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Two Approaches to State Estimation for a Class of Piecewise Affine Systems [1]

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Abstract

In this paper we present two approaches to state estimation for a class of discrete time bi-modal piece-wise affine systems.The proposed approaches have the characteristic feature that they do not require information on the currently active dynamics of the piecewise affine system. We propose a Luenberger-type observer, and derive sufficient conditions for the observation error to be globally asymptotically stable, in the case when the system dynamics is continuous over the switching plane. When the dynamics is discontinuous, we derive conditions that guarantee that the estimation error will be bounded with respect to the state bound. Second, we propose to apply particle filtering algorithm, which aims at approximating the a posteriori probability density of the state. The presented approaches are compared and illustrated with examples.

1 Introduction

In this paper we present two approaches to the state estimation problem for a class of discrete time bi-modal piece-wise affine systems. The systems of the considered class comprise two linear dynamics with the same input distribution matrix B, and the active dynamics is chosen depending on the half-space in which the state resides. The characteristic feature of our approach is that the state reconstruction is performed on the basis of input and measured output signals only, while the information on the active linear dynamics (or mode) is not available. The presented ideas and the main line of reasoning can be extended to more general classes of piece-wise affine systems.

Observer design for the case when the mode of the hybrid system is known is presented in [1,2,10]. The proposed observer is of Luenberger type, and, if feasible, achieves global asymptotic stability of the observation error. A more difficult case, when the discrete mode is not known, was considered in [4]. The proposed observers use discrete inputs and outputs of the hybrid plant, augmented with discrete signals derived from the continuous measurements when necessary, to obtain the estimate of the mode. Subsequently, the estimate of the continuous state can be obtained, for example, using the techniques of [1,2,10]. The designed observers correctly identify the mode of the plant after a finite number of time steps, and the continuous observation error exponentially converges to the bounded set. The class of systems considered in this paper does not have discrete inputs and outputs and therefore we propose a more direct approach for state estimation.

In this paper, the first proposed state estimator has the form of a Luenberger-type observer. The design procedure is devised for choosing observer gains, based on finding feasible solutions to a set of linear matrix inequalities. We distinguish two cases. When the vector field of the overall system is continuous over the switching plane and the derived matrix inequalities are feasible global asymptotic stability of the observation error can be guaranteed. In the case of the discontinuous vector field the estimation error is guaranteed to be bounded, relative to the state bound. A Luenberger observer approach to state estimation problem for continuous time systems of similar type was considered in [12].

In addition to the deterministic approach, based on Luenberger observers, we also consider a stochastic setup, in which we propose to use the particle filtering algorithm. The output of the algorithm is an approximation of the a posteriori probability density of the state, conditioned on all available output measurements. A tutorial on particle filtering can be found in [3], and an overview of convergence results for some classes of systems is available in [8]. Particle filtering has been considered before in the context of hybrid state estimation, for instance for state estimation of jump Markov linear systems [9], in the target tracking applications [6, 14] and fault detection and diagnostics [13]. In these works the discrete dynamics is assumed to evolve according to a Markov chain, while in our case the switching is completely deterministic.

The main advantage of the particle filtering algorithms is that they are able to successfully cope with non-Gaussian probability density functions. This can be of particular interest in hybrid state estimation, where non-Gaussian distributions can arise as an inherent property of the considered estimation problem. This will be illustrated by the example.

The paper is organized as follows. In section 2 we present the problem formulation, with a discussion on some special cases. In section 3 we present a deterministic approach to state estimation based on observers of Luenberger type. In section 4 we present a state estimation procedure based on the particle filtering approach. In section 5 two examples are presented, illustrating the derived theory. Conclusions and future work are presented in section 6.

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2 Problem statement

Consider the following dynamical system:

$$x(k+1) = \begin{cases} A_1 x(k) + Bu(k), & \text{if } H^\top x(k) \leq 0 \\ A_2 x(k) + Bu(k), & \text{if } H^\top x(k) > 0 \end{cases} \quad (1a)$$

$$y(k) = Cx(k) \quad (1b)$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^p$, $u \in \mathbb{R}^m$, $A_1, A_2 \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$ and $H \in \mathbb{R}^n$. The hyperplane defined by $\ker H^\top$ therefore separates the state space into two half-spaces in which one of the two dynamics is active.
System (1) is a bi-modal piece-wise affine system, with the same input distribution matrix B for both modes. Furthermore, depending on the values of A_1, A_2 we distinguish two situations:

1. the vector field of the system is continuous over the switching plane, i.e. $A_1 x = A_2 x$, when $H^\top x = 0$. It is straightforward to show that in this case:

   $$A_2 = A_1 + GH^\top \quad (2)$$

   for some vector G of appropriate dimensions. In this case equation (1a) can be rewritten as:

   $$x(k+1) = A_1 x(k) + G \max(0, H^\top x(k)) + Bu(k).$$

   Also, from (2), $\text{rank}(\Delta A) = 1$, where $\Delta A = \text{rank}(A_1 - A_2)$.

2. the vector field of the system is not continuous over the switching plane, i.e. a parameterization as in (2) can not be found

The problem at hand is to design a state estimation procedure, which, on the basis of the known system model, input $u(k)$, and measured output $y(k)$ provides a state estimate $\hat{x}(k)$.
In the sequel we will need the following definition.

Definition 2.1 The sequence $(x(0), x(1), x(2), \ldots)$ is said to be *bounded by* x_{max} if

$$\forall k \geq 0 \quad \|x(k)\| \leq x_{max}.$$

The sequence $(x(0), x(1), x(2), \ldots)$ is said to be *eventually bounded by* x_{max} if

$$\forall \delta > 0 \quad \exists k_0 > 0 \quad \forall k \geq k_0 \quad \|x(k)\| \leq x_{max} + \delta.$$

i.e. $\limsup_{k\to\infty} \|x(k)\| \leq x_{max}$

In matrices $(*)$ at position (i, j) denotes the transposed matrix element at position (j, i), e.g.

$$\begin{bmatrix} A & B \\ (*) & C \end{bmatrix} \text{ means } \begin{bmatrix} A & B \\ B^\top & C \end{bmatrix}.$$

3 Deterministic approach

As an observer for the system (1), we propose a bi-modal system with the following structure:

$$\hat{x}(k+1) = \begin{cases} A_1\hat{x}(k) + Bu(k) + L_1(y(k) - \hat{y}(k)), \\ \qquad \text{if } H^\top \hat{x}(k) \leq 0 \\ A_2\hat{x}(k) + Bu(k) + L_2(y(k) - \hat{y}(k)), \\ \qquad \text{if } H^\top \hat{x}(k) > 0 \end{cases} \quad (3a)$$

$$\hat{y}(k) = C\hat{x}(k) \quad (3b)$$

where $\hat{x} \in \mathbb{R}^n$ and L_1 and $L_2 \in \mathbb{R}^{n\times p}$ are matrices. The design problem consists of determining observer gains L_1, L_2 so that the state estimate $\hat{x}$ approximates x as good as possible, e.g. $\hat{x}$ should asymptotically converge to x, or the estimation error

$$e(k) = x(k) - \hat{x}(k). \quad (4)$$

should be bounded. The dynamics of the state estimation error is then described by

$$e(k+1) = \begin{cases} (A_1 - L_1C)e(k), \\ \qquad H^\top x(k) \leq 0, H^\top \hat{x}(k) \leq 0 \\ (A_2 - L_2C)e(k) + \Delta A x(k), \\ \qquad H^\top x(k) \leq 0, H^\top \hat{x}(k) > 0 \\ (A_1 - L_1C)e(k) - \Delta A x(k), \\ \qquad H^\top x(k) > 0, H^\top \hat{x}(k) \leq 0 \\ (A_2 - L_2C)e(k), \\ \qquad H^\top x(k) > 0, H^\top \hat{x}(k) > 0 \end{cases} \quad (5)$$

where $x(k)$ satisfies (1a) and $\hat{x}(k)$ satisfies (3a). By substituting $\hat{x} = x - e$ in (5), we see that the right-hand side of the state estimation error dynamics is piece-wise linear in the variable $v := \text{col}(e, x)$. (Here, col stacks subsequent entries of its argument in a column matrix).
In order to obtain stable error dynamics we search for a Lyapunov function of the form

$$V(x) = x^\top Px \quad (6)$$

where $P = P^\top > 0$, such that:

$$V(e(k+1)) - V(e(k)) < 0, \quad (7)$$

for $e(k) \neq 0$. Considering the first and the fourth mode of error dynamics (5), (7) becomes:

$$e^\top\{(A_1 - L_1C)^\top P(A_1 - L_1C) - P\}e < 0 \quad (8a)$$

$$e^\top\{(A_2 - L_2C)^\top P(A_2 - L_2C) - P\}e < 0 \quad (8b)$$

for each $e \neq 0$. Considering the second and third mode of the error dynamics (5) we get the following inequalities:

$$\begin{bmatrix} e \\ x \end{bmatrix}^\top \begin{bmatrix} (A_2 - L_2C)^\top P \times \\ \times (A_2 - L_2C) - P & (*) \\ \Delta A^\top P(A_2 - L_2C) & \Delta A^\top P \Delta A \end{bmatrix} \begin{bmatrix} e \\ x \end{bmatrix} < 0 \quad (8c)$$

$$\begin{bmatrix} e \\ x \end{bmatrix}^\top \begin{bmatrix} (A_1 - L_1C)^\top P \times \\ \times (A_1 - L_1C) - P & (*) \\ -\Delta A^\top P(A_1 - L_1C) & \Delta A^\top P \Delta A \end{bmatrix} \begin{bmatrix} e \\ x \end{bmatrix} < 0 \tag{8d}$$

Note that (8c),(8d) can not be negative definite if $\Delta A \neq 0$, because the term in the lower right corner is always at least positive-semidefinite.

Therefore, in order to obtain feasible equations conditions (8a)-(8d) have to be relaxed. We will consider two ways of relaxing the requirements, using the S-procedure [7, 11].

First, note that (8a)-(8d) do not need to hold in the whole (e, x) space, but only when the respective modes of the error dynamics are active. The second and the third mode of (5) are active only when:

$$x^\top H H^\top (x - e) < 0. \tag{9}$$

Combining (9) with (8c),(8d), and taking the Schur complement of the obtained matrices leads to the following theorem.

Theorem 3.1 *The state estimation error dynamics (5) is globally asymptotically stable if there exist matrices $P = P^\top > 0, L_1, L_2$ and constants $\lambda_1, \lambda_2 \geq 0$ such that the following set of matrix inequalities is satisfied:*

$$\begin{bmatrix} P & (A_i - L_iC_i)^\top P \\ P(A_i - L_iC_i) & P \end{bmatrix} > 0 \tag{10a}$$

$$\begin{bmatrix} P & 0 & (A_i - L_iC)^\top P & 0 \\ 0 & P & 0 & (*) \\ (*) & 0 & P & (*) \\ 0 & \Delta A^\top P & \begin{matrix} -\frac{1}{2}\lambda_i H H^\top \\ -(-1)^i \Delta A^\top \times \\ \times P(A_i - L_iC) \end{matrix} & \lambda_i H H^\top \end{bmatrix} \geq 0 \tag{10b}$$

for $i = 1, 2$.

The previous result is applicable only to systems with a continuous vector field. Indeed, the term in the lower right corner is positive semidefinite by construction, and of rank at most 1. The following inclusion must hold (see [12] for details):

$$\ker H^\top \subseteq \ker \Delta A$$

which implies:

$$A_2 = A_1 + G H^\top$$

for some G of suitable dimensions.

In order to overcome this limitation we search for another way to relax the requirements (8a)-(8d). Condition (7) will be required only when:

$$\|e\|^2 \geq \varepsilon^2 \|x\|^2. \tag{11}$$

Combining (11) with (8c),(8d) we get the following theorem:

Theorem 3.2 *The state estimation error e is eventually bounded by e_{max}, under the assumption that x is bounded by x_{max} if there exist matrices $P = P^\top > 0$, L_1, L_2, and constants $\lambda_1, \lambda_2 > 0$ such that the following set of matrix inequalities is satisfied:*

$$\begin{bmatrix} P & 0 & (A_i - L_iC)^\top P & 0 \\ 0 & P & 0 & (*) \\ (*) & 0 & P - \lambda_i I & (*) \\ 0 & \Delta A^\top P & \begin{matrix} -(-1)^i \Delta A^\top \times \\ \times P(A_i - L_iC) \end{matrix} & \lambda_i \varepsilon I \end{bmatrix} \geq 0 \tag{12}$$

for $i = 1, 2$. Moreover, if

$$\gamma_1 I \leq P \leq \gamma_2 I \tag{13}$$

then

$$e_{max} \leq \sqrt{\frac{\gamma_2}{\gamma_1}} \varepsilon x_{max}. \tag{14}$$

The proof of the previous theorem is similar to the continuous time case, that has been studied in [12]. Note that we can combine both relaxations (9) and (11), to obtain the same properties of the estimation error e as in theorem 3.2.

Equation (14) explicitly gives the eventual upper bound of the estimation error. The observer gains L_1, L_2 can be determined so as to minimize this upper bound , which amounts to minimizing γ_2/γ_1 and ε, under (12). If it is possible to design Luenberger observers for both constituting linear dynamics with a common Lyapunov function of the form (6), equations (12) can always be made feasible for large enough ε.

Some examples of observer design using developed theory will be shown in section 5. First, we present the particle filtering approach to the state estimation for the considered class of systems.

4 Particle filtering approach

Particle filtering belongs to the class of sequential Monte Carlo methods. The underlying system is assumed to be stochastic, in the sense that process noise w and measurement noise v are assumed to be present. The probability densities of the noise (p_w and p_v) are assumed to be known. No other assumption on the nature of the noise is made. In our case, consider the system, augmented with process and measurement noise terms:

$$x(k+1) = \begin{cases} A_1 x(k) + Bu(k) + w(k), & \text{if } H^\top x(k) \leq 0 \\ A_2 x(k) + Bu(k) + w(k), & \text{if } H^\top x(k) > 0 \end{cases} \tag{15a}$$

$$y(k) = Cx(k) + v(k) \tag{15b}$$

The idea of the method is roughly the following. The state space is initially populated with a finite number of particles N, which are distributed according to the (assumed) a priori probability density of the state. At each time step each particle is propagated according to the system dynamics. For each particle the likelihood (particle mass) related to the measured output is computed, and the particles are resampled (resampling step has been shown to be necessary, in order to prevent all of the particle masses asymptotically going to zero (see e.g. [3])). The a posteriori density of the state is constructed in the number of points, on the basis of particle masses. A detailed statement of the described algorithm, as applied to considered class of systems, follows.

We assume the process noise w in the system description (15) to be absent. The set of measured outputs up to time k is denoted by

$$Y(k) := \{y(0), \dots, y(k)\}.$$

Algorithm 4.1 Particle filtering:

1. Initialization: Set k=1, and draw N samples $\{\bar{x}_i(0)\}_{i=1,\dots,N}$ from the initial distribution $p(x(0))$
2. Prediction: Compute $\{\bar{x}_i(k)\}_{i=1,\dots,N}$, using system model (1), from $\{\bar{x}_i(k-1)\}_{i=1,\dots,N}$
3. Update: Compute the likelihood for each sample:
$$\bar{q}_i(k) = p(y(k)|\bar{x}_i(k)) = p_v(y(k) - C\bar{x}_i(k)), \quad (16a)$$
for $i = 1, \dots, N$.
4. Normalization:
$$q_i(k) = \frac{\bar{q}_i(k)}{\sum_{i=1}^{N} \bar{q}_i(k)} \quad (16b)$$
5. Resampling: draw N samples from
$$\hat{p}(x) = \sum_{i=1}^{N} q_i(k)\delta(x - \bar{x}_i(k)) \quad (16c)$$
(where $\delta(\cdot)$ is the Dirac delta function) to obtain a new set $\{\bar{x}_i(k)\}_{i=1,\dots,N}$, and construct
$$\hat{p}(x(k)|Y(k)) = \sum_{i=1}^{N} \frac{1}{N}\delta(x(k) - \bar{x}_i(k)) \quad (16d)$$
Set $k := k + 1$, and go to step 2.

The tuning parameters of the algorithm are the number of samples (particles) N, and the choice of the measurement noise probability density function p_v.

A state estimate $\hat{x}(k)$ can be obtained from the constructed approximate a posteriori probability density of the state $\hat{p}(x(k)|Y(k))$, for example, as the minimum variance estimator:

$$\hat{x}_{MV}(k) = E_{\hat{p}(x|Y(k))}x \quad (17)$$

or the maximum a posteriori probability estimator:

$$\hat{x}_{MAP}(k) = \arg\max_{x} \hat{p}(x|Y(k)). \quad (18)$$

In general, convergence of the estimated a posteriori density of the state to the true a posteriori density of the state can be expected, as the number of particles $N \to \infty$ (see [8] for more details). In this moment it is unclear whether these results transfer to the piecewise affine systems of the considered type, and it is a topic of the future research. Simulation examples show good properties of this methodology. Convergence of the state estimate obtained with (17) of (18) to the true state of the system depends also on the system properties (i.e. on system observability, as with any other state estimation scheme). This is shown in example 5.2.

5 Examples

Example 5.1 Consider the system (1) with the following parameter values:

$$A_1 = \begin{bmatrix} \cos(\frac{\pi}{3}) & \sin(\frac{\pi}{3}) \\ -\sin(\frac{\pi}{3}) & \cos(\frac{\pi}{3}) \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \cos(\frac{2\pi}{3}) & \sin(2\frac{\pi}{3}) \\ -\sin(\frac{2\pi}{3}) & \cos(\frac{2\pi}{3}) \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, H = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

which is discontinuous over the switching plane. Note that the first state determines the mode, while the second state is measured. Hence, the discrete mode can not be determined directly from the measurements.

Solving (12) with $\varepsilon = 0.1$ we obtain the following values for the gains of observer (3):

$$L_1 = \begin{bmatrix} 0.8662 \\ 0.5031 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.8662 \\ 0.4982 \end{bmatrix}.$$

with $e_{max} \le 0.13x_{max}$ (equation (14)).

The simulation results are depicted in figure 1. The input is chosen as a sequence of normally distributed random numbers, with zero mean and variance 1. The initial state of the system (1) is $x(0) = [-1 \quad -1]^{\top}$, and the the initial state of the observer (3) is $\hat{x}(0) = [5 \quad 5]^{\top}$

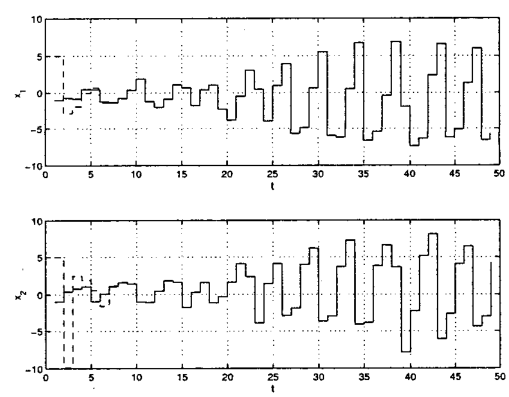

Figure 1: The system (solid) and observer (dashed) response (upper: state x_1, lower: state x_2)

The minimum variance state estimate (17), obtained from a particle filter with 500 particles, for the same input sequence, is depicted in figure 2. The distribution of the measurement noise is chosen to be Gaussian, with zero mean, and variance 0.1. The estimation error for both filters is depicted in figure 3.

We see that both state estimation procedures give good estimates of the state. Simulation shows that, for this example, the estimation error of the Luenberger-type observer converges to zero. The estimation error of the particle filter is present but small. This is due to the uncertainty added by

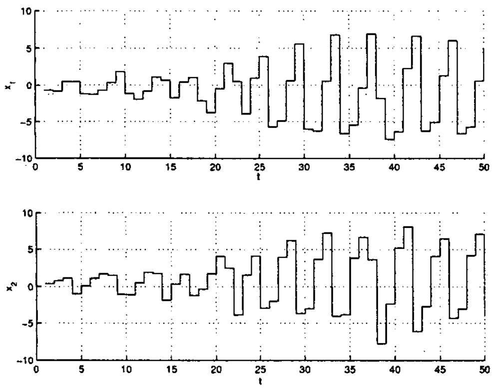

Figure 2: The system (solid) response and particle filter output (upper: state x_1, lower: state x_2) (figures overlap after $t \approx 3$)

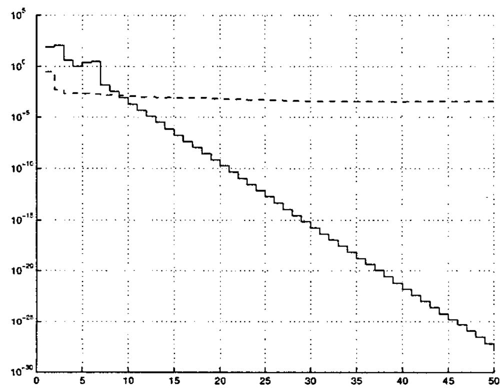

Figure 3: Norm of the estimation error of Luenberger observer (solid) and particle filter (dashed)

introducing measurement noise v. Transient performance of the Luenberger-type observer depends on the initial state estimate, while with particle filters transient performance depends on the properties of noise.

Example 5.2 Consider the system (1) with the following parameter values:

$$A_1 = \begin{bmatrix} 0.95 & 0.0475 \\ -0.0475 & 0.95 \end{bmatrix}$$

$$A_2 = A_1^{\mathsf{T}}$$

$$B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, H = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

The input matrix B is taken to be zero, so the system evolution is governed only by the initial state $x(0)$. Both pairs (A_1, C), (A_2, C) are observable. Consider two initial state vectors $x^1(0) = [a \quad b]^{\mathsf{T}}$ and $x^2(0) = [-a \quad b]^{\mathsf{T}}$, where $a \geq 0$. The output sequences $y_1(k)$ and $y_2(k)$ generated from $x^1(0)$ and $x^2(0)$, respectively, are the same for any $k > 0$, while the state trajectories are not, when $a \neq 0$. In other words, the system is unobservable (in the sense that state can not be uniquely determined from the measured output) whenever the first component of the state differs from 0.

Remark 5.3 Observability of piecewise affine and hybrid systems is a complex issue. Discussion on the topic and some computational tests for checking the observability can be found in [5].

The Luenberger observer is designed, using the methodology described in section 3. The following observer gains were obtained:

$$L_1 = \begin{bmatrix} -0.0495 \\ 0.8387 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.0455 \\ 0.7495 \end{bmatrix}.$$

while the best found error bound that can be guaranteed is $e_{max} \leq 24x_{max}$. The simulation is depicted in the figure 4, with initial states $x(0) = [0.2 \quad 4]^{\mathsf{T}}$, and $\hat{x}(0) = [-0.3 \quad 4]^{\mathsf{T}}$. We see that the state estimate $\hat{x}$ converges towards the other possible state trajectory, starting in $[-0.2 \quad 4]^{\mathsf{T}}$.

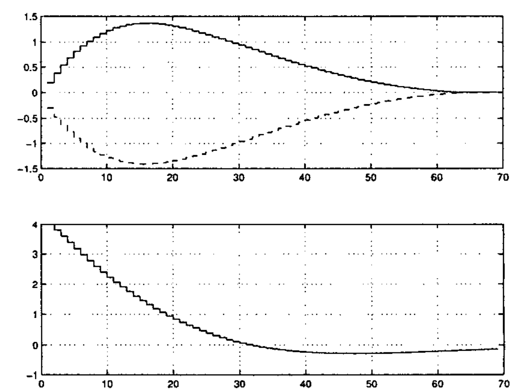

Figure 4: System (solid) and observer (dashed) response (upper: state x_1, lower: state x_2)

The distribution of the particles in the particle filter is shown in figure 5, at different times k. We see that the filter assigns approximately equal probabilities to both possible trajectories.

The estimates of the state obtained using either (17) or (18) will not converge to the true state. These state estimators give one estimate of the state, and can not correctly interpret the information contained in the reconstructed a posteriori probability density, which is by the problem nature bi-modal. The estimate obtained with (17) is the average of both possible trajectories, while the estimator (18) picks randomly one of the trajectories at each time instant. In both cases the obtained estimates are not possible in the system with dynamics (15).

A solution is to compute a set of possible state estimates, on the basis of the particle filter output. This problem is not trivial, and is similar, for example, to the problem of multiple target tracking with radar [14].

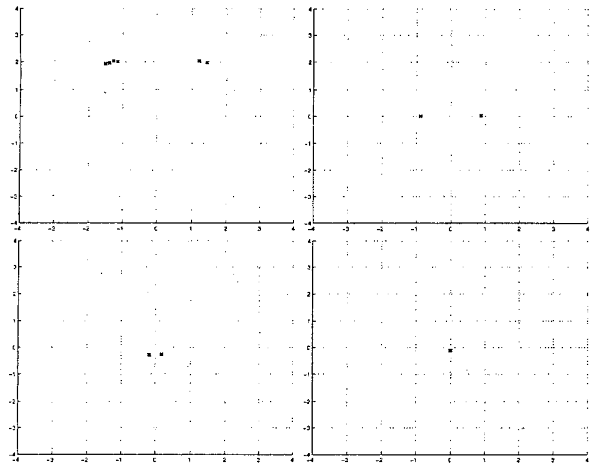

Figure 5: Distribution of particles at times $k = 10, 30, 50, 70$

6 Conclusions

Two approaches to the state estimation problem for a class of discrete-time piecewise affine systems were presented. A deterministic approach based on observers of Luenberger type was shown to achieve globally asymptotically stable error when the vector field of the system is continuous and the derived set of linear matrix inequalities is feasible. In the case when global stability can not be achieved (e.g. when the vector field is not continuous, or the previous approach is not feasible) the estimation error can be asymptotically bounded, relative to the state bound. Under the assumption that both constituting linear dynamics allow the design of the classical Luenberger observer with a common quadratic Lyapunov function this design is always feasible.

The achievable relative upper bound of the estimation error can be optimized, but the optimal value depends on the properties of the system. This was clearly illustrated by an example of the unobservable system (while both constituting linear dynamics are observable), where the design is feasible, but the best found relative upper bound is much larger than 1.

The Luenberger-type observer is computationally easy to implement and easy to compute (the required gains can be computed off-line). The main line of reasoning can be applied to more general classes of piece-wise affine systems, with the requirement that the input distribution matrix B is the same for all constituting affine dynamics.

The second proposed approach is a particle filtering algorithm. The output of particle filtering algorithm is the approximate a posteriori density of the state, which can be used to obtain a state estimate. The type of convergence which can be expected is the convergence of the approximate a posteriori probability density, when the number of particles N goes to infinity. In this moment it is not clear whether the convergence analyses (as e.g. in [8]) transfer to the considered class of systems. Precise convergence properties of particle filtering algorithm for the considered class of systems are the topic of the future research.

In an example the performance of the particle filter turned out to be comparable to the performance of the Luneberger-type observer. However, on the unobservable example the particle filter is able to follow both possible trajectories, in the sense that reconstructed probability density of the state reflects the two equally probable system trajectories. Luenberger-type observer converges to one of the possible trajectories, depending on the initial state estimate.

The particle filtering method is computationally expensive (where the computational effort depends on the used number of particles), but can be applied to more general types of systems structures (cf. [14]).

The development and application of particle filtering methods to other types of hybrid systems will be the focus of the future research. Moreover, both presented approaches will be experimentally tested on a practical setup.

References

[1] A. Alessandri and P. Coletta. Switching observers for continuous-time and discrete-time linear systems. In *Proc. of the American Control Conference*, pages 2516–2521, 2001.

[2] A. Alessandri and Colleta P. Design of Luenberger observers for a class of hybrid linear systems. In *Proc. of Hybrid Systems: Computation and Control*, Rome, 2001.

[3] M.S. Arulamapalam, S. Maskell, Gordon N., and T. Clapp. A tutorial on particle filters for online nonlinear/non-gaussian bayesian tracking. *IEEE Transactions on Signal Processing*, 50:174–188, 2002.

[4] A. Balluchi, L. Benvenuti, M.D. Di Benedetto, and A.L. Sangiovanni-Vincentelli. Design of obesrvers for hybrid systems. In *Proc. of Hybrid Systems: Computation and Control: 2002*, pages 76–89. Springer-Verlag, Berlin, 2002.

[5] A. Bemporad, G. Ferrari-Trecate, and M. Morari. Observability and controllability of piecewise affine and hybrid systems. *IEEE Transactions on Automatic Control*, pages 1864–1876, 2000.

[6] Y. Boers and H. Driessen. Hybrid state estimation: A target tracking application. *Automatica*, 38:2153–2158, 2002.

[7] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan. *Linear Matrix Inequalities in Control Theory*, volume 15 of *Studies in Applied Mathematics*. SIAM, 1994.

[8] D. Crisan and A. Doucet. A survey of convergence results on particle filtering methods for practitioners. *IEEE Transactions on Signal Processing*, 50:736–746, 2002.

[9] A. Doucet, A. Logothetis, and V. Krishnamurty. Stochastic sampling algorithms for state estimation of jump Markov linear systems. *IEEE Transactions on Automatic Control*, 45:188–202, 2000.

[10] G. Iulia Bara, J. Daafouz, F. Kratz, and C. Iung. State estimation for a class of hybrid systems. In *Proc. of 4th Int. Conf. on Automation of Mixed Processes: Hybrid Dynamic Systems*, pages 313–316, Dortmund, 2000.

[11] M. Johansson and A. Rantzer. Computation of piecewise quadratic lyapunov functions for hybrid systems. *IEEE Transactions on Automatic Control*, 43(4):555–559, 1998.

[12] A. Juloski, W.P.M.H. Heemels, and S. Weiland. Observer design for a class of piecewise affine systems. In *Proc. of CDC 2002*, pages 2606–2611, Las Vegas, USA, 2002.

[13] X. Koutsoukos, J. Kurien, and F. Zhao. Monitoring and diagnosis of hybrid systems using particle filtering methods. In *Proc. of Conference on Mathematical Theory of Networks and Systems*, 2002.

[14] F. Verschure. Multiple modal particle filters for track before detect. Master's thesis, Eindhoven University of Technology, The Netherlands, 2003.