IDENTIFICATION OF AN EXPERIMENTAL HYBRID SYSTEM [1]

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Abstract: In this paper we present an experimental study in the identification of an industrial hybrid system. Piecewise ARX models, that consist of a number of ARX models, together with the partition of the regressor space into regions where each of the models is valid, were identified. Effects of dry friction, and mechanical constraints in the experimental setup are demonstrated, and their influence on the identification procedure is discussed. Comparison of the simulated responses of the identified models with the responses of the real system shows that the obtained models are able to describe relevant aspects of the dynamics of the experimental setup. Ways to improve the identification procedure are proposed.

Keywords: hybrid systems, piecewise affine systems, identification

1. INTRODUCTION

In this paper we present an experimental study in the identification of an electronic component placement process in the pick-and-place machines. Pick-and-place machines are used to automatically place electronic components on the printed circuit board (PCB), and form a key part of an automated PCB assembly line. The pick-and-place machine works as follows: the PCB is placed in the working area of the mounting head; the mounting head, carrying an electronic component (using, for instance, a vacuum pipette), is navigated to the position where the component should be placed on the PCB; the component is placed, released, and the process is repeated with the next component. A fast component mounter, consisting of 12 mounting heads working in parallel is shown in figure 1. The throughput of such configuration can be up to 96.000 placed components per hour (Assembleon (2002)).

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Fig. 1. Fast component mounter (courtesy of Assembleon)

The control of the pick-and-place machine is a complex hierarchical problem. In the sequel we turn our attention to the mounting head, i.e. to the subproblem of the component placement on the PCB. Assuming that the mounting head, carrying the component, is in the right position above the PCB, the component is pushed down, until it comes in contact with the PCB, and released. The PCB is not rigid, but, depending on the material, has certain elasticity properties. The whole operation should be as fast as possible (to achieve maximal throughput), while satisfying technological and safety constraints (e.g. the exerted forces must not damage the component). For the purpose of analysis, control design and simulation, models of the placement process are needed. In this paper we will show that suitable models can be identified from experimental data.

During the placement process switching between several different situations (modes) occurs (as discussed in section 2). This motivates the search for the model in the area of hybrid systems. In this paper we will use piecewise affine (PWA) framework (Sontag (1981)), for modelling and identification. For further analysis and control design one can choose between several equivalent frameworks (Heemels *et al.* (2001)), such as linear complementarity (LC) framework (Heemels *et al.* (2000)), mixed logic dynamic (MLD) framework (Bemporad and Morrari (1999)), or piecewise affine framework (Johansson and Rantzer (1998); Sontag (1981)).

Recently several techniques for identification of hybrid systems were proposed (Bemporad *et al.* (2000, 2003); Ferrari-Trecate *et al.* (2001)). In this paper we will apply the technique developed in Ferrari-Trecate *et al.* (2001, 2002), to the experimental setup made around the mounting head of the pick-and-place machine. Experimental setup is described in section 2. Brief summary of the identification procedure is given in the section 3. Identification results and discussion are presented in sections 4,5. Conclusions and discussion on possible improvements of the identification procedure when applied in practice are presented in section 6.

2. EXPERIMENTAL SETUP

In order to study the placement process an experimental setup was made as depicted in figure 2. The schematic of the setup is presented in figure 3. The setup consists of the mounting head, from an actual pick-and-place machine, which is fixed above the impacting surface (small disc, figure 2). The impacting surface is in contact with the ground via the spring (spring c_2, figure 3, within the outer tube in figure 2), which is intended to simulate elasticity properties of the real PCB. The mechanical construction under the impacting surface is such that only movement on the vertical axis is enabled (inner tube, which can slide inside the outer tube, figure 2). This construction provides linear friction (damper d_2, figure 3), and dry friction (block f_2, figure 3), as will be discussed later.

The mounting head contains a vacuum pipette, which can move on the vertical axis (depicted by mass M, figure 3), which is connected via the spring to the casing (spring c_1, figure 3), an electrical motor, which enables the movement (depicted by force $\boldsymbol{F}$, in figure 3), and a position sensor, which measures the position of the pipette, relative to the upper retracted position. The position axis is pointed downwards (i.e. the value of the position increases when the pipette moves downwards). The motion of the pipette is also subject to friction (damper d_1 and a dry friction block f_1, figure 3).

Fig. 2. Experimental setup

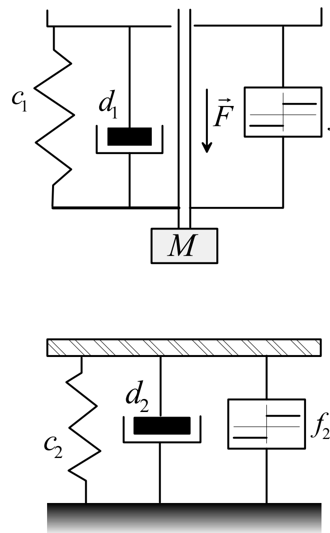

Fig. 3. Model of the mounting head

We distinguish the following situations:

upper saturation: the pipette is in the upper retracted position (i.e. can not move upwards, due to the physical constraints)

free mode: the pipette is not in contact with the impacting surface, but is not in the upper saturation

impact mode: the pipette is in contact with the impacting surface, but is not in lower saturation

lower saturation: the spring below the impacting surface is in saturation, the pipette can not move downwards due to the physical constraints

Control input of the experimental setup is the voltage applied to the motor (which is, with a negligible time constant, converted to the proportional force $\boldsymbol{F}$). Input signal for the identification experiment should be chosen in a way that all modes are sufficiently excited (Ferrari-Trecate *et al.* (2002)). Exact conditions that the input signal should satisfy are in general not available. To obtain the data for identification, input signal $u(t)$ is chosen as:

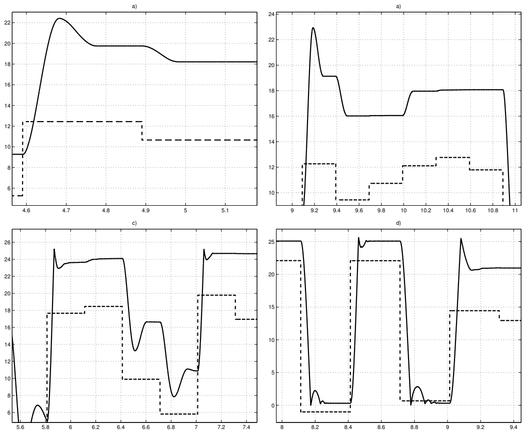

Fig. 4. Some features of the experimental data set a),b) effects of the dry friction c)lower saturation d)upper saturation (solid - system response, dashed - scaled input)

$$u(t) = a_k \quad \text{when} \quad t \in [kT, (k+1)T)$$

where T is a fixed time step, and the amplitude a_k is a random variable, with uniform distribution in the interval $[a, b]$. By properly choosing the boundaries of the interval $[a, b]$ only ceratin modes of the system are excited (e.g. only free and impact modes can be excited, without reaching upper and lower saturations).

Some features of the data sets obtained with this type of input signal are shown in figure 4. In figure 4a an effect of dry friction damping on the system response is depicted. In figure 4b small changes in the input signal produce no change in position (dry friction in stick phase). In figure 4c system is excited so that lower saturation is reached. The lower saturation effectively acts as a velocity reset map, active when certain position is reached ($\approx$ 25 in figures 4c,d). In figure 4d both upper and lower saturations were reached. The bouncing effect can be observed when reaching upper saturation, due to the elastic impact with the mechanical constraints.

3. IDENTIFICATION ALGORITHM

We consider the problem of reconstructing a Piece-Wise Affine (PWA) map from a finite number of noisy data points. A PWA map $f : \mathbb{X} \mapsto \mathbb{R}$ is defined by the equations

$$f(x) = f_q(x) \quad \text{if} \quad x \in \bar{\mathcal{X}}_q \tag{1}$$

$$f_q(x) = \begin{bmatrix} x^T & 1 \end{bmatrix} \bar{\theta}_q \tag{2}$$

where $\mathbb{X} \subset \mathbb{R}^n$ is a bounded polyhedron, $\{\bar{\mathcal{X}}_q\}_{q=1}^s$ is a polyhedral partition of $\mathbb{X}$ in s regions and $\bar{\theta}_q \in \mathbb{R}^{n+1}$ are Parameter Vectors (PVs). Therefore, a PWA map is composed of s affine submodels defined by the pairs $(\bar{\theta}_q, \bar{\mathcal{X}}_q)$. The data set $\mathcal{N}$ collects the samples $(x(k), y(k))$, $k = 1, \ldots, N$, generated by the model

$$y(k) = f(x(k)) + \eta(k) \tag{3}$$

where $\eta(k)$ are noise samples. We assume that the number s of submodels is known. Then, the aim of PWA regression is to estimate the PVs and the regions by using the information provided by $\mathcal{N}$.

When considering hybrid systems, an input/output description of a PWA system (see Sontag (1981) for a definition) with inputs $u(k) \in \mathbb{R}^m$ and outputs $y(k) \in \mathbb{R}$ is provided by Piece-Wise ARX (PWARX) models that are defined by equation (3) where k is now the time index and the vector of regressors $x(k)$ is given by

$$x(k) = \big[\, y(k-1) \;\; y(k-2) \;\ldots\; y(k-n_a) \\ u^T(k-1) \;\; u^T(k-2) \ldots \; u^T(k-n_b) \,\big]^T .$$

It is apparent that, if the orders n_a and n_b are known, the identification of a Piece-Wise ARX model amounts to a PWA regression problem.

Hereafter we summarize the identification procedure reported in Ferrari-Trecate *et al.* (2001, 2002) that is structured in three steps.

1. Local Regression. For $j = 1, \ldots, N$ a Local Dataset (LD) $\mathcal{C}_j$ is formed. It collects $(x(j), y(j))$ and the samples $(x, y) \in \mathcal{N}$ including the $c-1$ nearest neighbors x to $x(j)$. The cardinality c of an LD is a parameter of the algorithm satisfying $c > n+1$. LDs collecting only datapoints associated to a single submodel are referred to as *pure* LDs. Otherwise the LD is termed *mixed*. Linear regression is performed on each LD $\mathcal{C}_j$ to obtain the Local Parameter Vectors (LPVs) θ_j. The LD centers $m_j = \frac{1}{c}\sum_{(x,y)\in\mathcal{C}_j} x$ are also computed and the Feature Vectors (FVs) $\xi_j = [\theta_j', m_j']'$ are formed. As for the LDs, FVs are either pure or mixed.

Intuitively, if c and the noise are "small" enough, pure FVs (that capture characteristics of the true submodels) are expected to form s dense clouds in the FV-space whereas mixed FVs form a pattern of isolated outliers.

2. Clustering. The FVs are partitioned in s groups through clustering. For this purpose, a K-means algorithm (see Duda and Hart (1973)) exploiting suitably-defined confidence measures on the FVs can be used. Confidence measures allows to assign little influence to the mixed FVs so that the clustering results mainly depend on pure FVs. The resulting clusters are denoted with $\{\mathcal{D}_q\}_{q=1}^s$.

3. Estimation of the submodels. By using the bijective maps $(x(j), y(j)) \leftrightarrow \mathcal{C}_j \leftrightarrow \theta_j$, sets $\{\mathcal{F}_i\}_{q=1}^s$ of data points are built according to the rule: $(x(j), y(j)) \in \mathcal{F}_q \leftrightarrow \theta_j \in \mathcal{D}_q$. The points in each final set $\mathcal{F}_q$ are then used for estimating the PVs of each submodel through weighted least squares. The regions $\{\mathcal{X}_q\}_{q=1}^s$ are reconstructed on the basis of the final sets by resorting to multicategory pattern recognition algorithms (see Vapnik (1998)) that find the hyperplanes separating $\{x : (x, y) \in \mathcal{F}_q\}$ and $\{x : (x, y) \in \mathcal{F}_{q'}\}$ for all indices $q \neq q'$.

As pointed out in Ferrari-Trecate *et al.* (2002, 2001), if the signal-to-noise ratio is "high" enough, it is expected that the sets $\mathcal{F}_q$ correctly classify the largest part of the datapoints, i.e. those corresponding to pure FVs. Misclassified datapoints can be also detected and re-attributed a posteriori through residuals analysis. This will improve the overall quality of the reconstructed model.

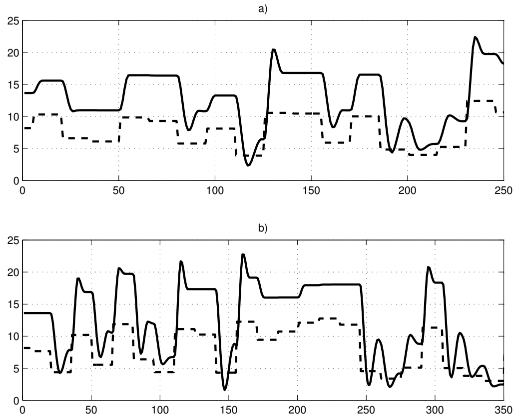

Fig. 5. Data sets used for a)identification and b)validation (solid - system response, dashed - scaled input)

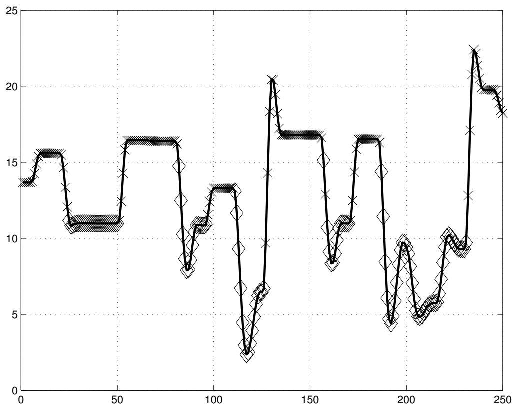

Fig. 6. Classified data points, 2+1 model, c=55, 2 modes (crosses - mode 1, diamonds - mode 2)

4. IDENTIFICATION WITH FREE/IMPACT MODES

In the following experiment the parameters of the input signal were chosen so that only free and impact modes are excited (i.e. no upper/lower saturations are reached). The obtained data-set was divided in the two sets: one is used for identification, and the second is used for validation. The two sets of data together with the scaled input signals are shown in figure 5. The effects of the friction nonlinearity are clearly observable, for instance in figure 5b, on the time interval (175, 250).

PWARX models with two modes are identified, with parameters $n_a = 2, n_b = 1$ and $n_a = 2, n_b = 2$, respectively. The results of the identification algorithm are presented in figures 6-8. The parameter c, i.e. the size of the local data cluster in the first step of the identification algorithm, determines the quality of the obtained model. For minimal theoretical values of c ($c = 4$, resp. $c = 5$) the obtained models are not usable (i.e. simulated responses are completely dissimilar to the measured ones). With a wide range of values of c obtained models differ in quality, and good models are obtained for $c \geq 40$. It is interesting to note that even for a large values of c (i.e. $c = 90$, for a data set of 250 points) good models can be obtained.

When the PWARX model with two modes is identified identification procedure makes an attempt to distinguish two major groups (clusters) of data points, and to fit an ARX model to each of them. Results of the classification procedure for $2 + 1$ model, with $c = 55$ are shown in figure 6. Intuitively, this two groups correspond to the free and impact modes. Because of the presence of dry friction (see figures 4a,4b) responses in both modes are nonlinear. Therefore, local data sets (LDs) with small cardinality (small c) will produce scattered parameter estimates, and clustering

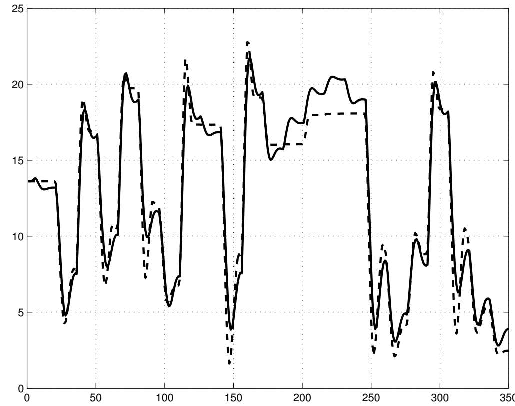

Fig. 7. Response of the identified 2+1 model, 2 modes, $c = 55$ (solid - model response, dashed-system output)

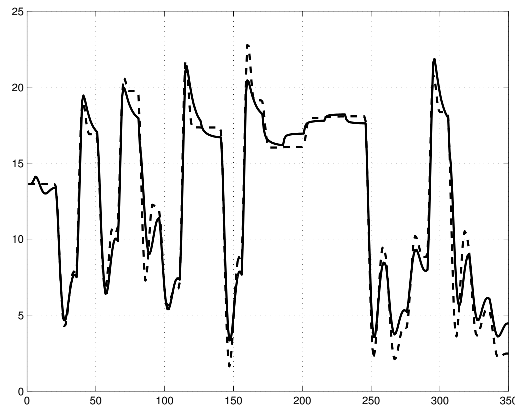

Fig. 8. Responses of the identified 2+2 model, 2 modes, $c = 55$ (solid - model response, dashed-system output)

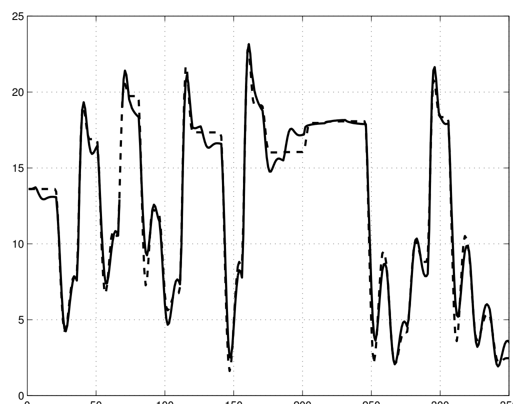

Fig. 9. Responses of the identified 2+2 model, 3 modes, $c = 35$ (solid - model response, dashed-system output)

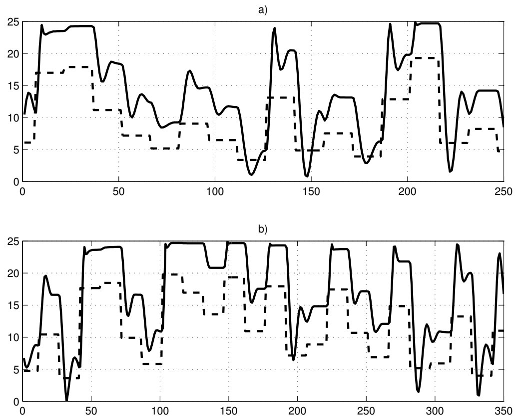

Fig. 10. Data sets used for a) identification b)validation (solid-system response, dashed-scaled input)

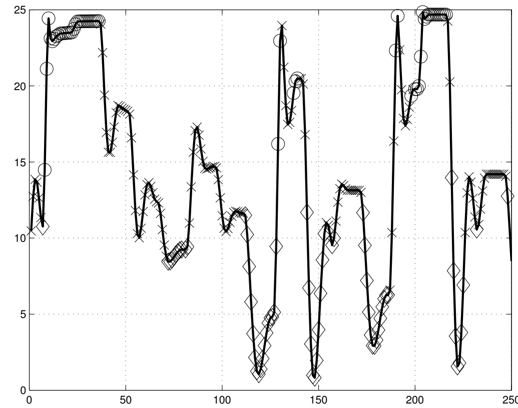

Fig. 11. Classified data points, 2+2 model, $c = 25$, 3 modes (circles - mode 1, crosses - mode 2, diamonds - mode 3)

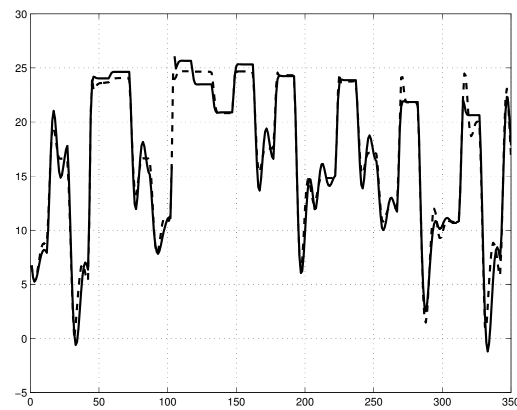

Fig. 12. Response of the identified 2+2 model, $c = 25$, 3 modes (solid - model response, dashed - system response)

will not be successful. LDs collecting large number of data points (large c) will produce parameter estimates corresponding to the "averaged" linear model. Such parameter estimates form clusters in the parameter space. Effect of "averaging" is noticeable in figure 7, where responses to the large step signals are predicted correctly, but responses to the small step signals are incorrect (time interval $(175, 250)$), and in figure 8, where the compromise is made between responses to large and small step signals.

The previous discussion motivates the attempt to identify a piece-wise affine model with more modes, on the same data set. Results of the identification when $s = 3$ are shown in figure 9. Points on time interval $(200, 250)$ are classified as belonging to the third mode, and the simulated response is correct. Points on the interval $(150, 200)$ are classified as belonging to the other mode, and the response is not correct. Identification with more modes using the same experimental data was not successful.

Identification with higher model orders, with two or three modes shows no significant difference on the response quality.

5. IDENTIFICATION WITH SATURATIONS

In order to get a model which is valid in wider range of operating conditions, the input was chosen so that impacts between the head and the spring occur and the lower saturation of the spring is reached. Data was again divided in two sets, one used for identification, and other for validation, as depicted in figure 10.

PWARX models with three modes were identified, with $n_a = 2, n_b = 2$. The result of the classification of the data is shown in figure 11, and the simulated response is shown in figure 12. Three groups of data points can be distinguished in figure 12, which can be associated with the free mode, impact mode and the lower saturation. The additional difficulty with the identification of the lower saturation lies in the fact that the saturation occurs precisely at one value of the head height; in other words the model of the saturation mode is valid on a plane in a state space only, rather than in a full dimensional part of the state space. Hence, all the data points that belong to this mode will lie on the boundary of the mode region. A few points that (intuitively) do not belong to the saturation mode, but are classified as such can be noticed in the figure 11. A plausible interpretation is that the data points near the true lower saturation value with high incoming velocity are attributed to the saturation mode. The simulated response corresponds well to the real system output. Absence of simulated output on figure 12 at time 100 is due to the fact that the identified partition of the regressor space does not cover the whole regressor space. This effect is due to the properties of the used classification procedure (support vector machines). This effect was also noticed in Ferrari-Trecate *et al.* (2001), and the remedy is to use another (numerically more intensive) classification procedure.

In the next experiment parameters of the input were chosen such that both upper and lower saturations are reached. The data sets used for identification and validation are depicted in figure 13.

A PWARX model with four modes has been identified. The simulated response is shown in figure 14. Difficulties with the identification of the two saturation modes are apparent. Also, responses in the modes when the head is free and in contact with the spring, but not saturated are not predicted well. This can be attributed to the fact that the two modes are not sufficiently excited in the data set used for identification.

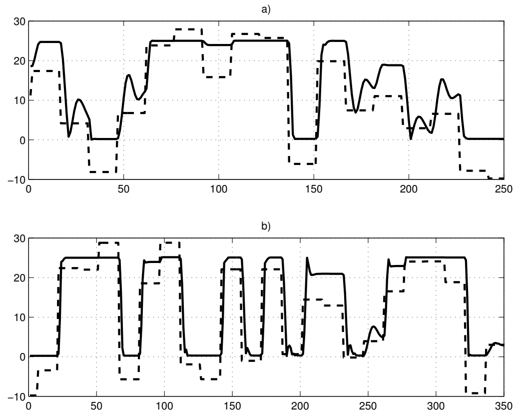

Fig. 13. Data sets used for a)identification b)validation (solid - system response, dashed - scaled input)

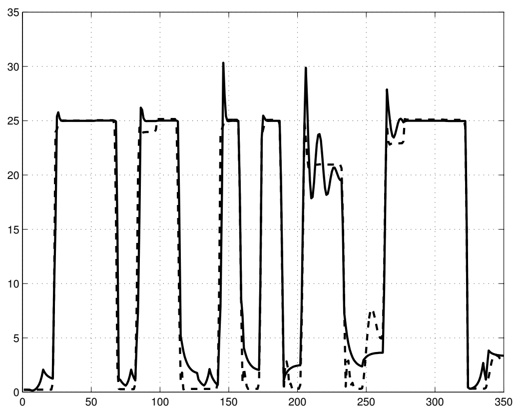

Fig. 14. Response of the identified 2+2 model, $c = 40$, 4 modes (solid - model response, dashed - system response)

6. CONCLUSIONS

In this paper the identification of the experimental setup, made around the mounting head of a pick-and-place machine was discussed. Piecewise ARX (PWARX) models of the system were identified, using the methodology introduced in Ferrari-Trecate *et al.* (2002). The obtained models can be used, for instance, for control design, diagnostics and fault detection.

The identified models consist of a certain number of ARX models (modes), together with the partition of the regressor space into regions where each of the models is valid. Initial parameters of the identification procedure (number of modes and model orders) can be determined for instance, by physical insight in the process to be identified. Another input parameter of the algorithm is the size of the local data cluster c, and it is demonstrated that c plays a crucial role in obtaining good models.

Non-smooth nonlinear effects due to dry friction were observed in the collected experimental data, especially in the mode when the head is in contact with the spring. Effects of the friction can be "averaged out", using large local data sets, but good response prediction can not be achieved. Effects of the friction can be taken into account by allowing additional modes in the identified model. In this case special care has to be taken about the input design, in order to sufficiently excite all modes. This is the subject of the further research.

Mechanical constraints in the mechanical setup can be reached for certain values of the input signal. Effects of saturations were shown in obtained experimental data sets. It is shown that it is possible to apply the identification methodology for the identification of the dynamics while in saturation. Problems with the classification of the points that belong to saturated modes were observed, due to the geometry of the saturation regions. Classification procedures, used to determine the boundaries of the regions are sensitive to improperly classified data points. A region boundary, determined in the wrong way, can have a disastrous effect on the validity of the identified model.

In practical situations a lot of a-priori information on the nature of the system to be identified is usually available before the identification experiment (e.g. number of modes, model orders, saturation values, correlation between certain parameters in linear models...). In the present moment only a limited amount of such information can be supplied to the identification procedure. Future research will focus on the possibilities of including more a priori information in the identification procedure (gray box modelling), and on determining structural models (like the one depicted in figure 3), and their parameters (white box modelling). Application of emerging new methods for hybrid identification to our experimental setup (like the one developed in Bemporad *et al.* (2003)) is also the subject of the future research.

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