



Switched LQG control for linear systems with multiple sensing methods[☆]

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ARTICLE INFO

Article history:

Received 31 March 2017

Received in revised form 7 October 2018

Accepted 15 January 2019

Available online 21 February 2019

Keywords:

Data processing

LQG control

Stochastic systems

Probabilistic models

Time-delay

Probability of information loss

Measurement noise

Switched systems

Self-triggered control

ABSTRACT

In many control contexts, such as vision-based control, data-processing methods are needed to distill information from measurement data (such as images). These data-processing methods introduce several undesirable effects such as delays, measurement inaccuracies and possible absence of information, which limit closed-loop performance. Typically, a single processing method with an appropriate compromise between these effects is chosen in practice. Instead of settling for a compromise using only one fixed processing method, we propose to break the design trade-off by switching on-line between several data-processing methods having different delay, accuracy, and data-loss characteristics. We provide a modeling framework for sensing and data-processing methods that is suitable for control applications and incorporates the characteristics of the undesirable effects mentioned above. Using the models provided by the framework, we provide explicit policies for switching on-line between sensing methods with different characteristics based on a modified rollout strategy. Our approach formally guarantees that an LQG-type infinite horizon performance is better than, or at least not worse than, non-switching approaches. The advantages of the proposed methodology are further highlighted via a numerical example.

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1. Introduction

Data-processing units are an important but often unmodeled component of many control loops. In many high-end control systems, however, large numbers and complex types of sensors provide large quantities of data, which must be processed to distill control-relevant information. Such *data-intensive* control systems arise in many robotic applications, where position and velocity information is often obtained from data from several sensors which typically includes (a series of) camera images (Corke, 2011; Malis, 2002; Oda, Ito, & Shibata, 2009), but also arise in, e.g., visual navigation (Chakraborty, Mehta, Curtis, & Dixon, 2016; Chaumette & Hutchinson, 2006), medical imaging (Albers, Suijs, & de With,

2009), and big-data applications (Chen & Zhang, 2014). An important fact is that in most real-life applications the processing unit is constrained in hardware cost and/or in physical space, leading to limitations on the processing capability of the data-processing unit.

Under classical control design without proper modeling of the data-processing unit, i.e., assuming that the processing unit is ideal or accounting for the worst case, the constraints on the processing unit lead to unexpected or conservative behavior when implemented in practice. Hence, this approach often leads to performance degradation of the closed-loop system. Instead, in this work, we propose to model the data-processing methods by several characteristic properties and use those characteristics explicitly in the control design. Based on this modeling setup, we propose a switching control mechanism to further improve performance beyond classical results (that use only a single processing method). Nowadays, this is enabled by flexibility in software and hardware, which allows us to take advantage of the fact that a data-processing unit may deploy several different implementations of data-processing methods with a trade-off in characteristics.

To be more precise, we introduce in this paper a general framework to model a large class of sensing methods in a control loop, which incorporates three characteristic properties of the sensing methods; (i) the delay before the information is available; (ii) the accuracy of the provided information; and (iii) the probability of actually acquiring relevant information. Data-processing is a typical core component of a sensing method. Typically, for a given

[☆] This work is supported by the Netherlands Organisation for Scientific Research (NWO-TTW) under grant number 12697 “Control based on Data-Intensive Sensing”, and by the Innovational Research Incentives Scheme under the VICI grant “Wireless control systems: A new frontier in automation” (No. 11382) awarded by NWO-TTW. The material in this paper was partially presented at the 53rd IEEE Conference on Decision and Control, December 15–17, 2014, Los Angeles, CA, USA, 6th IFAC Workshop on Distributed Estimation and Control in Networked Systems NecSys’16, September 8–9, 2016, Tokyo, Japan, and the 2017 American Control Conference, May 24–26, 2017, Seattle, WA, USA. This paper was recommended for publication in revised form by Associate Editor Zhihua Qu under the direction of Editor Daniel Liberzon.

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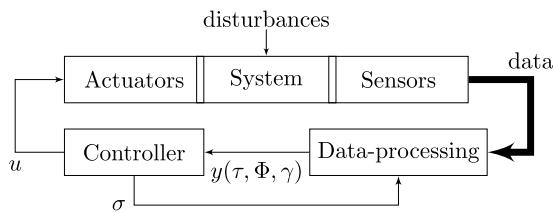


Fig. 1. Control loop with data-processing (van Horssen et al., 2015).

data-processing unit, there is a trade-off between the sensing characteristics (Chakraborty et al., 2016; Corke, 2011). For instance, a more elaborate data-processing method can provide higher accuracy (or a higher probability of providing useful information), but requires more computational time and thus incurs a larger delay than a simple(r) method. In addition, in this case, hard deadlines on processing time often enforce the trade-off with respect to delay by causing data absence due to deadline misses.

We consider a closed-loop system consisting of the interconnection of a physical system with one or multiple sensors and actuators, a data-processing unit with several available processing methods, and a digital controller in a feedback structure, as depicted in Fig. 1. Sensor data is processed in batches by the data-processing unit before control-relevant information is obtained. We consider this type of system in the context of stochastic optimal sampled-data control (Åström, 1970; Åström & Wittenmark, 2013; Chen & Francis, 1995) of linear continuous-time plants with stochastic state disturbances with infinite horizon average quadratic cost criteria (Bertsekas, 2005), i.e., the linear quadratic Gaussian (LQG) setting. To improve performance beyond classical results, we propose to break the trade-off in data-processing characteristics by switching on-line between multiple processing methods with different characteristics as already mentioned. This leads to a general switching and control *co-design* problem where the (real-valued) decisions are related to the actuation input and the (integer-valued) decisions regarding which processing algorithm to use at each decision time. Because the general optimal switching and control *co-design* problem is computationally hard to solve, our approach is based on an (approximate) dynamic programming approach. In particular, we use rollout techniques (Bertsekas, 2005) and derive explicit switching criteria and an LQG-type feedback policy. This control design approach leads to novel control policies that guarantee performance improvement when compared to implementations that only use one processing method. We emphasize that the plant is considered in continuous time, as well as the cost function to reflect the true cost. The controller is in discrete time (with non-equidistant time intervals) in order to reflect the actual digital implementation in which processing delays are also taken into account.

We propose a switching method that can circumvent the trade-off between characteristics for an important general setup with deterministic delays. Results for many cases, such as those in our preliminary works (van Horssen et al., 2015; van Horssen, Antunes, & Heemels, 2016), can be recovered from the results for this general setup. As a secondary result, we show how a case with stochastic delays can be addressed using the result for deterministic delays by a deadline-driven approach. The framework to model data-processing methods is presented with generality to facilitate further studies in this appealing and relevant research area.

In our preliminary works van Horssen et al. (2015, 2016), the trade-offs between delay and measurement accuracy, and between delay and probability of data acquisition were investigated in isolation, respectively, under the assumptions that measurements of the full state are available and that the delays are deterministic. Building upon these preliminary works, in this paper, we extend

these results leading to a general modeling and control approach that simultaneously addresses the interplay between delay, measurement accuracy and probability of data acquisition. We recover the results of van Horssen et al. (2015, 2016) as special cases. To broaden the applicability of the methods, we extend the results to the general case of output feedback with partial information, for which we provide solutions based on both Luenberger-type observers and time-varying Kalman filters. Furthermore, we allow the delays in the loop to become stochastic, which was not the case in preliminary works, and apply the ideas our recently proposed self-triggered deadline-driven implementation (Prakash, van Horssen, Antunes, & Heemels, 2017) with additional optimization opportunities. Moreover, full details on the derivation of the results are given, which are not available in van Horssen et al. (2015, 2016).

The present work builds upon the seminal and classical works (Åström, 1970; Bertsekas, 2005; Doob, 1953; de Koning, 1982), and insights from Schenato, Sinopoli, Franceschetti, Poolla, and Sastry (2007). The modeling of data-processing methods and the control design ideas are, to our best knowledge, novel. LQG-type sampled-data control is studied since the early works of Kabamba and Hara (1993) and Khargonekar and Sivashankar (1991) and many other approaches to delay different from this work, such as loop-shifting (Mirkin, Shima, & Tadmor, 2014) for fixed time-delays and Lyapunov-based methods such as Fridman (2014), have been studied in recent years. The probability of data-acquisition characteristic also appears in networked control context as data loss, or ‘dropouts’, see, for instance, Demirel (2015), Gommans, Heemels, Bauer, and van de Wouw (2013), Quevedo, Silva, and Goodwin (2008) and Schenato et al. (2007) and the references therein. Many of those works consider data loss/acquisition with the purpose of analyzing the trade-off between control and communication rate when controlling, or ‘scheduling’, when and how often to (re-)transmit, in order to reduce or limit the communication rate (Al-Areqi, Görges, Reimann, & Liu, 2013; Antunes & Heemels, 2014; Antunes, Heemels, Hespanha, & Silvestre, 2012; Demirel, Aytakin, Quevedo, & Johansson, 2015; Kouchiyama & Ohmori, 2010; Molin & Hirche, 2009; Reimann, Al-Areqi, & Liu, 2013). Other papers considering trade-offs between different characteristics of data-processing and communication are (Demirel, 2015; Wu, Lou, Chen, Hirche & Kuhnlenz, 2013) and Wu, Jia, Johansson and Shi (2013). In Wu and Jia et al. (2013), a method to balance the trade-off between communication rate and estimation quality under energy constraints is given. An example of vision-based control causing delay can be found in Wu and Lou et al. (2013), where scheduling is used to reduce the communication rate with a cloud-computing platform. The communication rate trade-off with stochastic delays is considered in Demirel (2015). Note that in the current work we consider communication channels to be ideal and the probability of data-acquisition characteristic only models data loss due to the data-processing unit, although such an extension can be envisioned based on the framework presented in this paper. While reducing communication is important, typically to preserve energy or reducing network congestion, this is not the main concern in data-intensive applications that are considered here. Instead, we aim to fully utilize the available processing power to improve closed-loop performance by deciding which processing method is most suited to be used for any newly acquired data. This is a different problem than considered in the papers mentioned above, where no *co-design* problem for control and switching of multiple data-processing methods is solved. Recent works Bolzern, Colaneri, and Nicolao (2010, 2016) have considered a different control and switching *co-design* problem for the same class of linear systems with stochastic and deterministic switches (switched Markov Jump Linear Systems) in a discrete-time state-feedback context. The relevance of providing

suboptimal switching policies with performance guarantees was also underlined by works such as Geromel, Deaecto, and Daafouz (2013), where the concept of consistency was introduced. In different context, Picasso, Vito, Scattolini, and Colaneri (2010) also considered complex supervisors to switch among different time-scales and configurations by optimization.

The organization of the remainder of the paper is as follows. The control set-up with data-processing, the control design, and the problem formulation are detailed in Section 2. The proposed switching and control policy is given in Section 3. Section 4 contains the main analytical results. A numerical example in Section 5 illustrates the results. Concluding remarks are given in Section 6. The proofs of the main results can be found in the Appendix.

2. Framework and problem formulation

We discuss in Section 2.1 the plant model, in Section 2.2 the measurement and sensing framework, and in Section 2.3 the digital control update scheme. In Section 2.4, we state the performance index and the main control problem that we are interested in.

2.1. Plant model

The plant model of the system that is to be controlled takes the form of the continuous-time stochastic differential equation

$$dx_C(t) = (A_C x_C(t) + B_C u_C(t))dt + B_\omega d\omega(t), \quad t \in \mathbb{R}_{\geq 0}, \quad (1)$$

where $x_C(t) \in \mathbb{R}^{n_x}$ is the state and $u_C(t) \in \mathbb{R}^{n_u}$ is the control input at time $t \in \mathbb{R}_{\geq 0}$, and $\omega(t)$ is an n_ω -dimensional Wiener process with incremental covariance $I_{n_\omega} dt$ (cf. Åström (1970)). Here, $A_C \in \mathbb{R}^{n_x \times n_x}$, $B_C \in \mathbb{R}^{n_x \times n_u}$, and $B_\omega \in \mathbb{R}^{n_x \times n_\omega}$ are constant matrices describing the plant behavior. We make the standard assumptions that (A_C, B_C) is controllable and B_C has full rank (which implies full column rank if $n_x > n_u$). The initial condition $x_C(0)$ with realization $x_0 \in \mathbb{R}^{n_x}$ is assumed to be a multivariate Gaussian random variable with mean \bar{x}_0 and covariance Φ^{x_0} , i.e., $x_C(0) = x_0 \sim \mathcal{N}(\bar{x}_0, \Phi^{x_0})$.

2.2. Measurements and sensing model

For illustrative purpose, we focus on data-processing as the main sensing element that is to be modeled. At *sampling times* t_k , $k \in \mathbb{N}$, with $t_0 = 0$, a new *sample of raw data* pertaining to the plant is taken (instantaneously). At this time, one *data-processing method* $\sigma_k \in \mathcal{M} := \{1, 2, \dots, M\}$ is (immediately) activated to distill information regarding the system state $x_C(t_k)$ that is relevant for feedback control. As discussed in the introduction, only one method may be active at any given time. Next, we propose a modeling framework incorporating three characteristics of sensing or data-processing that are relevant for control. We present the modeling framework with mathematical generality and present control design ideas for a large subset of the cases that can be identified in the framework. This subset encompasses many practically relevant cases and trade-offs that are of interest. An exhaustive analysis of all possible cases and trade-offs between the characteristics of data-processing is a hard problem and beyond the scope of this paper.

After a *processing delay* $\tau_k \in \mathbb{R}_{>0}$, which depends on the processing method $\sigma_k \in \mathcal{M}$ that is chosen, the raw data is processed and the *output* $y_k \in \mathbb{R}^{n_y}$ becomes available to the controller. Each processing method $m \in \mathcal{M}$ is characterized by three properties, modeled by a tuple $(F_m, \Phi_m^v, \bar{\gamma}_m)$ containing a cumulative distribution function (cdf) $F_m : \mathbb{R}_{>0} \rightarrow [0, 1]$ for the delay, a measurement noise covariance $\Phi_m^v \in \mathbb{R}^{n_y \times n_y}$ modeling accuracy, and a probability of data-acquisition $\bar{\gamma}_m \in [0, 1]$, respectively, which are detailed next. In this work, we assume that improving one of the characteristics deteriorates another one.

Moreover, we assume that two realizations of data-processing characteristics are conditionally independent of one another given the method $\sigma_k \in \mathcal{M}$. In the remainder of this paper this conditional independence is a standing assumption.

The processing delays τ_k , $k \in \mathbb{N}$, are assumed to be independent and identically distributed (i.i.d.) with cdf F_{σ_k} defined by the probability measure μ_{σ_k} in the sense that $F_m(\tau) := \mu_m((0, \tau))$, $\tau \in \mathbb{R}_{(0, \infty)}$ for all $m \in \mathcal{M}$. The support of μ_m , $m \in \mathcal{M}$, may be unbounded, but we assume that $\mu_m((0, \infty)) = 1$. The measure μ_m can be decomposed into continuous and discrete components as in $\mu_m = \mu_m^c + \mu_m^d$, with $\mu_m^c([0, s]) = \int_0^s f_m^c(\tau) d\tau$, where f_m^c is a measurable function, and μ_m^d is a discrete measure that captures possible point masses at $b_i \in \mathbb{R}_{>0}$, $i \in \mathbb{N}_{\geq 1}$, such that $\mu_m^d(\{b_i\}) = w_i^m \in \mathbb{R}$. The (Lebesgue–Stieltjes) integral of some function W with respect to the measure μ_m is defined as

$$\int_0^t W(s) \mu_m(ds) := \int_0^t W(s) f_m^c(s) ds + \sum_{i \in \mathbb{N}_{\geq 1} : b_i \in (0, t]} w_i^m W(b_i).$$

Sometimes, we will also use the probability distribution function (pdf) f_m , which is such that $F_m(\tau) = \int_0^\tau f_m(s) ds$, where $f_m(\tau)$ can be seen as $f_m^c(\tau) + \sum_{i \in \mathbb{N}_{\geq 1}} w_i^m \delta(\tau - b_i)$ for $\tau \in \mathbb{R}_{>0}$ where $\delta(\cdot)$ denotes the Dirac delta function. The means are denoted by $\bar{\tau}_m$, $m \in \mathcal{M}$, i.e., $\bar{\tau}_{\sigma_k} := \mathbb{E}[\tau_k]$ for any $k \in \mathbb{N}$. In this work, we will focus on control design for deterministic delays and illustrate how the case of stochastic delays can be addressed using the insights provided by the deterministic case.

The *accuracy* of the obtained output y_k is modeled by additive perturbations on the measurements $Cx_C(t_k)$, where the constant matrix $C \in \mathbb{R}^{n_y \times n_x}$ determines the measured variables. The perturbations v_k are assumed to be independent realizations of n_y -dimensional random variables, i.e., $v_k \in \mathbb{R}^{n_y}$. For all $k \in \mathbb{N}$, v_k has zero-mean multivariate Gaussian distribution with covariance Φ_m^v if method $m \in \mathcal{M}$ is activated at time t_k , i.e., $v_k \sim \mathcal{N}(0_{n_y}, \Phi_m^v)$ if $\sigma_k = m$.

The possibility that the processing method does not provide a useful result, i.e., the *possibility of no information*, is modeled by the variable $\gamma_k \in \{0, 1\}$. If the new output information y_k is useful, we have $\gamma_k = 1$, otherwise $\gamma_k = 0$. The indicators $\gamma_0, \gamma_1, \gamma_2, \dots, \gamma_k, \dots$ are independent realizations of Bernoulli random variables with probability

$$\bar{\gamma}_m = \Pr(\gamma_k = 1 \mid \sigma_k = m)$$

for any given $m \in \mathcal{M}$. Thus, the random variables γ_k , $k \in \mathbb{N}$, are independent, and, for any given $m \in \mathcal{M}$, identically distributed. In the remainder of the paper we will use the term ‘switch’ for changes in σ_k and ‘jump’ for changes in the data-loss parameter γ_k . Note that we consider the communication channels to be ideal, in the sense that they do not introduce data losses, and the γ parameter purely models the data losses introduced by the data-processing unit.

The model properties lead to the following structure for the measurement output. The output

$$y_k = \begin{cases} Cx_C(t_k) + v_k, & \gamma_k = 1, \\ \emptyset, & \gamma_k = 0, \end{cases} \quad (2)$$

becomes available to the controller at time $t_k + \tau_k$, for all $k \in \mathbb{N}$, where the symbol \emptyset denotes ‘no data available’. We assume that the moment at which a data-processing method completes (and provides the output) coincides with the moment at which a new measurement starts being processed. Thus,

$$t_{k+1} = t_k + \tau_k, \quad k \in \mathbb{N}. \quad (3)$$

Optimization of the actuation moment may be possible in some settings and is discussed in Section 6. This presents a complementary problem that is beyond the scope of this paper.

Remark 1. Note that we explicitly consider the delay to be nonzero. This allows us to obtain a discrete-time representation of the continuous-time system and formulate a decision-making problem in discrete-time. In fact, in our framework, it is not possible to consider zero delays since this would entail zero time length between two consecutive decisions, making the problem ill-posed.

2.3. Digital control

In this work, the plant is controlled digitally. We consider the actuator update times to coincide with the sampling times, see also (3). As it is standard in most digital or sampled-data control schemes (Åström & Wittenmark, 2013; Chen & Francis, 1995), a zero-order hold actuation strategy is implemented. Hence, u_C is a staircase signal, given by

$$u_C(t) = u_k := u_C(t_k), \quad \text{for all } t \in [t_k, t_{k+1}), k \in \mathbb{N}. \quad (4)$$

By the previously mentioned assumption, the actuator update times coincide with the sampling times, i.e., when y_{k-1} becomes available at time t_k , u_k can be instantaneously updated.

A switching strategy determines the switching signal given by the sequence of processing methods $\sigma := \{\sigma_k\}_{k \in \mathbb{N}}$ pertaining to which data processing method σ_k to use in each interval $[t_k, t_{k+1})$. The main results of this work pertain to novel switching strategies combined with optimal actuation strategies.

To simplify our analysis, we assume the computational time of the controller to be significantly smaller than the processing delay such that it is negligible. As we will show, the proposed control strategy does not require a significant computational load and therefore this assumption is reasonable in many cases. In addition, a known computational or actuation delay may be directly taken into account by slight adaptation of the main results.

2.4. Control problem statement

Performance is measured by an expected average cost

$$J_C^a := \limsup_{T \rightarrow \infty} \frac{1}{T} \int_0^T \mathbb{E} [g_C(x_C(t), u_C(t))] dt, \quad (5)$$

which depends on the control input function u_C with $g_C(x, u) := x^T Q_C x + u^T R_C u$, where $Q_C \in \mathbb{R}^{n_x \times n_x}$ is a positive semi-definite matrix with $(A_C, Q_C^{\frac{1}{2}})$ observable, and $R_C \in \mathbb{R}^{n_u \times n_u}$ is a positive definite matrix. This performance index corresponds to that of a standard LQG average cost problem.

A control policy π is defined as a sequence

$$\pi = (\iota_0, \iota_1, \dots)$$

of multivariate functions $\iota_k := (\iota_k^\sigma, \iota_k^u)$ that determine at sampling times t_k , $k \in \mathbb{N}$, the switching and actuation inputs in the sense that

$$(\sigma_k, u_k) = (\iota_k^\sigma(\mathcal{I}_k), \iota_k^u(\mathcal{I}_k)) = \iota_k(\mathcal{I}_k), \quad k \in \mathbb{N}, \quad (6)$$

based on the information vector

$$\mathcal{I}_k := \{\mathcal{I}_{k-1}, \sigma_{k-1}, u_{k-1}, y_{k-1}, \tau_{k-1}, \gamma_{k-1}\},$$

available at the controller at that time, with initial information $\mathcal{I}_0 := \{\bar{x}_0, \Phi^{x_0}\}$. Recall that at time t_k , $k \in \mathbb{N}$, the output y_{k-1} has just arrived at the controller.

Let the cost J_C^a for a given policy π be denoted by J_π . A conventional design approach is to apply a non-switching control policy, here referred to as a base policy, for which a-priori a processing method, say $b \in \mathcal{M}$, is fixed, such that $\iota_k^\sigma(\mathcal{I}_k) = b$ (and thus $\sigma_k = b$) for all $k \in \mathbb{N}$, and use known tools (see, e.g., Åström (1970)) to find a policy ι_k^u that provides the optimal control inputs u_k , $k \in \mathbb{N}$, to the plant. Let base policies with $b \in \mathcal{M}$ also be indicated by $b \in \mathcal{M}$

(such that \mathcal{M} also represents the set of base policies). For cost (5), the best achievable performance J_b of a base policy, with $b \in \mathcal{M}$, is given by

$$J_b := \min_{\iota_k^u \text{ for all } k \in \mathbb{N}} J_C^a |_{\sigma_k = b \text{ for all } k \in \mathbb{N}}, \quad b \in \mathcal{M}.$$

For such non-switching base policies $b \in \mathcal{M}$, the optimal control inputs u_k , $k \in \mathbb{N}$, are provided by LQG-type control policies (see, e.g., Åström (1970)) and the optimal value J_b is known to be independent of the initial state. Let one optimal choice of policy $b \in \mathcal{M}$ be denoted by b^* , such that $J_{b^*} \leq J_b$ for all $b \in \mathcal{M}$, i.e.,

$$b^* := \arg \min_{b \in \mathcal{M}} J_b. \quad (7)$$

The control problem considered in this paper is to determine, for system (1),(2),(3), a control policy π that provides both an actuation signal u_C and a switching signal σ , to obtain for cost (5) a smaller value J_π than can be achieved for any non-switching base policy, in the sense that

$$J_\pi \leq J_{b^*} \leq J_b, \quad \text{for all } b \in \mathcal{M}. \quad (8)$$

Note that finding optimal switching and actuation policies, i.e., finding $\pi^* = \arg \min_\pi J_\pi$, is a hard problem in general (see, e.g., Antunes and Heemels (2014) and Zhang, Hu, and Abate (2012)) for which paths-on-graphs-type approaches as in Lee (2009) and Lee and Dullerud (2011) become computationally intractable. Therefore, we propose a suboptimal switching policy design approach, explained in Section 3, which leads to computationally efficient control policies that provide performance guarantee (8) and often outperform all base policies. By this we mean that guaranteeing (8) in itself is not the main goal of the paper, but the goal is also to show that in practice we can obtain strict performance improvement for J_π in practical settings. The performance guarantee (8) is formally proven and performance improvement is illustrated by a numerical example in Section 5. The proposed actuation policies are based on the time-varying LQG design and we mainly present novel switching policies. Existing results do not provide a solution for this problem.

In the framework presented in Section 2.2, many special cases of interest can be identified. In this work, first, we derive our first main result for the most general case with deterministic delays, i.e., when $f_m(\tau) = \delta(\tau - \bar{\tau}_m)$, $\tau \in \mathbb{R}_{>0}$. The derivation of the explicit switching condition is detailed in Section 4. The (proof of) the main result also applies to van Horssen et al. (2015, 2016) and their generalizations, which are recovered as special cases. For reasons of space, the explicit derivation of those special cases is omitted. Subsequently, we show in our second main result that the first main result can also be used to tackle stochastic delays by a deadline-driven approach, which uses a reformulation of the problem set-up to one in which the delays are again deterministic.

3. Proposed control policy

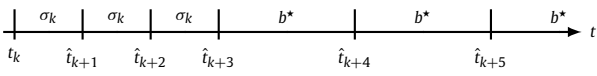
In this section, the control policy is proposed. First, the policy structure is given. Subsequently, a discrete-time system formulation is introduced and the state estimation and actuation problems are addressed. These preliminaries are then used to derive explicit switching and control solutions in Section 4. As mentioned, we present the technical parts of our control policy for the cases with deterministic delays, i.e., $f_m(\tau) = \delta(\tau - \bar{\tau}_m)$ for each $m \in \mathcal{M}$.

3.1. Proposed switching and actuation policy

The control policy that we propose is based on stochastic approximate dynamic programming and is known as a rollout method (see Bertsekas (2005)). Note that the classical roll-out method, which has an infinite horizon lookahead, only applies to

switched systems where the sampling interval τ_k is equal to the same fixed constant for any method $\sigma_k \in \mathcal{M}$ in order to have the same cost-to-go, which is not the case in our setting. We use the ideas behind the classical rollout approach to establish our new results, which extend the rollout methods to systems with time-varying sampling intervals.

We propose to use the following policy for the decisions. A switching decision is made at time t_k on which processing method $\sigma_k \in \mathcal{M}$ to choose next, assuming that it would be chosen in several subsequent instants as well and assuming that after those instants the optimal base policy b^* method is always selected. The number of instants is given by a discrete-time horizon of size $H_k \in \mathbb{N}_{>0}$ for all $k \in \mathbb{N}$. Thus, the policy assumes the use of σ_k for $t_k, t_{k+1}, t_{k+2}, \dots, t_{k+(H_k-1)}$, while supposing that after the horizon, i.e., at times $t_{k+H_k}, t_{k+(H_k+1)}, \dots$, the optimal base policy b^* method is selected. This is schematically depicted in the following diagram, where $H_k = 3$ and where instants $\hat{t}_{k+h}, h \in \mathbb{N}$, indicate the (expected) future sampling times.



Then the decision-making procedure is then repeated in a receding horizon fashion, i.e., the same procedure is repeated at times t_{k+1}, t_{k+2}, \dots .

Note that typically in the classical rollout methods the horizon H_k is a fixed constant for all $k \in \mathbb{N}$. We further extend the classical results by allowing the horizon H_k to change on-line, i.e., it can change with $k \in \mathbb{N}$. In particular, we will include the horizon in the optimization of the switching decision. A larger switching horizon can create more or better switching options, which can improve the obtained performance. To accommodate this additional optimization variable in view of guaranteeing (8), we restrict our updating procedure with the following condition.

$$\text{If } H_k > 1, \text{ then } \sigma_{k+1} = \sigma_k \text{ and } H_{k+1} = H_k - 1, \quad (9)$$

which means that switching whilst being on a previously selected horizon is not allowed. We define $\mathcal{H}_m := \{1, 2, \dots, H_m\}$, with $H_m \in \mathbb{N}_{>0}$ as the set of admissible horizons H for each $m \in \mathcal{M}$, and, for all $k \in \mathbb{N}$.

Clearly, restriction (9) is not needed if $\mathcal{H}_m := \{1\}$ for each $m \in \mathcal{M}$, i.e., if $H_k = 1$ for all $k \in \mathbb{N}$.

As mentioned in Section 2.4, the proposed actuation policy is based on the standard optimal LQG controller design (see e.g. Åström (1970)), as detailed in Section 3.4.

For some policy π , let the cost incurred on the interval $[t_k, t_k+T)$ be denoted

$$J_{C, [t_k, t_k+T)}^a := \frac{1}{T} \int_{t_k}^{t_k+T} \mathbb{E} [g_C(x_C(t), u_C(t)) | \pi] dt. \quad (10)$$

In accordance with the proposed control policy, we define the switching criterion as the choice of the method $m \in \mathcal{M}$ and horizon $H \in \mathcal{H}_m$ that minimizes (10) when that method is selected on the horizon of length H assuming that afterwards method b^* is always selected, while the actuation policy is given by the (time-varying) LQG regulator, denoted $l^u = LQG$. Formally, for all $k \in \mathbb{N}$ subject to (9),

$$j_k^a : (\sigma_k, H_k) := \arg \min_{m \in \mathcal{M}, H \in \mathcal{H}_m} J_k^{m,H} \quad (11)$$

where, for all $k \in \mathbb{N}$,

$$\begin{aligned} J_k^{m,H} &:= J_{C, [t_k, t_k+T)}^a | \mathcal{I}_k, l^u = LQG, \\ \sigma_l &= m \text{ for all } l \in \mathbb{N}_{[k, k+H)}, \\ \sigma_l &= b^* \text{ for all } l \in \mathbb{N}_{\geq k+H}. \end{aligned} \quad (12)$$

While (11) defines a family of policies since $J_k^{m,H}$ is parameterized by T , later, we will let $T \rightarrow \infty$ and consider only the specific resulting policy. It is important to realize that taking the limit in (12) directly, would leave (11) ill-defined since $J_k^{m,H}$ would be equal for each switching option. To provide insights under well-defined policies, we first assume that T is a large number. Finally, using the insights, we can let $T \rightarrow \infty$ whilst keeping (11) well-defined.

The main result of this paper is that the switching and actuation policy as described in this section (for $T \rightarrow \infty$) improves upon non-switching policies $b \in \mathcal{M}$ in the sense of (8) for the cases described in Section 2.4. The derivation of $J_k^{m,H}$ for those special cases leads to explicit switching conditions and is detailed in Section 4. The analysis uses the assumption that the time needed to evaluate the control policy, i.e., the computation time needed to compute values for σ_k and u_k , is significantly less than the time needed for data-processing, such that the computation time can be neglected. The established policies allow this assumption since the computational complexity can be kept small, as will be shown.

Remark 2. We consider the restriction (9) for ease of exposition in the current paper. We believe that it is possible to find additional conditions on the switching rules in this paper that will allow switching whilst being on the horizon. Establishing such conditions is beyond the scope of this paper and left as future work. The limitation of having only one method on the horizon can be lifted, but such an approach may quickly become computationally problematic due to the increasing number of possible combinations of methods.

3.2. Discrete-time formulation

To establish the envisioned results, the system behavior is described in discrete time at the sampling instants.

The state and measurements at instants $t_k, k \in \mathbb{N}$, are described by

$$x_{k+1} = A_k x_k + B_k u_k + \omega_k, \quad (13)$$

$$y_k = \begin{cases} Cx_k + v_k, & \gamma_k = 1, \\ \emptyset, & \gamma_k = 0. \end{cases} \quad (14)$$

where $x_k := x_C(t_k) \in \mathbb{R}^{n_x}$ and $u_k := u_C(t_k) \in \mathbb{R}^{n_u}$ are the state and control input in discrete time for $k \in \mathbb{N}$, respectively. The matrices corresponding to the discrete time dynamics depend on realizations of the intersampling intervals, i.e., by exact discretization (see e.g. Åström (1970) and Åström and Wittenmark (2013))

$$A_k := A(\tau_k), \text{ where } A(\tau) := e^{A_C \tau},$$

$$B_k := B(\tau_k), \text{ where } B(\tau) := \int_0^\tau e^{A_C s} B_C ds.$$

The disturbances ω and v are sequences of zero-mean independent random vectors, $\omega_k \in \mathbb{R}^{n_\omega}$ and $v_k \in \mathbb{R}^{n_v}$, respectively, with covariances $\mathbb{E}[\omega_k(\omega_k)^\top] = \Phi_k^\omega$ and $\mathbb{E}[v_k(v_k)^\top] = \Phi_k^v$, for all $k \in \mathbb{N}$, with

$$\Phi_k^\omega := \Phi^\omega(\tau_k), \text{ where } \Phi^\omega(\tau) := \int_0^\tau e^{A_C s} B_\omega B_\omega^\top e^{A_C^\top s} ds.$$

The cost function (5) can be written in terms of the discrete-time system, for $N(T) := \{\min L \mid \sum_{k=0}^L \tau_k > T\}$ the number of sampling intervals up to time T , as

$$J_C^a = \limsup_{T \rightarrow \infty} \frac{1}{T} \mathbb{E} \left[\sum_{k=0}^{N(T)-1} g(x_k, u_k, \tau_k) \right], \quad (15)$$

where $g(x, u, \tau) := x^\top Q(\tau)x + 2x^\top S(\tau)u + u^\top R(\tau)u + \alpha(\tau)$, for

$$\begin{bmatrix} Q(\tau) & S(\tau) \\ (\star)^\top & R(\tau) \end{bmatrix} := \int_0^\tau e^{\begin{bmatrix} A_C & B_C \\ 0 & 0 \end{bmatrix} s} \begin{bmatrix} Q_C & 0 \\ 0 & R_C \end{bmatrix} e^{\begin{bmatrix} A_C & B_C \\ 0 & 0 \end{bmatrix} s} ds.$$

and, for $\text{tr}(\cdot)$ the trace of a matrix,

$$\alpha(\tau) := \text{tr}(Q_C \int_0^\tau \int_0^t e^{A_c s} B_\omega B_\omega^\top e^{A_c^\top s} ds dt),$$

which is the cost associated with the continuous-time behavior of the Wiener process between update instances. Let $Q_k := Q(\tau_k)$, $S_k := S(\tau_k)$, $R_k := R(\tau_k)$ and $\alpha_k := \alpha(\tau_k)$.

Note that we consider deterministic delays such that the matrices take deterministic values, but for stochastic delays the matrices would be random variables. We will show in Section 4.6 that for the deadline-driven approach to deal with stochastic delays, the problem can be converted into an equivalent problem where the matrices take deterministic values as well. In the remainder of the paper we present equations for the deterministic case. To accommodate further study, the chosen formulation often also applies to the stochastic delay case.

3.3. Estimation of the current state

As in standard LQG control, we will use a state estimate to determine the control input. Recall that the separation principle holds (Schenato et al., 2007) when the switching signal $\{\sigma_k\}_{k \in \mathbb{N}}$ is known. Since measurements are delayed, may be absent, and provide only partial and/or noisy information, the current state is estimated. At time t_k , the realizations of τ_l and γ_l , for all $l \in \mathbb{N}_{<k}$ are known. Hence, an estimate \bar{x}_k of the state x_k from the information \mathcal{I}_k can be obtained by the linear estimator–predictor

$$\begin{aligned} \bar{x}_k &= A_{k-1} \bar{x}_{k-1} + B_{k-1} u_{k-1} \\ &+ \gamma_{k-1} L_{k-1} (y_{k-1} - C \bar{x}_{k-1}), \quad k \in \mathbb{N}_{>0}, \end{aligned} \quad (16)$$

with initial condition $\bar{x}_0 := \mathbb{E}[x_0 | \mathcal{I}_0]$. The gain matrices $L_k \in \mathbb{R}^{n_x \times n_y}$, $k \in \mathbb{N}$ will be given by a chosen estimation policy that can be either a time-varying Kalman filter or a Luenberger observer, leading to two types of control policies. In alignment with the policy notation in Section 2.4, we denote the non-switching and switching control policies that use a time-varying Kalman filter by b_K and π_K , respectively, and we denote control policies with a Luenberger observer analogously by b_L or π_L . Expressions for the gains L_k , $k \in \mathbb{N}$, are given next.

The time-varying Kalman filter (see, e.g., Åström (1970)) provides the best estimates \bar{x}_k of x_k in the least-squares sense. It is given by (16) with time-varying gain

$$L_k = A_k \Theta_k C^\top (C \Theta_k C^\top + \Phi_{\sigma_k}^v)^{-1} \quad \text{for all } k \in \mathbb{N}, \quad (17)$$

that depends on the error covariance $\Theta_k = \mathbb{E}[(x_k - \bar{x}_k)(x_k - \bar{x}_k)^\top | \mathcal{I}_k]$. For the Kalman filter, Θ_k is equal to the covariance of $x_k - \mathbb{E}[x_k | \mathcal{I}_k]$ since $\bar{x}_k = \mathbb{E}[x_k | \mathcal{I}_k]$. The realizations of the error covariance are given by

$$\begin{aligned} \Theta_k &= (A_{k-1} - \gamma_{k-1} L_{k-1} C) \Theta_{k-1} (A_{k-1} - \gamma_{k-1} L_{k-1} C)^\top \\ &+ \gamma_{k-1} L_{k-1} \Phi_{\sigma_{k-1}}^v (L_{k-1})^\top + \Phi_{k-1}^\omega, \\ &= \begin{cases} (A_{k-1} - L_{k-1} C) \Theta_{k-1} (A_{k-1} - L_{k-1} C)^\top \\ \quad + L_{k-1} \Phi_{\sigma_{k-1}}^v (L_{k-1})^\top + \Phi_{k-1}^\omega, & \text{if } \gamma_{k-1} = 1, \\ A_{k-1} \Theta_{k-1} (A_{k-1})^\top + \Phi_{k-1}^\omega, & \text{if } \gamma_{k-1} = 0, \end{cases} \end{aligned} \quad (18)$$

where the initial condition $\Theta_0 = \Phi^{\omega_0}$ is equal to the covariance matrix of the initial estimate. Note that innovation of the estimates only occurs if $\gamma_{k-1} = 1$, otherwise (16) is a pure prediction step.

Instead of the Kalman filter, we can select a fixed-structure Luenberger observer for which the gain is constant for each chosen method $m \in \mathcal{M}$. Let $\bar{L}_m \in \mathbb{R}^{n_x \times n_y}$ denote the value of the gain for each method $m \in \mathcal{M}$, then the gain of the estimator–predictor (16) is given by

$$L_k = \bar{L}_{\sigma_k} \quad \text{for all } k \in \mathbb{N}. \quad (19)$$

While for the most general case with data loss this does not provide the best linear estimator when the choice of method is fixed (Schenato et al., 2007), a value of the gain can be derived for the policies b_L that is optimal (in the least-squares sense) in the class of fixed-structure observers (see Costa, Fragoso, and Marques (2006), Schenato (2008) and Silva and Solis (2013)). Furthermore, there are many cases where an optimal Luenberger observer performs equally well as a Kalman filter, as is the case in the standard LQG solution. The computation of the ‘optimal’ gains $\bar{L}_m \in \mathbb{R}^{n_x \times n_y}$ will be detailed in Section 4. Note that the covariance is computed using the Joseph form (18) for the Luenberger observer, which reduces to the Riccati equation only for the Kalman filter (17).

The estimation mechanisms proposed in this section will also be considered when predicting future realizations of the state estimate. While future realizations of the state estimate depend on the future realizations of τ_k and γ_k , which are not known at time t_k , we can use the rollout scheme discussed before to fix their probabilities and compute predictions of the future state estimates and error covariances. While the time-varying Kalman filter provides the best estimate in the least-squares sense, predicting future error covariances is not always possible (Sinopoli, Schenato, Franceschetti, Poolla, Jordan, & Sastry, 2003). Therefore, the Luenberger observer is a useful alternative, as will be shown. In particular, for policies b_K with $\gamma_k = 1$ for all $k \in \mathbb{N}$ and deterministic delay, there exists a stationary Kalman filter gain, i.e., there exists a cost-equivalent Luenberger observer, that is optimal. However, if, for example, $\gamma_k \neq 1$ but random, then predicting covariances is not possible due to the nonlinear dependency on previous covariances. For the policies b_L , this relationship remains linear. Therefore, for mean-square stabilizing Luenberger observer gains, the expected covariance of the future state estimates converges to a constant value $\bar{\Theta}_b := \lim_{N \rightarrow \infty} \mathbb{E}[\Theta_N | \mathcal{I}_0, b]$ if $b = b_L$, as we show in the next sections.

3.4. Actuation policy

The proposed switching policy (11) compares, at time t_k , $k \in \mathbb{N}$, the expected future cost for different scheduling options whilst assuming that the future choices of the sequence $\{\sigma_l\}_{l \in \mathbb{N}_{\geq k}}$ are known. Recall that we consider $f_m(\tau) = \delta(\tau - \bar{\tau}_m)$ for each $m \in \mathcal{M}$. For any arbitrary known switching sequence $\{\sigma_l\}_{l \in \mathbb{N}_{\geq k}}$, for $T \rightarrow \infty$ in (12), and given a state estimate based on the information \mathcal{I}_k , the optimal actuation policy u^u is given by the time-varying LQR controller (see, e.g., Åström (1970)) where the state is replaced by its estimate, i.e., the LQR part of the LQG regulator.

The proposed actuation policy u^u is to implement, on the horizon after t_k , the control inputs corresponding to the optimal actuation policy u^u for the switching option and horizon (11) selected at t_k . Define, for all $m \in \mathcal{M}$, the realizations of the system matrices for a given $\bar{\tau}_m$, as

$$\begin{aligned} \bar{A}_m &:= A(\bar{\tau}_m), & \bar{B}_m &:= B(\bar{\tau}_m), & \bar{\Phi}_m^\omega &:= \Phi^\omega(\bar{\tau}_m), \\ \bar{Q}_m &:= Q(\bar{\tau}_m), & \bar{R}_m &:= R(\bar{\tau}_m), & \bar{S}_m &:= S(\bar{\tau}_m), \\ \bar{\alpha}_m &:= \alpha(\bar{\tau}_m). \end{aligned}$$

Formally, at times t_{k+h} for $h = 0, 1, \dots, H_k - 1$, actuation updates take place according to

$$u_{k+h}^u : u_{k+h} = -K_{\sigma_k, h} \bar{x}_{k+h}, \quad h = \{0, 1, \dots, H_k - 1\}, \quad (20)$$

where the control gains $K_{m, h} \in \mathbb{R}^{n_u \times n_x}$, $m \in \mathcal{M}$, can be derived from the Riccati equation

$$\begin{aligned} G_{m, h} &= \bar{R}_m + \bar{B}_m^\top P_{m, h+1} \bar{B}_m, \\ K_{m, h} &= (G_{m, h})^{-1} ((\bar{B}_m)^\top P_{m, h+1} \bar{A}_m + (\bar{S}_m)^\top), \\ P_{m, h} &= \bar{A}_m^\top P_{m, h+1} \bar{A}_m + \bar{Q}_m - K_{m, h}^\top G_{m, h} K_{m, h}, \end{aligned} \quad (21)$$

solved recursively with $P_{m,H} = \bar{P}$ with $\bar{P} := P_{b^*}$ the stationary solution to (21) for $m = b^*$, i.e., $P_{b^*} = P_{b^*,0}$ when $H \rightarrow \infty$ under policy b^* (i.e., when $T \rightarrow \infty$ in (12) and $m = b^*$). Let $G_{m,H} = \bar{G} := G_{b^*}$ and $K_{m,H} = \bar{K} := K_{b^*}$ be computed analogously. For the case where $H = 1$, we drop the additional subscript h in $P_{m,h}$.

Now, we have provided the mechanics needed to evaluate (12) in a meaningful way. In Section 4, we derive from (12) explicit switching conditions and the main results. For comparison, the cost of the base policies can be computed as shown next.

3.5. Cost of base policies and selecting b^*

To establish the best base policy b^* a-priori, we require an analytic expression of the performance of the base policies. As explained in Section 3.3, for our general framework, which includes probabilistic data loss, it is possible to derive an optimal control scheme with the Kalman filter, but predicting covariances is not possible. Therefore, an analytic expression for the performance, in terms of index (5), is not available for the general model (see, e.g., Schenato et al. (2007)). As a result, our main results build on the predictions using a Luenberger-type observer with an LQR-type controller. For the general case with Luenberger observer under deterministic delays, we can establish an analytic expression for the performance which has the same form as the classical expression for LQG performance. For several special cases, such as the case without data loss and the cases discussed in Remark 8, an expression can also be established for the Kalman filter, e.g., if a cost-equivalent fixed-gain observer exists.

For measurements with deterministic delay, i.e., $f_m(\tau) = \delta(\tau - \bar{\tau}_m)$, we have the following known result (Åström, 1970; Costa et al., 2006; Schenato et al., 2007) for the LQG-type policies when $\sigma_k = b$, $b \in \mathcal{M}$, for all $k \in \mathbb{N}$. If the expected covariance of the state $\lim_{N \rightarrow \infty} \mathbb{E}[\Theta_N | \mathcal{I}_0, b]$ converges to a constant value $\bar{\Theta}_b$, which is the case for the observers considered in the base policies in this paper (as discussed in Section 3.3), then the cost of a base policy b is given by

$$J_b := J_C^a(b) = \limsup_{N \rightarrow \infty} \mathbb{E} \left[\frac{1}{N} \sum_{k=0}^{N-1} g(x_k, u_k, \tau_k) \right] \\ = \frac{1}{\bar{\tau}_b} c_b, \text{ where } c_b := \text{tr}(\bar{\Phi}_b^\omega \bar{P}_b + \bar{\Theta}_b \bar{K}_b^\top \bar{G}_b \bar{K}_b) + \bar{\alpha}_b \quad (22)$$

where the matrices $\bar{P}_b, \bar{K}_b, \bar{G}_b$ follow from the stationary solution to (21). For the base policies considered in this paper we will make an assumption (Assumption 1) to guarantee that the limit $\bar{\Theta}_b := \lim_{N \rightarrow \infty} \mathbb{E}[\Theta_N | \mathcal{I}_0, b]$ of the expected future covariance exists. The value of $\lim_{N \rightarrow \infty} \mathbb{E}[\Theta_N | \mathcal{I}_0, b]$ can, e.g., be computed using the procedure in Section 4.2 for the Luenberger observer (computing the stationary solution to (25), e.g., via the expression after (26)) or, for the LQG problem without data loss, by solving the standard algebraic Riccati equations. From the base policy costs, b^* can be computed according to (7).

4. Main results

The main results are in the form of explicit switching and control solutions with performance guarantees. This section details for two cases the derivation of $J_k^{m,H}$, described by (12), such that (30) is well-posed. The first case considers the trade-off between the three modeling parameters when delays are different but deterministic for each case. It is a mixture and generalization of the cases in van Horssen et al. (2015, 2016). Specifically, we consider the set of characteristics with trade-off between deterministic delays, constant (non-zero) accuracy and constant loss probability for each

$m \in \mathcal{M}$, i.e., $f_m(\tau) = \delta(\tau - \bar{\tau}_m)$ for $\tau \in \mathbb{R}_{\geq 0}$, $\Phi_m^\nu \geq 0$, and $\bar{\gamma}_m \leq 1$ for each $m \in \mathcal{M}$. This model represents a large class of systems with data-processing. The second case details how a deadline-driven approach together with the results for the first case can be used to address the case of stochastic delays.

4.1. Cost prediction

In order to evaluate the switching condition (11) at time t_k , an expression for the expected future cost (12) is needed. For a given choice of $m \in \mathcal{M}$, the future choices of σ_k are assumed to be known in accordance with the policy proposed in Section 3. Since the values of τ_k are assumed to be deterministic in this case, all future instances of system matrices (A_l, B_l , etc. for $l \in \mathbb{N}_{>k}$) are known. Let the predictions of the future state estimates for $\pi \in \{\pi_K, \pi_L\}$ be denoted $\hat{x}_{k,h} := \mathbb{E}[\bar{x}_{k+h} | \mathcal{I}_k]$, with initial condition $\hat{x}_{k,0} = \bar{x}_k$. Additionally, let $\hat{\Theta}_{k,h} := \mathbb{E}[(x_{k+h} - \bar{x}_{k+h})(x_{k+h} - \bar{x}_{k+h})^\top | \mathcal{I}_k]$ denote the predictions of the future covariance of the state at time $k+h$, computed at time k , with $\hat{\Theta}_{k,0} := \Theta_k$. In accordance with our switching policy as described in Section 3.1, the expected future cost (12) up to time $t_k + T$, for $T = N\bar{\tau}_{b^*} + H\bar{\tau}_m$, $N \in \mathbb{N}$ can be determined for each $m \in \mathcal{M}$. Following optimal control arguments which can be found in Åström (1970) and Schenato et al. (2007), we find that, for $N \rightarrow \infty$ which implies $T \rightarrow \infty$ in (12),

$$J_k^{m,H} = \limsup_{N \rightarrow \infty} \frac{[\bar{x}_k^\top P_{m,0} \bar{x}_k + \chi_m^H(\Theta_k, N) + H\bar{\alpha}_m + N\bar{\alpha}_{b^*}]}{H\bar{\tau}_m + N\bar{\tau}_{b^*}} \quad (23)$$

where

$$\chi_m^H(\Theta_k, N) := \text{tr}(\hat{\Theta}_{k,0} P_{m,0} + \hat{\Theta}_{k,0} K_{m,0}^\top G_{m,0} K_{m,0}) \\ + \sum_{h=1}^{H-1} \text{tr}(\bar{\Phi}_m^\omega P_{m,h} + \hat{\Theta}_{k,h} K_{m,h}^\top G_{m,h} K_{m,h}) \\ + \text{tr}(\bar{\Phi}_m^\omega \bar{P} + \hat{\Theta}_{k,H} \bar{K}^\top \bar{G} \bar{K}) \\ + \sum_{h=H+1}^{H+N-1} \text{tr}(\bar{\Phi}_m^\omega \bar{P} + \hat{\Theta}_{k,h} \bar{K}^\top \bar{G} \bar{K}), \quad (24)$$

and where $\hat{\Theta}_{k,h}$ are forward predictions of the covariance, starting with $\hat{\Theta}_{k,0} = \Theta_k$, while assuming that $\sigma_{k+h} = m$, for $h = 0, 1, \dots, H-1$, and $\sigma_l = b^*$ for all $l \in \mathbb{N}_{>k+H}$.

For the most general case with data loss, evaluating (11) is still not possible because, for the Kalman filter (17), (18) is a nonlinear function of previous values of γ_k , hence the predictions $\hat{\Theta}_{k,h}$ cannot be easily computed since $\gamma_l, l \in \mathbb{N}_{>k}$, are unknown. Furthermore, for $N \rightarrow \infty$, (24) is unbounded, leaving (11) ill-defined. We show next that, for the Luenberger-type observer, these issues can be resolved, leading to our main result.

4.2. Predictions with Luenberger observer

Here, using the Luenberger observer, we establish an alternative expression for (24) such that an equivalent form of (11) is well-defined. To compute values of $\hat{\Theta}_{k,h}$, the observer gain can be fixed to a constant value for each $h \in \mathbb{N}$ depending on the value of σ_{k+h} , $k \in \mathbb{N}$, i.e., assume that $L_{k+h} = \bar{L}_{\sigma_{k+h}}$ as in (19) for all $h \geq 0$. Then, the dependence on γ_{k+h} becomes linear and exact values of $\hat{\Theta}_{k,h}$ can be computed as follows.

Define the expectation over the right-hand side of (18) with respect to γ_k as

$$\mathcal{F}_m(\Theta) := \bar{\gamma}_m \bar{A}_m \Theta (\bar{A}_m)^\top + (1 - \bar{\gamma}_m) \bar{A}_m \Theta (\bar{A}_m)^\top + \Psi_m,$$

where $\tilde{A}_m := \bar{A}_m - \tilde{L}_m C$ and $\Psi_m := \bar{\gamma}_m \tilde{L}_m \Phi_m^v (\tilde{L}_m)^\top + \tilde{\Phi}_m^\omega$. Then, for all $h \in \mathbb{N}$ with initial condition $\hat{\Theta}_{k,0}$,¹

$$\begin{aligned} \hat{\Theta}_{k,h+1} &= \mathcal{F}_{\sigma_{k+h}}(\hat{\Theta}_{k,h}), \\ &= \text{vec}^{-1} \left(\mathcal{T}_{\sigma_{k+h}} \text{vec}(\hat{\Theta}_{k,h}) \right) + \Psi_{\sigma_{k+h}}, \end{aligned} \quad (25)$$

where, using the Kronecker product \otimes ,

$$\mathcal{T}_m := \bar{\gamma}_m \tilde{A}_m \otimes \tilde{A}_m + (1 - \bar{\gamma}_m) \tilde{A}_m \otimes \bar{A}_m.$$

The following assumption guarantees that the covariance is bounded for any non-switching policy using only method $m \in \mathcal{M}$ with a chosen observer gain \tilde{L}_m , and therefore the corresponding non-switching estimator is stable. The assumption can easily be checked by computing the eigenvalues of \mathcal{T}_m for all $m \in \mathcal{M}$.

Assumption 1. \mathcal{T}_m is Schur for all $m \in \mathcal{M}$.

As mentioned in Section 3.3, gains that satisfy [Assumption 1](#) can be obtained, for example, by the methods in [Costa et al. \(2006\)](#), [Schenato \(2008\)](#) and [Silva and Solis \(2013\)](#).

If \mathcal{T}_{b^*} is Schur, then for $h \in \mathbb{N}_{>H}$,

$$\begin{aligned} \hat{\Theta}_{k,h} &= \text{vec}^{-1} \left((I - \mathcal{T}_{b^*})^{-1} (I - \mathcal{T}_{b^*}^{(h-H)}) \text{vec}(\Psi_{b^*}) \right. \\ &\quad \left. + \mathcal{T}_{b^*}^{(h-H)} \text{vec}(\hat{\Theta}_{k,H}) \right), \end{aligned} \quad (26)$$

which converges when $N \rightarrow \infty$ to

$$\limsup_{N \rightarrow \infty} \hat{\Theta}_{k,N} = \text{vec}^{-1} \left((I - \mathcal{T}_{b^*})^{-1} \text{vec}(\Psi_{b^*}) \right) =: \tilde{\Theta}_{b^*}.$$

In the sequel, when deriving switching conditions, we will use the fact that this limit $\tilde{\Theta}_{b^*}$ is independent of the value of the expected covariance at the end of the horizon $\hat{\Theta}_{k,H}$, which follows from (26).

Observe in (26) that the contributions of Ψ_{b^*} to $\hat{\Theta}_{k,h}$ for $h \in \mathbb{N}_{>H}$, i.e., after the horizon, are independent of the choice of σ_k . Thus, in the last summation in (24), i.e., for $h \in \mathbb{N}_{H+1, H+N-1}$, the contributions of Ψ_{b^*} are equal for each choice in (11) and can therefore be neglected as they do not play a role in evaluating (11). Furthermore, the summation of $\mathcal{T}_{b^*}^{(h-H)} \text{vec}(\hat{\Theta}_{k,H})$ for all $h \in \{H, H+1, \dots, H+N\}$ has an analytic solution for $N \rightarrow \infty$, namely $\text{vec}^{-1} \left((I - \mathcal{T}_{b^*})^{-1} \text{vec}(\hat{\Theta}_{k,H}) \right)$. This allows us to define the method-dependent part (the terms that depend on a particular choice of m) of $\lim_{N \rightarrow \infty} \chi_m^H(\Theta_k, N)$ as

$$\begin{aligned} \tilde{\eta}_m^H(\Theta_k) &:= \text{tr}(\Theta_k P_{m,0} + \Theta_k K_{m,0}^\top G_{m,0} K_{m,0}) \\ &\quad + \sum_{h=1}^{H-1} \text{tr}(\tilde{\Phi}_m^\omega P_{m,h} + \hat{\Theta}_{k,h} K_{m,h}^\top G_{m,h} K_{m,h}) \\ &\quad + \text{tr}(\tilde{\Phi}_m^\omega \bar{P}) + \text{tr}(Z_{b^*}(\hat{\Theta}_{k,H}) \bar{K}^\top \bar{G} \bar{K}), \end{aligned} \quad (27)$$

$$\text{where } Z_{b^*}(\hat{\Theta}_{k,H}) = \text{vec}^{-1} \left((I - \mathcal{T}_{b^*})^{-1} \text{vec}(\hat{\Theta}_{k,H}) \right).$$

4.3. Luenberger gain selection

Note that the choice of \tilde{L}_m is free. As discussed in [Sinopoli et al. \(2003\)](#), the steady-state Kalman gain for the case without information loss, i.e., when $\bar{\gamma}_{b^*} = 1$, is a natural choice. However, the non-switching system with data loss can be interpreted as a Markov jump linear system (MJLS) ([Costa et al., 2006](#)) with two modes. Using MJLS theory ([Costa et al., 2006](#), Def. 5.7), a mean expected value of the covariance $\tilde{\Theta}_b$ can be determined as the

solution to $\Theta = \mathcal{F}_b(\Theta)$ for given \tilde{L}_b , by either an iterative procedure or by solving a semidefinite program, as the solution to

$$\begin{aligned} \tilde{\Theta}_b &= \bar{A}_b \tilde{\Theta}_b (\bar{A}_b)^\top + \tilde{\Phi}_b^\omega \\ &\quad - \bar{\gamma}_b \bar{A}_b \tilde{\Theta}_b C^\top (C \tilde{\Theta}_b C^\top + \Phi_b^v)^{-1} C \tilde{\Theta}_b \bar{A}_b^\top, \end{aligned} \quad (28)$$

obtained when using the Luenberger-type gain

$$\tilde{L}_b := \bar{A}_b \tilde{\Theta}_b C^\top (C \tilde{\Theta}_b C^\top + \Phi_b^v)^{-1}, \quad (29)$$

which is optimal in the class b_L in the sense that $\tilde{\Theta}_b(\tilde{L}_b) \preceq \tilde{\Theta}_b(L)$ for any $L \in \mathbb{R}^{n_x \times n_y}$, such that $\tilde{\Theta}_{b^*}$ is given by $\tilde{\Theta}_b(\tilde{L}_{b^*})$. The expression for the cost of the base policies with Luenberger observer, which we denote J_{b_L} , is given by (22) for $\tilde{\Theta}_b$ replaced by $\tilde{\Theta}_b$.

4.4. Switching condition and performance guarantee

To present the first main result, we use the next proposition, which provides the proposed switching policy (11) for $T \rightarrow \infty$ (i.e., when $N \rightarrow \infty$) when the expected covariances in (23) evolve according to (25) for the optimal Luenberger gains.

Proposition 1. *Suppose that [Assumption 1](#) holds and that states are estimated using (16) with (19) where $L_{k+h} = \tilde{L}_m$ for $h \in \mathbb{N}_{[0, H-1]}$ and $L_{k+h} = \tilde{L}_{b^*}$ for $h \in \mathbb{N}_{\geq H}$. Then, for $T \rightarrow \infty$, the switching condition (11) is equivalent to*

$$\begin{aligned} (\sigma_k, H_k) &:= \\ \arg \min_{m \in \mathcal{M}, H \in \mathcal{H}_m} &\bar{\chi}_k^1 P_{m,0} \bar{\chi}_k + \tilde{\eta}_m^H(\Theta_k) - H \frac{\bar{\tau}_m}{\bar{\tau}_{b^*}} \tilde{c}_{b^*} + H \bar{\alpha}_m \end{aligned} \quad (30)$$

where $\tilde{c}_{b^*} := \text{tr}(\tilde{\Phi}_{b^*}^\omega \bar{P} + \tilde{\Theta}_{b^*} \bar{K}^\top \bar{G} \bar{K}) + \bar{\alpha}_{b^*}$. \square

The proof is provided in [Appendix A](#).

Remark 3. The last term in (30) corrects for the misalignment of the expected sampling times such that the term $\frac{1}{H \bar{\tau}_m + N \bar{\tau}_{b^*}}$ in (23) equals to the same value of T in (12) for each $m \in \mathcal{M}$. Hence, the divisor T can be eliminated from the arg min in (11) and the case $T \rightarrow \infty$ becomes well-defined.

We can compute the right-hand side of (30) explicitly for each $m \in \mathcal{M}$, $H \in \mathcal{H}_m$ and thus our switching condition has become explicit. We can now state our first main result.

Theorem 1. *If the assumptions of [Proposition 1](#) are satisfied and if for π_L the control policy functions (6) are defined by (20) and (30) with restriction (9), it holds that*

$$J_{\pi_L} \leq J_{b_L^*} \leq J_{b_L}, \text{ for all } b \in \mathcal{M}, \quad (31)$$

where $b_L^* = \arg \min_{b \in \mathcal{M}} J_{b_L}$, and J_{b_L} given by (22) with $\tilde{\Theta}_b = \tilde{\Theta}_b$, and $\tilde{\Theta}_b$ the solution to (28). \square

[Theorem 1](#) means that our proposed policy achieves the improvement (8) with respect to its non-switching counterparts for the Luenberger observer. The switching policy (30) is restricted to allow switching only after each chosen horizon.

Remark 4. While (30) can be evaluated after each sample, i.e., lifting restriction (9) and allowing switching on the horizon, an analogous result to [Theorem 1](#) for this case has not yet been established (see also [Remark 4](#)).

Observe that the switching condition (30) requires little computational effort compared to other scheduling approaches. In particular, many switching variables can be precomputed to limit computations and the number of switching options can easily be adapted to the amount of available resources. Hence, the policy is fast from a computational viewpoint and therefore especially useful in resource-constrained implementations of data-intensive systems.

¹ Define $\text{vec}(\cdot)$ as the stack operator transforming a matrix to a column vector and consider the identity $APB = \text{vec}^{-1}[(B^\top \otimes A)\text{vec}(P)]$.

Remark 5. Although of interest, it is beyond the scope of the present paper to establish a guarantee of strict performance improvement of the proposed strategies. However, we do prove $J_\pi \leq J_b$ and show the strict improvements via a numerical example. In addition, note that we believe that strict performance improvement guarantees could be derived by following a similar approach to the one in Antunes and Heemels (2014) where a switched system derived in a different context was studied. Such an approach entails rather long arguments, requiring concepts such as ergodicity, and it is therefore not pursued here.

Remark 6. Note that in the context of the rollout method, we explicitly compute infinite horizon cost differences such that approximations or Monte–Carlo methods are not needed. The general idea is that the use of a base policy creates a separation in the cost differences between switching options. One part, which is infinite over the infinite horizon, is equal for each switching option and is therefore not relevant in (30), and another part, which is finite over the infinite horizon, is different for each option and can be used for switching.

4.5. Kalman filter approach

While our policy with Luenberger observer already achieves improvement over J_{b_l} , it is useful to introduce a time-varying Kalman filter solution which can attain additional improvement. The use of the Kalman filter is two-fold, namely to estimate \bar{x}_k and Θ_k and to predict the future values of the covariance Θ_{k+h} . Note that we can use the Kalman filter for the estimation problem directly. For the predictions, consider the case where $\hat{\Theta}_{k,h}$ is the future expectation taken over (18) with the Kalman gain (17). The dependence of $\hat{\Theta}_{k,h}$ on γ_{k+h} is nonlinear (cf. Schenato et al. (2007)) for the time-varying Kalman filter, preventing the computation of future cost predictions. However, note that $\hat{\Theta}_{k,1}$ only takes expectation over γ_k , with linear dependence. Thus, $\hat{\Theta}_{k,1}$ has an exact solution

$$\hat{\Theta}_{k,1} = \bar{A}_m \Theta_k (\bar{A}_m)^T + \bar{\Phi}_m^\omega - \bar{\gamma}_m L_k (C \Theta_k C^T + \Phi_m^\nu) L_k^T, \quad (32)$$

where

$$L_k = \bar{A}_m \Theta_k C^T (C \Theta_k C^T + \Phi_m^\nu)^{-1}. \quad (33)$$

Let the policies that use a Kalman filter for estimation and for prediction on the switching horizon and a Luenberger observer for prediction after the switching horizon be denoted by π_{k+L} . Potentially, using the Kalman filter improves upon π_L in Theorem 1. While one may expect that $J_{\pi_{k+L}} \leq J_{\pi_L}$, it is not proven here as a relation between switched policies is nontrivial to establish. Instead, we establish that a result analogous to Theorem 1 can still be derived, i.e., that $J_{\pi_{k+L}} \leq J_{b_l^*}$. In fact, when the switching horizon is restricted to $\bar{H}_m = 1$, we obtain the following corollary.

Corollary 1. If $\bar{H}_m = 1$ for all $m \in \mathcal{M}$, and if the conditions of Proposition 1 are satisfied, with the difference that the observer gain L_k is chosen equal to (33), instead of $L_k = \tilde{L}_m$, such that $\hat{\Theta}_{k,1}$ is given by (32), then it holds that $J_{\pi_{k+L}} \leq J_{b_l^*}$.

Recall that \tilde{L}_m in Theorem 1 can be chosen freely. The proof follows from the fact that (33) is the optimal estimation gain in the least-squares sense, which is at least as good as \tilde{L}_m . Therefore, the covariance Θ_k and predicted cost $J_k^{\sigma_k=b, H_k=1}$ are smaller, in cost sense, when the Kalman filter is used rather than their respective counterparts for the Luenberger observer.

Remark 7. Note that the cost of a base policy with Kalman filter for estimation is less than the cost of the base policy considered in Theorem 1. Then considering our rollout method, with a Kalman filter

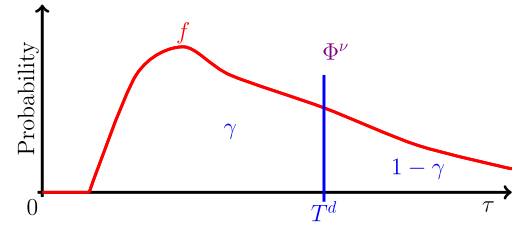


Fig. 2. Example of a delay probability distribution f and the connection between parameters of data-processing (delay τ , probability of data acquisition γ and accuracy modeled by Φ^ν) and a deadline T^d .

for estimation and for prediction on the switching horizon and a Luenberger observer for prediction after the switching horizon, can very well lead to a better cost than such a base policy with Kalman filter for estimation in the sense that $J_{\pi_{k+L}} \leq J_{b_k^*}$. Establishing conditions to formally guarantee this property is nontrivial. This is not further pursued in the present paper, but only illustrated by the numerical example in Section 5.

Remark 8. For the special cases in van Horssen et al. (2015, 2016) and the generalization of van Horssen et al. (2015), it is also possible to guarantee improvement in the sense $J_{\pi_k} \leq J_{b_k^*}$. In those cases, the base policies b_k have a time-invariant control policy with Luenberger observer, i.e., with a fixed observer gain, which attains the same cost as the time-varying LQG with Kalman filter, in the sense that $J_{b_k} = J_{b_l}$.

Remark 9. The restriction $\bar{H}_m = 1$ in Corollary 1 may be lifted, but then $\hat{\Theta}_{k,h}$ is predicted with $L_{k+h} = \tilde{L}_m$ for $h \in \mathbb{N}_{[2,H]}$ and (9) should be imposed.

4.6. Stochastic delay

The class of systems with random measurement delay introduces uncertainty in the arrival time of new information. One additional challenge that needs to be considered is when new information is required or should be used, which was studied in Prakash et al. (2017) for stochastic actuation delays by using the insights in de Koning (1982). Here, we discuss how a deadline-driven approach in Prakash et al. (2017) can be captured in the framework of Theorem 1.

Consider, for example, a data-processing method that uses a fixed accuracy threshold on the information, i.e., a minimum accuracy has to be achieved for the processing to complete. Since it may not be exactly unknown how much time is needed to achieve this accuracy, the completion time may be (at least approximately) modeled by a stochastic probability distribution. To limit processing duration, the processing task can be interrupted when a certain maximum processing time threshold is exceeded, i.e., when the delay is larger than a deadline $T_k^d \in \mathbb{R}_{>0}$. The probability of data loss then depends on the value of the deadline. Mathematically, this case with stochastic delay has a known distribution f_m , a fixed accuracy $\Phi_m^\nu \geq 0$, and loss probability linked to a chosen deadline $\bar{\gamma}_m = F_m(T_k^d)$, for each method $m \in \mathcal{M}$, where F_m follows from f_m . The inclusion of stochastic delay and a deadline allows to model many cases that are relevant in practice. The relation between the deadline and the probability of obtaining information, given by the integral of the delay distribution, is schematically depicted in Fig. 2. Note that if the accuracy is delay-dependent, i.e., to consider $\Phi_m^\nu(\tau)$, a more extensive analysis is required that is beyond the scope of this paper.

Table 1
Average cost.

Policy	Cost (numerical)	Theoretical
Only M_1 for $b_L(b^*)$	206.7122	206.8321
Only M_2 for b_L	209.0117	209.4537
Only M_1 for b_K	206.6253	not obtainable
Only M_2 for b_K^a	197.5367	not obtainable
Switched π_{K+L}	187.9246	not obtainable

^aIndicates the optimal non-switching policy.

We take a *deadline-driven* (or self-triggered) approach, in the sense that the system waits until the set deadline before (instantaneously) taking a new sample, i.e., $t_{k+1} = t_k + T_k^d$. The data-loss mechanism is given by

$$\gamma_k := \begin{cases} 1 & \text{if } \tau_k \leq T_k^d \\ 0 & \text{if } \tau_k > T_k^d \end{cases} \quad (34)$$

for a given value of the k th deadline T_k^d and realization τ_k of f_{σ_k} . The data-loss mechanism (34) converts the stochastic delay τ_k to a Bernoulli variable γ_k and a deterministic delay T_k^d . The main problem is the selection of method $\sigma_k = m$ and a processing deadline T_k^d at each decision instant t_k , $k \in \mathbb{N}$.

We assume that the deadline is taken from a finite set $\mathcal{D} \subset \mathbb{R}_{>0}$ which may be arbitrarily large. This set-up can be interpreted as having an extended set of processing methods $\mathcal{M} \times \mathcal{D}$ from which a given choice of m and T^d defines the loss probability as $Pr(\gamma_k = 1 | \sigma_k = m) = F_m(T^d)$. For each method $b \in \mathcal{M}$, one optimal constant value for the deadline $\bar{T}_b^d \in \mathcal{D}$ (that attains the minimal cost) can be found off-line by, e.g., nonlinear optimization or visual inspection as done in our preliminary work (Prakash et al., 2017). Subsequently, we can use Proposition 1 with each combination $(m, T^d) \in \mathcal{M} \times \mathcal{D}$ as a switching option, assuming that $T_{k+h}^d = \bar{T}_{b^*}^d$ and $\sigma_{k+h} = b^*$ for $h \in \mathbb{N}_{\geq 1}$. For reasons of space, the explicit derivation is omitted. Note that all variables become dependent of the chosen deadline. We obtain the second main result.

Theorem 2. *Under the assumptions of Theorem 1, for a set of non-deterministic distributions f_m , $m \in \mathcal{M}$, and for $H_m = 1$, the same guarantee as in Theorem 1 can be obtained if the control policy functions (6) are given by (20) and (30) where $m \in \mathcal{M} \times \mathcal{D}$ such that $\bar{\tau}_m$ and $\bar{\tau}_{b^*}$ are replaced by T^d and $\bar{T}_{b^*}^d$, respectively. \square*

The proof follows directly from Theorem 1. This result is illustrated in the next section.

Remark 10. If a loss probability is present aside from the loss probability due to the deadline, such an additional probability can be taken into account by a simple multiplication of the probabilities.

5. Numerical example

Numerical examples for the trade-offs between delay and accuracy and probability of data acquisition can be found in van Horssen et al. (2015, 2016), respectively. Here, we consider the trade-off between all three parameters, where the probability of data acquisition is governed by a deadline that can be selected on-line, i.e., the case in Section 4.6. We consider a numerical system (1) given by $A_C = \begin{bmatrix} 0 & 1 \\ -1 & 0.8 \end{bmatrix}$, $B_C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $B_\omega = \begin{bmatrix} 0.8 \\ 0.1 \end{bmatrix}$, $C = I$, with cost $Q_C = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}$, $R_C = 10$. Furthermore, the initial state distribution is given with the origin $x_0 = [0, 0]^T$ as mean with small initial covariance $\Phi^{x_0} = 10^{-2} I_{n_x \times n_x}$.

The data-processing methods that are available are indicated by M_1 and M_2 and are modeled by a uniform distribution f_1 with

support between 0.2 and 1.2 and $\Phi_1^y = I$ and by a Gamma distribution f_2 with shape 10 and scale 0.06 and $\Phi_2^y = 2.2 I$, respectively. We allow the deadline only to be chosen between 0.1 and 1.5 with 0.01 step increments. We compute, for each method and for each choice of the deadline, the optimal Luenberger estimator gains and the associated expected covariance limit Θ_b , the optimal LQR controllers and the resulting performance J_b . This theoretical cost is given in Table 1 which leads to $b^* = 1$. The optimal deadlines are found to be $\bar{T}_1^d = 1.2$ and $\bar{T}_2^d = 0.78$. We find that the base policies only have a stable Luenberger solution for $T_d \in [0.84, 1.5]$ for M_1 and $T_d \in [0.63, 1.5]$ for M_2 . For this system and these processing methods, the cost is a convex function of the deadline in these intervals and the optimal deadlines within these regions. To illustrate that the contribution of α to the cost is typically small, we compute $\alpha_1/\bar{T}_1^d = 0.375$ and $\alpha_2/\bar{T}_2^d = 0.248$. Since the optimal switched policy is unknown, we compare to the classical continuous-time LQG control policy without delay or data-loss, which is a theoretical lower bound on the achievable performance that is not attainable in our digital control setting. Note that only the measurement covariance is different for the two modes in this case. We find a lower bound on the cost of 46.7346 and 81.0930 for M_1 and M_2 , respectively. Clearly, the undesirable effects of data-processing incur a significant performance loss.

We allow switching between all methods that have stable base policies. We restrict the switching options to allow only the next instant to be selected, i.e., $H = 1$, and use a time-varying Kalman filter for estimation and a deadline-dependent gain for the first prediction step (see Corollary 1). To verify the performance gain for the system with random disturbances for an infinite horizon, we resort to the typical approach of Monte-Carlo simulations with a large simulation time. To reduce computational complexity, we recompute all system variables for each deadline value. To numerically illustrate the achievable performance gain, we run 100 Monte-Carlo simulations with randomly generated disturbances and delays for a simulation time of 4800s. After this time, the average cost has approximately converged to a constant for all control policies and the switched performance can be compared to the theoretical performance and to simulated performance for the base policies, as presented in Table 1.

We obtain a 9% gain compared to our base policy and the switched policy is shown to be better than both non-switching policies under a Kalman filtering policy (which is the optimal control policy in the non-switching case). It should be noted that M_2 with Kalman filter would have been a better base policy, but this cannot be computed a priori without numerical simulation. Nevertheless, our switching solution performs better. We see that, for this example, the loss due to data-processing (compared to the unapplicable continuous-time LQG controller) can be significantly reduced by switching. We observed recurrent switching between the methods with varying deadlines. Near the origin, M_1 is typically selected whereas M_2 (with a shorter deadline) is usually chosen when the state is further away from the origin. This phenomenon was also observed in van Horssen et al. (2015).

6. Conclusions

The proposed framework to model data-processing units with different characteristics in terms of delay, measurement accuracy, and probability of data acquisition is shown to enable new control designs for system with multiple sensing methods (such as vision-based control of robotic systems, medical imaging, big-data control, etc.). While the authors' main motivation came from the data-processing case, the results are also applicable to other sensing methods and many networked control applications with limitations on computational resources and communication, as similar characteristics can be identified. The explicit switching conditions,

that are derived based on a generalization of the rollout technique, allow to combine the benefits of more than one processing method in a switched control policy that guarantees improvement over non-switching policies. A numerical example shows that this policy can achieve significant improvement over non-switching policies. Several special trade-offs of interest are detailed and discussed and the main results are supported by complete proofs. Making the output matrix C method-dependent could be a starting point for a combination with sensor selection/management (Hero & Cochran, 2011) and sensor fusion methods. Adaptions are envisioned to allow switching on the horizon as in classical rollout, for time-scale separation (Picasso et al., 2010) of actuation moments, for multirate control, for using fixed switching sequences as base policies and to take into account model uncertainty. As can be observed from the above potential extensions, the framework laid down in this paper does not only already address several cases of interest for the control of systems with multiple sensing or processing methods, but can also be used as a starting point for considering relevant problems in a much broader scope.

Appendix A. Proof of Proposition 1

First, we start by justifying the correction term $-H \frac{\bar{\tau}_m}{\bar{\tau}_{b^*}} \tilde{c}_{b^*}$. From the assumption that $L_{k+h} = \tilde{L}_{b^*}$ for $h \geq H$, it follows that $\lim_{N \rightarrow \infty} \hat{\Theta}_{k,N} = \tilde{\Theta}_{b^*}$. Then, for (increasingly) large N , $\|\hat{\Theta}_{k,N} - \tilde{\Theta}_{b^*}\| < \epsilon$ for an arbitrarily small ϵ . Hence, the last term in the summation in (24) becomes arbitrarily close to $\tilde{c}_{b^*} - \bar{\alpha}_{b^*}$ for large N . Therefore we have that, independent of the choice of m , increasing N to $N + 1$ in (24) for sufficiently large N adds $\tilde{c}_{b^*} - \bar{\alpha}_{b^*}$. Hence, with an increment in the factor $N\bar{\alpha}_{b^*}$, \tilde{c}_{b^*} is added in one additional interval $\bar{\tau}_{b^*}$. Recall that, for $T \rightarrow \infty$, $N \rightarrow \infty$. Consider (12) for $T = N\bar{\tau}_{b^*} + \bar{\tau}_{b^*}$ which coincides with (23) for $m = b^*$ and $H = 1$, which is the base policy. Then (12) for $T = N\bar{\tau}_{b^*} + H\bar{\tau}_{b^*}$ for $m \neq b^*$ is given by (23) plus the difference $(N\bar{\tau}_{b^*} + \bar{\tau}_{b^*}) - (N\bar{\tau}_{b^*} + H\bar{\tau}_{b^*}) = (\bar{\tau}_{b^*} - H\bar{\tau}_m)$ times the cost per time unit after N , which is equal to $\frac{1}{\bar{\tau}_{b^*}} \tilde{c}_{b^*}$. Taking then the division of $(\bar{\tau}_{b^*} - H\bar{\tau}_m)$ by $\bar{\tau}_{b^*}$ leads to the difference term $(\tilde{c}_{b^*} - H \frac{\bar{\tau}_m}{\bar{\tau}_{b^*}} \tilde{c}_{b^*})$. Note that the term \tilde{c}_{b^*} gives a constant which does not depend on m and can be removed giving $-H \frac{\bar{\tau}_m}{\bar{\tau}_{b^*}} \tilde{c}_{b^*}$. Now, the divisor $\frac{1}{\bar{\tau}_{b^*}}$ is equal for each $m \in \mathcal{M}$ and can thus be removed from the arg min.

Second, observe that the term $\text{tr}(\Phi_m^\omega \bar{P}_{b^*})$ in the summation in (24) does not depend on m and can be eliminated. The factor $N\bar{\alpha}_{b^*}$ does not depend on m and can also be removed. This allows the switching condition (11), for $N \rightarrow \infty$, to be defined as

$$(\sigma_k, H_k) := \arg \min_{\substack{m \in \mathcal{M} \\ H \in \mathcal{H}_m}} \bar{x}_k^\top P_{m,0} \bar{x}_k + \eta_m^H(\Theta_k) - H \frac{\bar{\tau}_m}{\bar{\tau}_{b^*}} \tilde{c}_{b^*} + H\bar{\alpha}_m$$

with

$$\begin{aligned} \eta_m^H(\Theta_k) := & \text{tr}(\Theta_k P_{m,0} + \Theta_k K_{m,0}^\top G_{m,0} K_{m,0}) \\ & + \sum_{h=1}^{H-1} \text{tr}(\bar{\Phi}_m^\omega P_{m,h} + \hat{\Theta}_{k,h} K_{m,h}^\top G_{m,h} K_{m,h}) \\ & + \text{tr}(\bar{\Phi}_m^\omega \bar{P}) + \sum_{h=H}^{H+N-1} \text{tr}(\hat{\Theta}_{k,h} \bar{K}^\top \bar{G} \bar{K}), \end{aligned} \quad (\text{A.1})$$

where $\hat{\Theta}_{k,h}$ is given by (25) with initial condition Θ_k .

Finally, we show that the differences $\lim_{N \rightarrow \infty} \eta_{b^*}^H(\Theta_k) - \eta_m^H(\Theta_k) = \tilde{\eta}_{b^*}^H(\Theta_k) - \tilde{\eta}_m^H(\Theta_k)$ coincide. Note that, by the chosen switching scheme and the assumption that $L_{k+h} = \tilde{L}_{b^*}$ for $h \geq 1$, the evolution of $\hat{\Theta}_{k,h}$ according to (25) is equivalent for all $h \geq H$. Furthermore, \mathcal{F}_m is a linear function of Θ_k for which the contributions of Ψ_{b^*} in (25) do not depend on m as can be seen in (26). Let $X_{\alpha+l}$ and Y_{H+l} represent the value of $\hat{\Theta}_{k,H+l}$ for $\sigma_k = \beta$ and $H = \alpha$ (e.g. $\alpha = 1$ and $\beta = b^*$) and $\sigma_k = m$ (e.g. $m \neq b^*$) with $H \in \mathcal{H}_m$, respectively. The difference of the summations in (A.1) can then be represented

by

$$\begin{aligned} \text{vec} \left(\sum_{l=0}^N X_{\alpha+l} - Y_{H+l} \right) &= \sum_{l=0}^N (\mathcal{T}_{b^*})^l [\text{vec}(X_\alpha) - \text{vec}(Y_H)] \\ &= (I - \mathcal{T}_{b^*})^{-1} [\text{vec}(X_\alpha) - \text{vec}(Y_H)] \end{aligned}$$

where the limit for $N \rightarrow \infty$ is taken and it is used that \mathcal{T}_{b^*} is Schur. Observing that

$$Z_{b^*}(Y_H) = \text{vec}^{-1} \left((I - \mathcal{T}_{b^*})^{-1} \text{vec}(Y_H) \right)$$

is the part related to the choice (m, H) where Y_H is given by $\hat{\Theta}_{k,H}$ according to (25) completes the proof.

Appendix B. Proof of Theorem 1

Define the function

$$\begin{aligned} V_N(\bar{x}, \Theta) := & \bar{x}^\top \bar{P} \bar{x} + \text{tr}(\Theta \bar{P}) + \text{tr}(\Theta \bar{K}^\top \bar{G} \bar{K}) \\ & + \sum_{h=1}^N \text{tr}((\hat{\Theta}_{k,h} - \tilde{\Theta}_{b^*}) \bar{K}^\top \bar{G} \bar{K}), \end{aligned}$$

where $\hat{\Theta}_{k,h}$ are forward predictions according to (25) for $\sigma_{k+h} = b^*$ starting from $\hat{\Theta}_{k,0} = \Theta$. Note that, under Assumption 1, $\hat{\Theta}_{k,h}$ converges to $\tilde{\Theta}_{b^*}$ exponentially fast for $N \rightarrow \infty$, hence $V_N(\bar{x}, \Theta)$ is bounded for any bounded arguments. Consider, at time t_k , the function (cf. (A.1))

$$\begin{aligned} V_N(\bar{x}_k, \Theta_k) = & \bar{x}_k^\top \bar{P} \bar{x}_k + \eta_{b^*}^1(\Theta_k) \\ & - \text{tr}(\bar{\Phi}_{b^*}^\omega \bar{P}) - N \text{tr}(\tilde{\Theta}_{b^*} \bar{K}^\top \bar{G} \bar{K}) \end{aligned}$$

and, for $(\sigma_k, H_k) = (m, H)$, the function

$$\begin{aligned} \mathbb{E} \left[V_N(\bar{x}_{k+H}, \Theta_{k+H}) + \sum_{l=k}^{k+H-1} g(x_l, u_l, \bar{\tau}_m) \mid \mathcal{I}_k \right] \\ = & \bar{x}_k^\top P_{m,0} \bar{x}_k + \text{tr}(\Theta_k (P_{m,0} + K_{m,0}^\top G_{m,0} K_{m,0})) \\ & + \sum_{h=1}^{H-1} [\text{tr}(\Phi_m^\omega P_{m,h}) + \text{tr}(\Theta_{k+h} K_{m,h}^\top G_{m,h} K_{m,h})] \\ & + H\bar{\alpha}_m + \sum_{h=H}^{H+N-1} \text{tr}(\hat{\Theta}_{k,h} \bar{K}^\top \bar{G} \bar{K}) \\ & + \text{tr}(\hat{\Theta}_{k,H+N} \bar{K}^\top \bar{G} \bar{K}) - N \text{tr}(\tilde{\Theta}_{b^*} \bar{K}^\top \bar{G} \bar{K}) \\ = & \bar{x}_k^\top P_m \bar{x}_k + \eta_m^H(\Theta_k) + H\bar{\alpha}_m \\ & + \text{tr}(\hat{\Theta}_{k,H+N} \bar{K}^\top \bar{G} \bar{K}) - N \text{tr}(\tilde{\Theta}_{b^*} \bar{K}^\top \bar{G} \bar{K}) \\ = & \bar{x}_k^\top P_{m,0} \bar{x}_k + \eta_m^H(\Theta_k) + H\bar{\alpha}_m \\ & + \tilde{c}_{b^*} - \bar{\alpha}_{b^*} + \epsilon_k - \text{tr}(\bar{\Phi}_{b^*}^\omega \bar{P}) - N \text{tr}(\tilde{\Theta}_{b^*} \bar{K}^\top \bar{G} \bar{K}), \end{aligned}$$

where $\epsilon_k := \text{tr}((\hat{\Theta}_{k,H+N} - \tilde{\Theta}_{b^*}) \bar{K}^\top \bar{G} \bar{K})$, with $\epsilon_k \rightarrow 0$ for any $k \in \mathbb{N}$ for $N \rightarrow \infty$. The difference is then, for any large N , given by

$$\begin{aligned} \mathbb{E} \left[V_N(\bar{x}_{k+H}, \Theta_{k+H}) + \sum_{l=k}^{k+H-1} g(x_l, u_l, \bar{\tau}_m) \mid \mathcal{I}_k \right] \\ - V_N(\bar{x}_k, \Theta_k) - \epsilon_k \\ = & \tilde{c}_{b^*} + \bar{x}_k^\top (P_{m,0} - \bar{P}) \bar{x}_k + \eta_m^H(\Theta_k) - \eta_{b^*}^1(\Theta_k) \\ & + H\bar{\alpha}_m - \bar{\alpha}_{b^*} \\ = & \tilde{c}_{b^*} + \bar{x}_k^\top (P_{m,0} - \bar{P}) \bar{x}_k + \tilde{\eta}_m^H(\Theta_k) - \tilde{\eta}_{b^*}^1(\Theta_k) \\ & + H\bar{\alpha}_m - \bar{\alpha}_{b^*} \\ = & \tilde{c}_{b^*} - d_m^H(\bar{x}_k, \Theta_k) - (1 - H \frac{\bar{\tau}_m}{\bar{\tau}_{b^*}}) \tilde{c}_{b^*} \\ = & H \frac{\bar{\tau}_m}{\bar{\tau}_{b^*}} \tilde{c}_{b^*} - d_m^H(\bar{x}_k, \Theta_k), \end{aligned} \quad (\text{B.1})$$

where $d_m^H(x, \Theta) := x^\top (\bar{P} - P_{m,0}) x + \tilde{\eta}_{b^*}^1(\Theta) - \tilde{\eta}_m^H(\Theta) - (1 - H \frac{\bar{\tau}_m}{\bar{\tau}_{b^*}}) \tilde{c}_{b^*} + \bar{\alpha}_{b^*} - H\bar{\alpha}_m$. Notice that $d_m^H(x, \Theta) \geq 0$ when $(\sigma_k, H_k) = (m, H)$ by construction of (30).

Let all instants t_k at which (30) is evaluated be indexes k in the set $\Sigma \subseteq \mathbb{N}$. Then, we can write the cost sum in function (15) as

$G_{N(T)}$ where

$$G_N := \sum_{k \in \Sigma_{\leq N}} \sum_{l=k}^{k+H_k-1} g(x_l, u_l, \tau_l)$$

for which $\mathbb{E}[G_{N(T)}] = \mathbb{E}[E_{N(T)}]$ with

$$E_N := \sum_{k \in \Sigma_{\leq N}} \mathbb{E} \left[\sum_{l=k}^{k+H_k-1} g(x_l, u_l, \bar{\tau}_{\sigma_k}) \mid \mathcal{I}_k \right].$$

The equivalence of the expectations holds by the fact that the process $X := (X_k)_{k \in \mathbb{N}}$ with $X_k := G_k - E_k$ is a martingale with respect to the filtration associated with \mathcal{I}_{k+1} for which Doob's optional sampling theorem (see, e.g., [Grimmett and Stirzaker \(2001, Th. 9, Sec. 12.5\)](#) or [Doob \(1953, Th. 2.2, Ch. VII\)](#)) holds.

The optional sampling theorem holds by the fact that $N(T)$ is a stopping time with finite expectation for given T , i.e., $\mathbb{E}[N(T)] < \infty$ since $\tau_k > 0$ for all $k \in \mathbb{N}$, and that $N(T) \rightarrow \infty$ as $T \rightarrow \infty$, and the fact that there exists some constant $c \in \mathbb{R}$ such that $\mathbb{E}[|X_{k+1} - X_k| \mid \mathcal{I}_{k+1}] \leq c$, which corresponds to a bound on the expected cost per stage, which follows from mean-square stability of x_k . Let $g_k := \sum_{l=k}^{k+H_k-1} g(x_l, u_l, \bar{\tau}_{\sigma_k})$ and observe that $g_k \geq 0$. We have that

$$\begin{aligned} \mathbb{E} \left[\left| X_{k+1} - X_k \right| \mid \mathcal{I}_{k+1} \right] &= \\ &= \mathbb{E} \left[\left| g_{k+1} - \mathbb{E}[g_{k+1} \mid \mathcal{I}_{k+1}] \right| \mid \mathcal{I}_{k+1} \right] \\ &\leq \mathbb{E}[|g_{k+1}|] + \left| \mathbb{E}[g_{k+1} \mid \mathcal{I}_{k+1}] \right| \\ &= 2\mathbb{E}[g_{k+1} \mid \mathcal{I}_{k+1}]. \end{aligned}$$

From (B.1), we have, for all $k \in \mathbb{N}$, that

$$\begin{aligned} \mathbb{E}[g_k \mid \mathcal{I}_k] &\leq 2 \left(H_k \frac{\bar{\tau}_{\sigma_k}}{\bar{\tau}_{b^*}} \tilde{c}_{b^*} - V_N(\bar{x}_k, \Theta_k) \right. \\ &\quad \left. + \mathbb{E}[V_N(\bar{x}_{k+H_k}, \Theta_{k+H_k}) \mid \mathcal{I}_k] + \epsilon_k \right), \end{aligned}$$

and we are only interested in the case $N \rightarrow \infty$. The first term is bounded by the fact that all sampling intervals and horizons are limited. Recall that $\epsilon_k \rightarrow 0$ for any $k \in \mathbb{N}$ for $N \rightarrow \infty$. The functions V_N are bounded for bounded arguments. The fact that $\mathbb{E}[|X_{k+1} - X_k| \mid \mathcal{I}_{k+1}] \leq c$ follows then from boundedness of the arguments of $\mathbb{E}[V_N(\bar{x}_k, \Theta_k) \mid \mathcal{I}_0]$ for all $k \in \mathbb{N}$ as $k \rightarrow \infty$, which is proven below.

For $\Delta_k := \mathbb{E} \left[d_{\sigma_k}^{H_k}(\bar{x}_k, \Theta_k) \mid \mathcal{I}_k \right]$, by adding (B.1) for $k \in \Sigma_{\leq N(T)}$, we can write

$$\begin{aligned} E_{N(T)} &:= \sum_{k \in \Sigma_{\leq N(T)}} H_k \frac{\bar{\tau}_{\sigma_k}}{\bar{\tau}_{b^*}} \tilde{c}_{b^*} - \Delta_k + \epsilon_k + \nu_k \\ &\quad + V_N(\bar{x}_0, \Theta_0) - V_N(\bar{x}_{N(T)}, \Theta_{N(T)}), \end{aligned} \quad (\text{B.2})$$

where

$$\nu_k := V_N(\bar{x}_{k+H_k}, \Theta_{k+H_k}) - \mathbb{E}[V_N(\bar{x}_{k+H_k}, \Theta_{k+H_k}) \mid \mathcal{I}_k].$$

To compute J_π , we take $\limsup_{T \rightarrow \infty} \frac{1}{T} \mathbb{E}[E_{N(T)}]$ where conditioning on \mathcal{I}_0 is implied. Taking the expectation, we have by the tower property that $\mathbb{E}[\nu_k] = 0$ for all $k \in \mathbb{N}$. The first part then equals J_{b^*} , since

$$\limsup_{T \rightarrow \infty} \frac{1}{T} \mathbb{E} \left[\sum_{k=0}^{N(T)-1} \bar{\tau}_{\sigma_k} \right] = 1. \quad (\text{B.3})$$

The equivalence in (B.3) holds by the facts that $(\tilde{H}_k)_{k \in \mathbb{N}}$, with $\tilde{H}_N := \sum_{k=0}^N \tau_k - \bar{\tau}_{\sigma_k}$ is also a martingale with respect to the filtration associated with \mathcal{I}_{k+1} for which Doob's optional sampling theorem ([Doob, 1953](#)) holds and that the discretization error vanishes in the limit. Notice that if the expectations of the last two terms in (B.2) are bounded for any $N \in \mathbb{N}_\infty$ as $T \rightarrow \infty$ and any \mathcal{I}_0 , then, in

the limit, we have

$$\begin{aligned} J_\pi &= \frac{1}{\bar{\tau}_{b^*}} \tilde{c}_{b^*} - \limsup_{T \rightarrow \infty} \frac{1}{T} \mathbb{E} \left[\sum_{k=0}^{N(T)-1} \Delta_k + \epsilon_k \right] \\ &\leq J_{b^*} \leq J_{b_L} \text{ for all } b \in \mathcal{M}, \end{aligned}$$

where we take the limit $N \rightarrow \infty$ such that $\epsilon_k = 0$ for all $k \in \mathbb{N}$ and where $\Delta_k \geq 0$ by (30) and $\Delta_k = 0$ for the choice $(\sigma_k, H_k) = (b^*, 1)$, completing the proof.

We consider the limit case $N \rightarrow \infty$. To show that the last two terms in (B.2) are bounded under expectation with respect to \mathcal{I}_0 , we first notice that $\mathbb{E}[V_N(\bar{x}_0, \Theta_0) \mid \mathcal{I}_0]$ is bounded by boundedness of the arguments \bar{x}_0 and Θ_0 (Recall that $\hat{\Theta}_{k,N}$ converges to $\tilde{\Theta}_{b^*}$ exponentially fast for increasing N). Next, we prove that $\mathbb{E}[V_N(\bar{x}_L, \Theta_L) \mid \mathcal{I}_0]$ remains bounded as $L \rightarrow \infty$. By the positive semi-definite assumption on Q_C , the assumption that the pair $(A_C, Q_C^{\frac{1}{2}})$ is observable, and the assumption that R_C is positive definite, we have that

$$\mathbb{E} \left[\sum_{l=k}^{k+H_k-1} g(x_l, u_l, \bar{\tau}_{\sigma_k}) \mid \mathcal{I}_k \right] \geq \alpha \mathbb{E}[x_k^T x_k \mid \mathcal{I}_k] \quad (\text{B.4})$$

for some sufficiently small $\alpha > 0$. Note that $\mathbb{E}[x_k^T x_k \mid \mathcal{I}_k] = \bar{x}_k^T \bar{x}_k + \text{tr}(\Theta_k)$. Considering the definition of $V_N(\bar{x}, \Theta)$, there exists $\beta, \beta > \alpha > 0$, for which

$$V_N(\bar{x}_k, \Theta_k) \leq \beta \mathbb{E}[x_k^T x_k \mid \mathcal{I}_k]. \quad (\text{B.5})$$

We conclude using (B.1) that for $k \in \mathbb{N}$,

$$\mathbb{E}[V_N(\bar{x}_{k+H}, \Theta_{k+H}) \mid \mathcal{I}_k] \leq c_1 V_N(\bar{x}_k, \Theta_k) + c_2,$$

where $c_1 := 1 - \frac{\alpha}{\beta} < 1$ and $c_2 := \max_{m,H} H \frac{\bar{\tau}_m}{\bar{\tau}_{b^*}} \tilde{c}_{b^*}$, which in turn implies that for $L \in \mathbb{N}$ and $c_3(L) := \sum_{s=0}^{L-1} (c_1)^s c_2$,

$$\mathbb{E}[V_N(\bar{x}_L, \Theta_L) \mid \mathcal{I}_0] \leq (c_1)^L \mathbb{E}[V_N(\bar{x}_0, \Theta_0) \mid \mathcal{I}_0] + c_3(L),$$

leading to the conclusion that $\mathbb{E}[V_N(\bar{x}_L, \Theta_L) \mid \mathcal{I}_0]$ is bounded as $L \rightarrow \infty$, since $(c_1)^L \rightarrow 0$ and c_3 converges to a constant.

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