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An event-triggered policy for remote sensing and control with performance guarantees

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***Abstract*— We consider a networked control system in which a remote controller queries the plant's sensors for measurement data and decides when to transmit control inputs to the plant's actuators. The goal is to keep transmissions to a minimum while guaranteeing that the closed-loop performance is within acceptable bounds. Our approach extends a recent line of research where explicit event-triggered control policies with performance guarantees are derived using approximate dynamic programming. The proposed policy in this new setting can be separated into an offline scheme for sensor query in which the transmission instances are computed a priori and an online scheme to schedule control input transmissions. The usefulness of the results is illustrated through two numerical examples.**

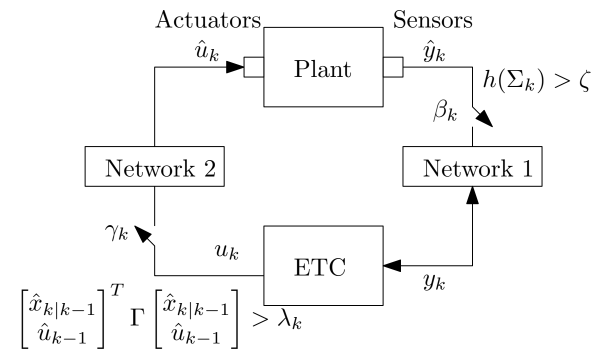

Fig. 1. Overall setup and proposed policies: sensor query depends only on the Kalman filter covariance matrices Σ_k, while the control update transmissions are scheduled based on the Kalman filter state estimate $\hat{x}_{k|k-1}$, and the previously sent control input $\hat{u}_{k-1}$.

I. INTRODUCTION

Efficient utilization of communication, computation and energy resources is one of the key challenges in many emergent networked control applications. Periodic sampling and control is the most common approach in control systems technology but lacks the flexibility to utilize these resources in an efficient way. Recent research proposes to depart from the periodic control paradigm in favor of event-triggered control (ETC) [1]. The fundamental idea behind ETC is that transmissions should be triggered by events inferred from the state or the output of the plant. This in general leads to an improvement of the trade-off between average transmission rate and control performance when compared to periodic control, since in ETC the resources are used only when required.

Desirably, the communication protocols corresponding to ETC should still be insightful and simple to implement and guarantee some performance criteria for the control system. However, the analysis of most basic event-triggered schemes proposed in the literature focuses on fundamental system notions like Lyapunov stability, $\mathcal{L}_2$-norm gain guarantees and minimum inter-event time (see, e.g., [2]–[7]). These simple rules include transmitting only when a Lyapunov function would otherwise exceed a desired decay level (e.g., [2]), transmitting when the error between current measurements or control values and most recently sent ones exceeds an absolute or relative threshold (e.g. [4], [8]), or transmitting when the norm of the covariance of the Kalman filter exceeds a certain threshold (e.g. [7]). In turn, there are optimization-based schemes that guarantee closed-loop performance (see e.g., [9]–[18]). The performance is usually defined through a quadratic cost, as in the celebrated LQR problem, which is often combined with an additive or multiplicative penalty for transmissions. However, many of the proposed ETC policies following this approach are hard to implement in practice, and others lack the insight and simplicity of the basic policies just described.

The aim of this paper is to provide simple event-triggered control policies with guaranteed performance in terms of a quadratic cost. We consider a networked control system in which a remote controller queries the plant's sensors for measurement data and decides when to transmit control inputs to the plant's actuators. The proposed event-triggered control policies for sensor query and control input transmissions are derived using approximate dynamic programming (in particular, rollout techniques) for an optimization problem which includes a quadratic performance cost defined in terms of state and input variables and penalty terms for transmissions. The obtained main results extend our line of research presented in [14], [19], where an event-triggered policy was designed only for scheduling control input transmissions, assuming that sensors measurements are available at every time instant. The new policy can be separated into an offline scheme for sensor query and an online scheme to schedule control input transmissions. Both schemes are simple to implement and have insightful interpretation and thereby increasing the acceptance of practitioners (see Fig. 1 for a first impression of which the details follow later).

In the online scheme determining the controller-to-actuator transmissions, the decisions are based on the state and control estimates, which are not known a priori and can be obtained via the time-varying Kalman filter. In the offline scheme for sensor query, transmissions are based on the covariances of the Kalman filter state estimates, which are known a priori. Interestingly, we can interpret this offline scheme as a policy in which transmissions occur when a function of the covariance of the Kalman filter exceeds a given threshold, which connects well to some policies proposed in this area of research (see, e.g., [7]).

The main advantage of our approach is that we can show that this event-triggered policy is stable (in a mean-square sense) and leads to performance guarantees in terms of the cost of all-time transmission policy, which was not the case in most other optimization-based ETC schemes. We discuss how this policy can be tuned to trade closed-loop performance guarantees and average transmission rates.

The usefulness of the results is illustrated through two numerical examples. The numerical results for both examples show that approximately 50% communication reduction is achieved while guaranteeing a performance bound when compared to periodic control implementations.

The remainder of the paper is organized as follows. Section II formulates the problem. In Section III we state the main result of the paper. Section IV presents two examples. The first example consists of remote control of a scalar system and the second example is concerned with the control of double integrator over a communication network. Section V provides concluding remarks.

A. Nomenclature

The trace of a square matrix $A \in \mathbb{R}^{n\times n}$ is denoted by $\mathrm{Tr}(A)$ and expected value vector and covariance matrix of a random $\eta \in \mathbb{R}^n$ are denoted by $\mathbb{E}[\eta]$ and $\mathrm{Cov}[\eta]$ respectively. For a symmetric matrix $Z \in \mathbb{R}^{n\times n}$, we write $Z \succ 0$ if Z is positive definite. The identity mapping is denoted by id and $\circ$ denotes composition operator.

II. Problem Formulation

We consider the remote control of a linear discrete-time system as depicted in Fig. 1. The plant model is assumed to be given by

$$\begin{aligned} x_{k+1} &= Ax_k + B\hat{u}_k + v_k \\ \hat{y}_k &= Cx_k + r_k, \end{aligned} \tag{1}$$

where $x_k \in \mathbb{R}^{n_x}$, $\hat{u}_k \in \mathbb{R}^{n_u}$ and $\hat{y}_k \in \mathbb{R}^{n_y}$ denote the state, the input, and the output, respectively and v_k and r_k denote the state disturbance and measurement noise at time $k \in \mathbb{N}_0$. We assume that the disturbance and noise processes are Gaussian zero-mean, independent sequences of random vectors with covariances Φ_v and Φ_r, respectively. The initial state is assumed to be either a Gaussian random variable with mean $\hat{x}_0$ and covariance Θ_0 or known in which case x_0 equals $\hat{x}_0$ and $\Theta_0 = 0$.

The transmissions in the networks are modeled by introducing $\sigma_k = (\beta_k, \gamma_k) \in \{0,1\}^2, k \in \mathbb{N}_0$, as a decision vector where $\beta_k = 1$ (or $\gamma_k = 1$) indicates the occurrence of a transmission through the network from sensor to controller (or controller to actuator) at time k and $\beta_k = 0$ (or $\gamma_k = 0$) otherwise. We also consider that a standard zero-order hold device holds the most recently received value of the control action at the actuator side (in case no new control input is transmitted). Let u_k (or y_k) denote the sent (or received) value by the controller at time $k \in \mathbb{N}_0$ when a transmission occurs. We write $u_k = \emptyset$ (or $y_k = \emptyset$) to denote the case when at time $k \in \mathbb{N}_0$ no new values are transmitted.

The transmission decisions in both networks connecting the sensors to the controller and the controller to the actuators are assumed to be taken by the controller. For the network connecting the controller to the actuators, respectively, the controller needs only to send data when desired. However, for the network connecting the sensors to the controller, the controller must first query the sensors and then receive measurement data. We assume that the delay introduced by this process is negligible and assume there that there are no packet drops in both networks.

We consider the following cost to be minimized

$$\mathbb{E}[\sum_{k=0}^{\infty} \alpha^k (x_k^T Q x_k + \hat{u}_k^T R \hat{u}_k)], \tag{2}$$

where $0 < \alpha < 1$ is the discount factor and $Q \succ 0, R \succ 0$ are proper (positive definite) weighting matrices. This cost is introduced for convenience as we are mostly interested in the minimization of the average cost defined as

$$\lim_{N\to\infty} \frac{1}{N} \mathbb{E}[\sum_{k=0}^{N-1} (x_k^T Q x_k + \hat{u}_k^T R \hat{u}_k)]. \tag{3}$$

Let I_{k-1} denote the information available to the controller at time k, i.e.,

$$I_{k-1} := (I_{k-2}, y_{k-1}, u_{k-1}, \sigma_{k-1})$$

for $k \in \mathbb{N}_{\geq 1}$ and $I_{-1} := (\hat{x}_0,\ \Theta_0)$. A policy $\pi := (\mu_0, \mu_1, \dots)$ is defined as a sequence of multivariate functions $\mu_k := (\mu_k^u, \mu_k^\sigma)$ that map the available information vector I_{k-1} into control actions u_k and scheduling σ_k, $k \in \mathbb{N}_0$.

We denote by $J_\pi^d(I_0)$ and J_π^a the costs (2) and (3), respectively, when

$$(u_k, \sigma_k) = \mu_k(I_{k-1}), \quad k \in \mathbb{N}_0. \tag{4}$$

We are interested in designing a policy that reduces the transmissions of the all-time transmission i.e. $\sigma_k = (1,1)$ for every time step $k \in \mathbb{N}_0$, while keeping the performance within a desired bound of the performance of the all-time transmission policy. We recall that the optimal control policy corresponding to the all-time transmission is given by

$$\mu_{all}^u(I_{k-1}) = L\hat{x}_{k|k-1} \tag{5}$$

where

$$\begin{aligned} L &= -(R + \alpha B^T K B)^{-1} B^T K A \\ K &= Q + \alpha A^T K A - P \\ P &= \alpha^2 A^T K B (R + \alpha B^T K B)^{-1} B^T K A. \end{aligned} \tag{6}$$

and $\hat{x}_{k|k-1} := \mathbb{E}[x_k | I_{k-1}]$ can be obtained by running a time-varying Kalman filter [20]. This results in the following cost-to-go in the discounted case

$$J_{\pi_{all}}^d(I_{k-1}) = \mathbb{E}[x_k^T K x_k | I_{k-1}] + \frac{\alpha}{1-\alpha} \mathrm{Tr}(K\Phi_v) + \sum_{s=k}^{\infty} \alpha^{s-k} \mathrm{Tr}(P\Sigma_s), \tag{7}$$

where $\Sigma_s = \text{Cov}[x_k|I_{k-1}]$ denotes the conditional covariance matrix of the estimation error that can be expressed as

$$\Sigma_s = \text{Ric}^{\,s}(\Sigma_0), \quad s \in \mathbb{N}_0 \tag{8}$$

where

$$\text{Ric}^{\,1}(\Sigma) = A\Sigma A^T + \Phi_v - A^T\Sigma C^T(C\Sigma C^T + \Phi_r)^{-1}C\Sigma A$$

and $\text{Ric}^s = \text{Ric}^{s-1} \circ \text{Ric}^1$, $\text{Ric}^0 = \text{id}$. Furthermore, for the average cost problem the minimizing control policy is as (5) with $\alpha = 1$ and the corresponding average cost is

$$J^a_{\pi_{all}} := \text{Tr}(K\Phi_v) + \text{Tr}(P\bar{\Sigma}), \tag{9}$$

where $\bar{\Sigma}$ is the steady state covariance of the Kalman filter estimate defined as

$$\bar{\Sigma} = \lim_{s\to\infty} \Sigma_s.$$

and given by discrete Algebraic Riccati equation.

III. Main Result

The main result of the paper is summarized in the following theorem. We say that (1) for a given policy π is mean square stable if $\sup_{k\in\mathbb{N}_0} \mathbb{E}[x_k^\intercal x_k] \le a$ for some positive constant a.

Theorem 1: Consider system (1) with policy π_{ro} parameterized by two non-negative scalars ζ, θ and defined for $k \in \mathbb{N}_0$ by

$$(u_k, \gamma_k) = \begin{cases} (L\hat{x}_{k|k-1}, 1), & \text{if } \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{u}_{k-1} \end{bmatrix}^T \Gamma \begin{bmatrix} \hat{x}_{k|k-1} \\ \hat{u}_{k-1} \end{bmatrix} > \lambda_k \\ (\emptyset, 0), & \text{otherwise} \end{cases} \tag{10}$$

$$\beta_k = \begin{cases} 1, & \text{if } h(\Sigma_k) > \zeta \\ 0, & \text{otherwise,} \end{cases} \tag{11}$$

where

$$\begin{aligned} \Gamma &= \begin{bmatrix} (1+\theta)P - \theta Q & \alpha(1+\theta)A^T KB \\ \alpha(1+\theta)B^T KA & R + \alpha(1+\theta)B^T KB \end{bmatrix} \\ \lambda_k &= \theta\,\text{Tr}(Q\Sigma_k) \\ \hat{x}_{k|k-1} &= \mathbb{E}[x_k|I_{k-1}] \\ \Sigma_k &= \text{Cov}[x_k|I_{k-1}] \end{aligned} \tag{12}$$

and

$$\begin{aligned} h(\Sigma) = \sum_{s=0}^{\infty} \alpha^{s+1}\,\text{Tr}\Big(P\big(&\text{Ric}^{\,s}(A\Sigma A^T + \Phi_v) \\ &- \text{Ric}^{\,s+1}(\Sigma)\big)\Big) \end{aligned} \tag{13}$$

is well defined in the sense that it is finite for every $\Sigma \in \mathbb{R}^{n_x\times n_x}$. Then

$$J^d_{\pi_{ro}}(I_0) \le (1+\theta)(J^d_{\pi_{all}}(I_0) + \frac{1}{1-\alpha}\zeta), \tag{14}$$

for every I_0. Furthermore, for $\alpha = 1$, the system (1), (4) is mean square stable for policy π_{ro} and the associated average cost satisfies

$$J^a_{\pi_{ro}} \le (1+\theta)(J^a_{\pi_{all}} + \zeta). \tag{15}$$

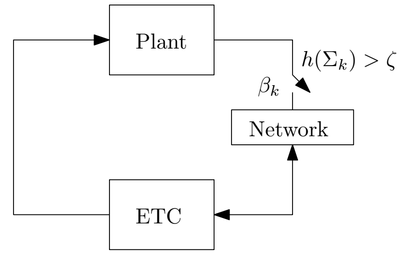

Fig. 2. Sensor query schematic

□

For the sake of brevity the proofs are omitted.

There are two options to compute $h(\Sigma)$. First, one can discretize the space of positive definite symmetric matrices of dimension $n_x \times n_x$ and compute beforehand the value of this function for a finite set of points Σ_i. Then, $h(\Sigma) \approx h(\Sigma_i)$ for a matrix Σ_i close to Σ. This is naturally only possible for small n_x. Second, we can approximate (13) by a finite summation (for a desirable precision), which must be computed online. The latter case requires more computational time and less memory resources than the former one.

We show in the state estimate subsection of current section that $\hat{x}_{k|k-1} = \mathbb{E}[x_k|I_{k-1}]$ and $\Sigma_k = \text{Cov}[x_k|I_{k-1}]$ can be obtained by the controller by running the time-varying Kalman filter. As we shall see $\Sigma_k = \text{Cov}[x_k|I_{k-1}]$ can be determined a priori, which entails that the scheduling sequence for sensor queries, triggered by condition (11) can be determined offline. In turn, the state estimate $\hat{x}_{k|k-1} = \mathbb{E}[x_k|I_{k-1}]$ depends on the noise realizations and therefore must be determined online. Consequently, the scheduling decisions, triggered by condition (10), must be determined by the controller online.

Remark 1: (Trade-off transmission versus performance guarantees) The two non-negative scalars θ and ζ should be seen as tunning knobs of the proposed method, which enable the adjustment of the transmission versus performance trade-off. It is clear that increasing ζ in (11) will make the sensor query triggering condition less stringent and thus one should expect less transmissions from sensors to the controller. This also leads to less tight performance guarantees (14),(15). It is also possible to see that increasing θ makes (10) less stringent and thus one should expect less transmissions from controller to actuators. Again, this also leads to less tight performance guarantees (14),(15). Note that if $\zeta = 0$ then transmissions from sensors to controller are triggered at every time step $k \in \mathbb{N}_0$, and if $\theta = 0$ then transmission from the controller to the actuators are triggered at every time step $k \in \mathbb{N}_0$. If $\zeta = 0$ and $\theta = 0$, we recover the all-time transmission control policy π_{all} and (14),(15) hold with equality.

Remark 2: When the controller is collocated with the actuators, as depicted in Fig. 2, there is only one communication network. The scheduling variable β_k determines if a new measurement should be obtained or not according to

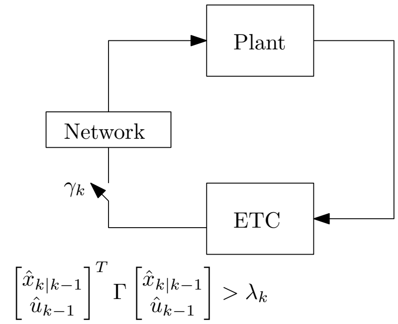

Fig. 3. ETC collocated with the sensors

the rule

$$\beta_k = \begin{cases} 1, & h(\Sigma_k) > \zeta \\ 0, & \text{otherwise} \end{cases}$$

and the control policy

$$\mu_k^{ro,u} = L\hat{x}_{k|k-1} \tag{16}$$

for $k \in \mathbb{N}_0$ that guarantees the performance bound of

$$J^d_{\pi_{ro}}(I_0) \leq J^d_{\pi_{all}}(I_0) + \frac{1}{1-\alpha}\zeta, \tag{17}$$

for every I_0 for the discounted cost. Furthermore, for $\alpha = 1$, the associated average cost satisfies

$$J^a_{\pi_{ro}} \leq J^a_{\pi_{all}} + \zeta. \tag{18}$$

Remark 3: Consider now the case where the controller is collocated with the sensors. This configuration, depicted in Fig. 3, is similar to the one considered in [19], where a relative triggering law with performance guarantees is proposed. The scheduling and control policies of Theorem 1, π_{ro}, become for $k \in \mathbb{N}_0$

$$\mu_k^{ro} = \begin{cases} \big(1, L\hat{x}_{k|k-1}\big), & \begin{bmatrix}\hat{x}_{k|k-1}\\ \hat{u}_{k-1}\end{bmatrix}^T \Gamma \begin{bmatrix}\hat{x}_{k|k-1}\\ \hat{u}_{k-1}\end{bmatrix} > \lambda_k \\ \big(0, \emptyset\big), & \text{otherwise,} \end{cases} \tag{19}$$

where

$$\begin{aligned} \Gamma &= \begin{bmatrix} (1+\theta)P - \theta Q & \alpha(1+\theta)A^T KB \\ \alpha(1+\theta)B^T KA & R + \alpha(1+\theta)B^T KB \end{bmatrix} \\ \lambda_k &= \theta \operatorname{Tr}(Q\Sigma_k). \end{aligned} \tag{20}$$

This policy guarantees the performance bound

$$J^d_{\pi_{ro}}(I_0) \leq (1+\theta) J^d_{\pi_{all}}(I_0), \tag{21}$$

for every I_0. Furthermore, for $\alpha = 1$, the associated average cost satisfies

$$J^a_{\pi_{ro}} \leq (1+\theta) J^a_{\pi_{all}}. \tag{22}$$

Hence, we recover here the results in [19] as a special case of the general framework summarized in Theorem 1.

State Estimate

To obtain $\hat{x}_{k|k-1} = \mathbb{E}[x_k|I_{k-1}]$ and $\Sigma_k = \operatorname{Cov}[x_k|I_{k-1}]$ the controller can run the time-varying Kalman filter

$$\begin{aligned} \hat{x}_{k+1|k} &= A\hat{x}_{k|k-1} + Bu_k + \beta_k G_k\big(y_k - C\hat{x}_{k|k-1}\big) \\ G_k &= A\Sigma_k C^T (C\Sigma_k C^T + \Phi_r)^{-1} \\ \Sigma_{k+1} &= \overline{\operatorname{Ric}}(\Sigma_k, \beta_k), \end{aligned} \tag{23}$$

where

$$\begin{aligned} \overline{\operatorname{Ric}}(\Sigma, j) = A\Sigma A^T + \Phi_v - \\ jA^T\Sigma C^T (C\Sigma C^T + \Phi_r)^{-1} C\Sigma A, \end{aligned} \tag{24}$$

β_k is known and determined by the controller and u_k is the input to the plant, also known to the controller and defined by the recursion

$$u_k = \begin{cases} u_{k-1}, & \text{if } \gamma_k = 0 \\ L\hat{x}_{k|k-1}, & \text{if } \gamma_k = 1. \end{cases}$$

This follows from the fact that the conditional distribution of x_k given I_{k-1} is Gaussian (the proof of this fact can be concluded from a similar proof in [21]).

Remark 4: Note that since β_k is a function of Σ_k, given the initial error covariance Σ_k the error covariance evolves autonomously according to

$$\Sigma_{k+1} = \overline{\operatorname{Ric}}(\Sigma_k, \beta_k(\Sigma_k)), k \in \mathbb{N}_0$$

As a result Σ_k does not depend on the output noise and state disturbances realizations of the plant and can be determined a priori. In particular, the scheduling sequence of sensor transmission $\{\beta_k\}_{k\in\mathbb{N}_0}$ can be determined offline. In turn, the state estimate obtained from (23) depends on the output noise and state disturbances through y_k and therefore the corresponding Kalman filter iterations must be computed online. As a consequence, the scheduling sequence $\{\gamma_k\}_{k\in\mathbb{N}_0}$ determining control input transmission cannot be determined a priori.

IV. Numerical Example

In this section, we present two numerical examples.

A. Scalar system

Consider the scalar dynamical system

$$\begin{aligned} x_{k+1} &= 2x_k + \hat{u}_k + v_k \\ \hat{y}_k &= 3x_k + r_k, \end{aligned} \tag{25}$$

where v_k and r_k are Gaussian zero-mean, independent random sequences with covariance $\Phi_s = 0.01$ and $\Phi_r = 0.02$. The system is connected to the remote controller through two separate networks as depicted in Fig. 1. We consider an average cost problem with $Q = 1$ and $R = 2$. We compare two cases, one with $\theta = \zeta = 0$ which corresponds to all-time transmission policy, π_{all}, and the other with $\theta = 0.2$ and $\zeta = 1.65$ implementing the triggering schemes, π_{ro}, (10)-(11). The method to compute the function h is based on discretization with a fine grid,

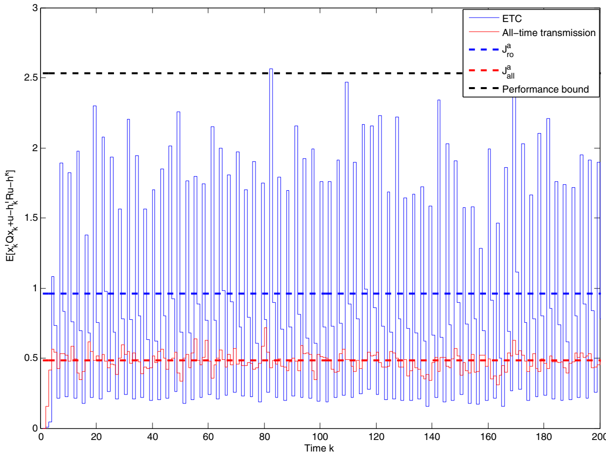

Fig. 4. $\mathbb{E}[x_k^T Q x_k + \hat{u}_k^T R \hat{u}_k]$ estimated via Monte Carlo simulation in two cases with $\theta = \zeta = 0$ and $\theta = 0.2,\ \zeta = 1.65$.

as discussed directly after the formulation of Theorem 1. The state estimation and related covariances are computed using a time-varying Kalman filter. Interestingly, by applying the proposed ETC scheme, the sensor and actuator network usage reduce to 66% and 90%, respectively, while preserving the stability of the closed-loop system. Fig. 4 shows the estimation of running cost $\mathbb{E}[x_k^T Q x_k + \hat{u}_k^T R \hat{u}_k]$ based on Monte Carlo simulations. As can be seen the average cost of the proposed algorithm satisfies the performance bound (15) as follow

$$0.97 = J^a_{\pi_{ro}} \leq 1.2(J^a_{\pi_{all}} + 1.63) = 2.53.$$

where $J^a_{\pi_{all}} = 0.4815$ computed using (9). Notice that the average cost of proposed ETC method $J_{\pi_{ro}}$ (blue dashed line in Fig. 1) is much less than the theoretical performance bound (black dashed line in Fig. 1) and close to the average cost of the all-time transmission policy $J^a_{\pi_{all}}$ (red dashed line in Fig. 1) which further emphasize that the reduction in network usage has been achieved with far less expected degradation in performance.

B. Double Integrator

We consider a discretized model of a double integrator controlled over network as depicted in Fig 1. The system and cost parameters are given by

$$A = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix},\ B = \begin{bmatrix} 0.005 \\ 0.1 \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad Q = 0.1I, \quad R = 0.01.$$

For this example, we compare two cases. The first one is with $\theta = 0,\ \zeta = 0$ i.e. the all-time transmission policy, π_{all}, and the other case based on π_{ro} as in Theorem 1 with $\theta = 0.1,\ \zeta = 0.5$. In both cases, a time-varying Kalman filter is used to provide a state estimation based on the available information at each iteration (i.e. the error covariance of the current state and the current state estimation). We compute h online using the method discussed right after Theorem 1. Fig. 5 shows the estimated running cost $\mathbb{E}[x_k^T Q x_k + \hat{u}_k^T R \hat{u}_k]$

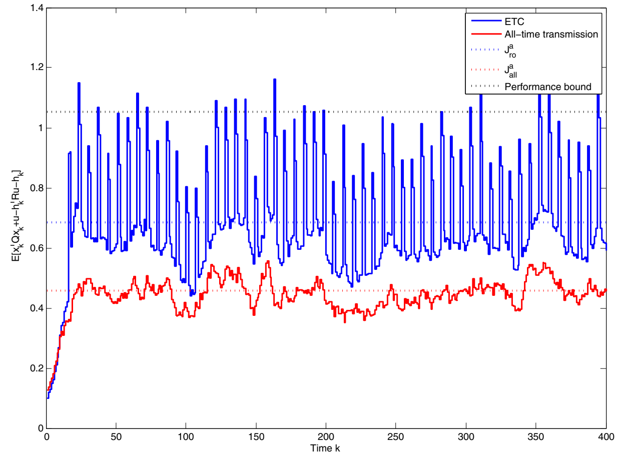

Fig. 5. $\mathbb{E}[x_k^T Q x_k + \hat{u}_k^T R \hat{u}_k]$ estimated via Monte Carlo simulation in two cases with $\theta = \zeta = 0$ and $\theta = 0.1,\ \zeta = 0.5$.

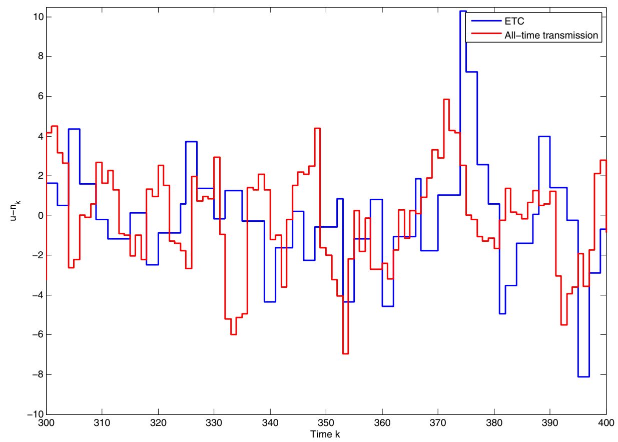

Fig. 6. Trajectory of actuation signal, $\hat{u}_k$, for two cases with $\theta = \zeta = 0$ and $\theta = 0.1,\ \zeta = 0.5$.

for $k \in \mathbb{N}_0$ based on Monte Carlo simulations in both cases. The blue and red dotted lines represent $J^a_{\pi_{ro}}$ and $J^a_{\pi_{all}}$ respectively. The black dotted line shows the theoretical bounds computed using (9) and (15) which satisfies

$$0.6842 = J^a_{\pi_{ro}} \leq 1.1(J^a_{\pi_{all}} + 0.5) = 1.05.$$

where $J^a_{\pi_{all}} = 0.45$. Similar to previous example, one can observe that the theoretical upper bound (black dotted line in Fig. 5) is not tight and the average cost of the proposed ETC scheme $J^a_{\pi_{ro}}$ (blue dotted line in Fig. 5) is much closer to the corresponding average cost of all-time transmission control policy $J^a_{\pi_{all}}$ (red dotted line in Fig. 5). Interestingly, the proposed ETC scheme not only reduces the network usage to 14% at the sensor side and 9% at the actuator side with respect to the all-time transmission case but also preserves the stability of the closed-loop system while guaranteeing a performance bound. To see the reduction in communications, the actuator signals of one realization of both cases with the same noise are shown in Fig. 6. As can be seen, the actuator holds its value more often (i.e. less transmissions occur) when applying the proposed method.

V. Conclusion

In this paper we proposed a simple and easy to implement event-triggered control strategy for linear discrete-time with performance guarantees compared to the all-time transmission control policy, while reducing the overall communication load. The considered setup consists of a networked control system in which a remote controller queries the plant's sensors for measurement data and decides when to transmit control inputs to the plant's actuators. Our proposed approach is obtained via an optimization-based scheme in which the performance is measured by a quadratic cost. The resulting policy can be separated into an offline scheme for sensor query and an online scheme to schedule control input transmissions. The usefulness of the results was illustrated through two numerical examples that led to significant communication savings with only a moderate loss in performance compared to the all-time transmission policy.

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