results to take into account other output feedback structures, e.g. the case where the controller is collocated with the actuators and the remote sensing and control where there exist communication networks between sensors and controller and between controller and actuators.

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