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Stability Analysis of Spatially Invariant Interconnected Systems with Networked Communication

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Abstract: In this paper tractable stability conditions are presented for a system consisting of an infinite number of spatially invariant subsystems interconnected through communication networks. The networks transmit packets asynchronously and independently of each other and are equipped with scheduling protocols that determine which actuator, sensor or controller node is allowed access to the network. The overall system is modeled as an interconnection of an infinite number of spatially invariant hybrid subsystems. Based on this framework, conditions leading to a maximally allowable transmission interval (MATI) for all of the individual communication networks is derived such that uniform global asymptotic stability (UGAS) of the overall system is guaranteed. These conditions only involve the local dynamics of a single hybrid subsystem in the interconnection. In the case of linear subsystems the conditions can be stated in terms of linear matrix inequalities thereby making them numerically tractable. An illustrative example of a platoon of vehicles is used to demonstrate the newly obtained results and their applications.

Keywords: Networked control systems, spatially invariant systems, linear matrix inequalities

1. INTRODUCTION

Many systems consist of interconnections of similar units or subsystems that only interact with their nearest neighbors. Despite that these units often exhibit simple behavior and interact with their neighbors in a predictable fashion, the resulting overall system often exhibits rich and complex behavior. Analysis and control design for these systems based on global monolithic models encounter severe limitations if many (or even an infinite number of) subsystems are interconnected due to the very high dimensionality and large number of inputs and outputs.

Therefore, a considerable amount of research effort has been targeted on analysis and design methods that aim to guarantee *global* system properties based on *local* conditions on the subsystems and information about the interconnection structure. An interesting line of work in this direction considering interconnections of an *infinite* number of subsystems can be found in Bamieh et al. (2002) and D'Andrea and Dullerud (2003). The focus in these works is on spatially invariant linear systems and local LMI conditions are derived that together with specific interconnection structures lead to UGAS and $\mathcal{L}_2$-stability guarantees of the overall system.

However, one of the main underlying assumptions in Bamieh et al. (2002) and D'Andrea and Dullerud (2003) is that the communication between the subsystems is perfect, where as in many applications this assumption does not hold. In systems with communication networks, network-induced artifacts such as time-varying transmission intervals and delays are present next to scheduling protocols that determine which sensor, controller or actuator nodes are allowed to communicate at a transmission time. Although a vast literature on networked control systems (NCSs) exists, see e.g., Hespanha et al. (2007) or Bemporad et al. (2010), analysis and design tools for interconnections of extremely large or even an infinite number of spatially invariant systems that communicate using packet-based communication networks, which operate independently and asynchronously, hardly exist.

In this paper, we study this particular problem starting from a general setup consisting of identical subsystems with communication networks described by nonlinear differential equations. The communication networks are subjected to time-varying transmission intervals, and scheduling protocols, such as the well-known round-robin and try-once-discard protocols, are present to determine the network access. Inspired by the research line of Walsh et al. (2002), Nešić and Teel (2004), Carnevale et al. (2007), and Heemels et al. (2010), the overall system is modeled as an interconnection of an infinite number of spatially invariant *hybrid* systems. Using this hybrid modeling setup, a maximally allowable transmission interval (MATI) for all of the individual communication networks is determined such that uniform global asymptotic stability (UGAS) of the overall system is guaranteed. We show that when the subsystems can be described by linear dynamical equations the conditions guaranteeing UGAS can be stated in terms of local LMIs making them amenable for computational verification. The conditions are local in the sense that they only involve the local dynamics of *one* subsystem in the interconnection and local conditions on the scheduling protocol to conclude UGAS of the overall *infinite-dimensional* system.

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The paper is organized as follows. First some preliminaries are presented in Section 2. The class of systems considered in this paper are described in Section 3, after which the conditions for UGAS for the overall system are presented in Section 4. In Section 5, we look at the special case of linear systems, and reformulate the conditions of Section 4 for linear systems into a LMI. Finally, in Section 6, a numerical example of a platoon of vehicles is provided, illustrating the application and the effectiveness of our results, and in Section 7 some conclusive remarks are given.

2. PRELIMINARIES

The notation $v \in \mathbb{R}^\bullet$ will denote real valued, finite vectors whose size is either clear from context or not relevant to the discussion. For vectors $v_1, v_2, \ldots, v_n \in \mathbb{R}^\bullet$, we denote by $(v_1, v_2, \ldots, v_n)$ the vector $\left[v_1^\top \; v_2^\top \; \ldots \; v_n^\top\right]^\top$, and by $|\cdot|$ and $\langle\cdot,\cdot\rangle$ the Euclidean norm and the usual inner product, respectively. Moreover, we use the notation $r(t^+) = \lim_{\tau \downarrow t} r(\tau)$. The space of real symmetric n by n matrices is denoted $\mathbb{R}^{n\times n}_{\mathbf{S}}$. A function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to be of class $\mathcal{K}$ if it is continuous, zero at zero, and strictly increasing. It is of class $\mathcal{K}_\infty$ if it is of class $\mathcal{K}$ and in addition it is unbounded. A function $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to be of class $\mathcal{KL}$ if for each $s \geq 0$ the function $\beta(s,\cdot)$ is strictly decreasing to zero in the second argument, and for each fixed $t \geq 0$ the function $\beta(\cdot,t)$ is of class $\mathcal{K}$. Moreover, the function β is said to be of class exp-$\mathcal{KL}$ if for each s there exist $K, c > 0$ such that $\beta(s,t) = K\exp(-ct)$.

In this paper, the state-space of the considered systems is infinite-dimensional, as we will see below. Therefore, we recall some definitions from D'Andrea and Dullerud (2003). Since the signals are often considered at a fixed time, it is convenient to separate the spatial and the temporal parts of a signal.

Definition 1. The space $\ell^{L,n}$ is the set of functions mapping $\mathbb{Z}^L$ to $\mathbb{R}^n$. The space $\ell_2^{L,n}$ is the set of functions in $\ell^{L,n}$ for which

$$\sum_{s_1 \in \mathbb{Z}} \cdots \sum_{s_L \in \mathbb{Z}} x(\mathbf{s})^\top x(\mathbf{s}) < \infty$$

holds with the inner product for $x, y \in \ell_2^{L,n}$ defined as

$$\langle x, y\rangle_{\ell_2} \coloneqq \sum_{s_1 \in \mathbb{Z}} \cdots \sum_{s_L \in \mathbb{Z}} x(\mathbf{s})^\top y(\mathbf{s}),$$

and the corresponding norm as $\|x\|_{\ell_2} \coloneqq \sqrt{\langle x, x\rangle_{\ell_2}}$.

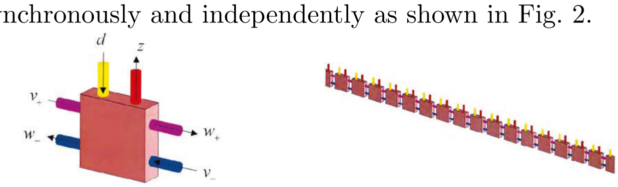
We will consider variables $d : \mathbb{R}_{\geq 0} \times \mathbb{Z}^L \to \mathbb{R}^\bullet$ that are vector-valued functions indexed by $L+1$ independent variables, i.e., $d = d(t, s_1, \ldots, s_L)$, where $t \in \mathbb{R}_{\geq 0}$ is the (continuous) time and $s_1, s_2, \ldots, s_L \in \mathbb{Z}$ are the spatial variables. The L-tuple $(s_1, s_2, \ldots, s_L)$ is denoted by $\mathbf{s}$. For fixed $t \in \mathbb{R}_{\geq 0}$ and $\mathbf{s} \in \mathbb{Z}^L$, a variable $d(t)$ can be considered as an element of $\ell^{L,n}$ or $\ell_2^{L,n}$ and $d(t,\mathbf{s})$ as an element of $\mathbb{R}^n$, i.e., a real-valued vector. For ease of notation, t is often omitted when considering variables, however, from the context it will be clear which space is considered. The spatial shift operators $\mathbf{S}_i$, acting on functions in $\ell_2^{L,n}$, are now for $i = 1, 2, \ldots, L$ defined as

$$(\mathbf{S}_i d)(\mathbf{s}) \coloneqq d(s_1, \ldots, s_{i-1}, s_i + 1, s_{i+1}, \ldots, s_L).$$

In the case that $L = 1$, which we consider in this paper, we denote $\mathbf{S}_1$ also as $\mathbf{S}$.

3. SPATIALLY INVARIANT INTERCONNECTED SYSTEMS WITH NETWORKED COMMUNICATION

In this section, the considered class of systems is introduced. The overall system consists of an infinite number of subsystems ("basic building blocks") that are all identical, see Fig. 1(a). These subsystems are interconnected according to a particular structure as indicated in Fig. 1(b), which was also considered in D'Andrea and Dullerud (2003). However, in D'Andrea and Dullerud (2003) it was assumed that the communication is perfect and infinitely fast, where as in this paper we consider the case where the communication between the subsystems occurs via packet-based (wireless) communication networks, see, e.g., Walsh et al. (2002) and Nešić and Teel (2004), that operate asynchronously and independently as shown in Fig. 2.



(a) Basic building block, one spatial dimension.

(b) Infinite interconnection.

Fig. 1. Spatially invariant interconnected systems from D'Andrea and Dullerud (2003).

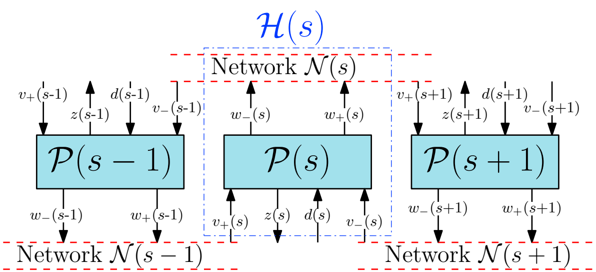


Fig. 2. Infinite networked interconnection, where each subsystem $\mathcal{P}(s)$ has its own communication network $\mathcal{N}(s)$ to communicate with its neighbors. The overall "networked" subsystem $\mathcal{H}(s)$ is the combination of subsystem $\mathcal{P}(s)$ and its network $\mathcal{N}(s)$.

To introduce the overall modeling setup in detail, we start by providing the dynamical model describing a subsystem $\mathcal{P}(s)$ indexed by $s \in \mathbb{Z}$. This is given by

$$\mathcal{P}(s) : \begin{bmatrix} \dot{x}(s) \\ w(s) \\ z(s) \end{bmatrix} = \begin{bmatrix} f_p(x(s), v(s), d(s)) \\ g_p(x(s)) \\ q_p(x(s), v(s), d(s)) \end{bmatrix}, \tag{1}$$

with the initial condition $x(0,s) = x_0(s) \in \mathbb{R}^{m_0}$, where $x_0 \in \ell_2^{m_0}$, and

$$v(s) = \begin{bmatrix} v_+(s) \\ v_-(s) \end{bmatrix} \quad \text{and} \quad w(s) = \begin{bmatrix} w_+(s) \\ w_-(s) \end{bmatrix}, \tag{2}$$

where $x(s) \in \mathbb{R}^{m_0}$ denotes the state, $v_+(s) \in \mathbb{R}^{m_+}$ and $v_-(s) \in \mathbb{R}^{m_-}$ the interconnected inputs, $w_+(s) \in \mathbb{R}^{m_+}$ and $w_-(s) \in \mathbb{R}^{m_-}$ the interconnected outputs, $d(s) \in \mathbb{R}^{m_d}$ the external (disturbance) input, and $z(s) \in \mathbb{R}^{m_z}$ the performance output of the subsystem $\mathcal{P}(s)$. Hence, $f_p : \mathbb{R}^{m_0} \times \mathbb{R}^{m_+ + m_-} \times \mathbb{R}^{m_d} \to \mathbb{R}^{m_0}$, $g_p : \mathbb{R}^{m_0} \to \mathbb{R}^{m_+ + m_-}$, and $q_p : \mathbb{R}^{m_0} \times \mathbb{R}^{m_+ + m_-} \times \mathbb{R}^{m_d} \to \mathbb{R}^{m_z}$ are nonlinear mappings, where it is assumed that g_p is continuously differentiable. Note that $x \in \ell_2^{1,m_0}$ is used to denote the state of the overall system, that $v_+(s)$ and $w_+(s)$ have the same size, and that $v_-(s)$ and $w_-(s)$ have the same size.

Based on identical copies of the building block (1)-(2), the infinite interconnected system of Fig. 1(b) is composed by defining

$$v_+(s) = \hat{w}_+(s-1) \quad \text{and} \quad v_-(s) = \hat{w}_-(s+1), \tag{3}$$

where, contrary to D'Andrea and Dullerud (2003), $\hat{w}_+(s)$ and $\hat{w}_-(s)$ are typically *not* equal to $w_+(s)$ and $w_-(s)$, but are their networked values, i.e., the latest broadcast values of $w_+(s)$ and $w_-(s)$, as will be explained below.

Define $m := (m_+, m_-)$, and define in a similar fashion as in D'Andrea and Dullerud (2003) the structured operator $\Delta_{\mathbf{S},m} : \ell_2^{1,m_++m_-} \to \ell_2^{1,m_++m_-}$ as

$$(\Delta_{\mathbf{S},m} r)(s) = \begin{bmatrix} [I_{m_+}\ 0] \left(\mathbf{S}^{-1} r\right)(s) \\ [0\ I_{m_-}] \left(\mathbf{S} r\right)(s) \end{bmatrix} = \begin{bmatrix} r_+(s-1) \\ r_-(s+1) \end{bmatrix} \tag{4}$$

for functions $r = (r_+, r_-) \in \ell_2^{1,m_++m_-}$. By using this operator, the interconnection (3) can be compactly expressed for every $s \in \mathbb{Z}$ as

$$v(s) = (\Delta_{\mathbf{S},m} \hat{w})(s). \tag{5}$$

Consider now a subsystem $\mathcal{P}(s)$ with its own local network $\mathcal{N}(s)$, $s \in \mathbb{Z}$, to communicate with its neighbors and its own collection of transmission/sampling times $t^s_{j^s}$, $j^s \in \mathbb{N}$, which satisfy $0 \le t^s_0 < t^s_1 < \ldots$. Note that all subsystems communicate asynchronously and independently, as each of them has its own sequence of transmission times. For the subsystem $\mathcal{P}(s)$, (parts of) the output $w(s)$ are sampled and transmitted over the network, at such a transmission time $t^s_{j^s}$ for some $j^s \in \mathbb{N}$. However, the local communication network $\mathcal{N}(s)$ is typically subdivided into several sensor, actuator and/or controller nodes, where each node corresponds to a subset of the entries in $w(s)$ (and thus $\hat{w}(s)$). A scheduling protocol, which is assumed to be taken the same for each network $\mathcal{N}(s)$, determines which of the nodes is granted access to the network at a transmission time. At transmission time $t^s_{j^s}$ for subsystem $s \in \mathbb{Z}$, the networked values are updated according to

$$\hat{w}\left(\left(t^s_{j^s}\right)^+, s\right) = w\left(t^s_{j^s}, s\right) + h\left(j^s, e\left(t^s_{j^s}, s\right)\right), \tag{6}$$

where $e(s)$ denotes the local network-induced error defined by

$$e(s) = \hat{w}(s) - w(s) = \begin{bmatrix} \hat{w}_+(s) - w_+(s) \\ \hat{w}_-(s) - w_-(s) \end{bmatrix} = \begin{bmatrix} e_+(s) \\ e_-(s) \end{bmatrix}. \tag{7}$$

Note that we followed here the modeling of scheduling protocols as in Nešić and Teel (2004) based on the protocol function $h : \mathbb{N} \times \mathbb{R}^{m_++m_-} \to \mathbb{R}^{m_++m_-}$. Indeed, in (6) it is determined on the basis of the local transmission counter j^s and the local network error $e\left(t^s_{j^s}, s\right)$ which node is allowed to communicate and typically the corresponding entries in h are zero, see Nešić and Teel (2004) for a detailed description. We assume the signals $\hat{w}$ to be left-continuous in the sense that for all $t \in \mathbb{R}_{\ge 0}$ and all $s \in \mathbb{Z}$, $\hat{w}(t,s) = \lim_{\tau \uparrow t} \hat{w}(\tau, s)$.

The transmission times satisfy $\delta \le t^s_{j^s+1} - t^s_{j^s} \le \tau_{mati}$ for all $j^s \in \mathbb{N}$, $s \in \mathbb{Z}$, where τ_{mati} denotes the maximally allowable transmission interval (MATI) for all the networks. It should be noted that $\delta > 0$ can be taken arbitrarily small since it is only imposed to prevent Zeno behavior (at least locally). Due to hardware limitations in reality such a lower bound $\delta > 0$ on the transmission intervals always exists. Moreover, it is assumed that each network operates in a zero-order-hold (ZOH) fashion, in the sense that $\hat{w}$ does not change between transmissions, i.e., $\dot{\hat{w}} = 0$.

Based on the above setup, each pair $(\mathcal{P}(s), \mathcal{N}(s))$ can be rewritten into the format of a hybrid system as described in Nešić and Teel (2004). To do so, the interconnection variables w need to be eliminated in the state and the error dynamics. From (1), (3) and (7) it follows that $v(s) = (\Delta_{\mathbf{S},m} e)(s) + (\Delta_{\mathbf{S},m} w)(s)$ with $w(s) = g_p(x(s), d(s))$, implying that by introducing the timers $\tau(s) \in \mathbb{R}_{\ge 0}$ and the counters $\kappa(s) \in \mathbb{N}$ for every fixed $s \in \mathbb{Z}$, each "networked" subsystem takes the form

$$\mathcal{H}(s): \begin{cases} \left.\begin{aligned} \dot{x}(s) &= f(x,e,d)(s) \\ \dot{e}(s) &= g(x,e,d)(s) \\ \dot{\tau}(s) &= 1 \\ \dot{\kappa}(s) &= 0 \end{aligned}\right\} & \begin{aligned} &\text{when} \\ &\tau(s) \in [0, \tau_{mati}] \end{aligned} \\ \left.\begin{aligned} x^+(s) &= x(s) \\ e^+(s) &= h(\kappa(s), e(s)) \\ \tau^+(s) &= 0 \\ \kappa^+(s) &= \kappa(s) + 1 \end{aligned}\right\} & \begin{aligned} &\text{when} \\ &\tau(s) \in [\delta, \tau_{mati}] \end{aligned} \end{cases} \tag{8}$$

with the new state of the subsystem indexed by $s \in \mathbb{Z}$ given by $\xi(s) = (x(s), e(s), \tau(s), \kappa(s))$ and the output equation

$$z(s) = q(x,e,d)(s),$$

where $f(x,e,d)(s) = f_p(x(s), (\Delta_{\mathbf{S},m} e)(s) + (\Delta_{\mathbf{S},m} w)(s), d(s))$, $g(x,e,d)(s) = -\frac{\partial g_p(x(s))}{\partial x(s)} f_p(x(s), (\Delta_{\mathbf{S},m} e)(s) + (\Delta_{\mathbf{S},m} w)(s), d(s))$, and $q(x,e,d)(s) = q_p(x(s), (\Delta_{\mathbf{S},m} e)(s) + (\Delta_{\mathbf{S},m} w)(s), d(s))$ with $w(s) = g_p(x(s))$. We assume that f_p, g_p, and q_p are such that $f : \ell_2^{1,m_0} \times \ell_2^{1,m_++m_-} \times \ell_2^{1,m_d} \to \ell_2^{1,m_0}$, $g : \ell_2^{1,m_0} \times \ell_2^{1,m_++m_-} \times \ell_2^{1,m_d} \to \ell_2^{1,m_++m_-}$, and $q : \ell_2^{1,m_0} \times \ell_2^{1,m_++m_-} \times \ell_2^{1,m_d} \to \ell_2^{1,m_z}$. The overall interconnection as depicted in Fig. 1(b) is now described by the hybrid system $\mathcal{H}$, being the infinite interconnection of subsystems $\mathcal{H}(s)$, $s \in \mathbb{Z}$. Using this hybrid framework, conditions for UGAS of the overall system will be derived in the next section.

Remark 1. Note that there are no direct feed-through terms in the networked interconnection of (1), meaning that $w(s)$ does not depend on $v(s)$. This condition is used to prevent that the jump of the subsystem s directly triggers a jump in the networked-induced errors $e(s-1)$ and/or $e(s+1)$ and thus possibly jumps of the subsystems $s-1$ and/or $s+1$, respectively. By adopting this condition, a chain reaction in the jumps of the overall system is prevented, and, hence, we are able to consider solutions for the overall system where the time $t \to \infty$. A similar condition was also adopted in the finite-dimensional case of Nešić and Teel (2004) and Carnevale et al. (2007), and Heemels et al. (2010). Moreover, in (1), it is assumed that the (disturbance) input $d(s)$ does not directly influence $w(s)$ to reduce the complexity of the networked setup.

4. STABILITY ANALYSIS

Here, we analyze UGAS of the overall system, in the case disturbances are absent, i.e., $d = 0$. Note that it is also possible to perform an $\mathcal{L}_2$-induced gain analysis for the case in which $d \ne 0$ using related techniques, see Heijmans (2015).

Definition 2. For the overall system $\mathcal{H}$, composed of the identical subsystems $\mathcal{H}(s)$, $s \in \mathbb{Z}$, as in (8), the set

$$\mathcal{E} = \{\xi \in \ell_2^{1,m_0} \times \ell_2^{1,m_+ + m_-} \times \ell^{1,1} \times \ell^{1,1} \mid x = 0 \wedge e = 0\} \quad (9)$$

is said to be uniformly globally asymptotically stable (UGAS) if there exists a function $\beta \in \mathcal{KL}$ such that for any initial condition $\xi(0) \in \ell_2^{1,m_0} \times \ell_2^{1,m_+ + m_-} \times \ell^{1,1} \times \ell^{1,1}$, all corresponding solutions $\xi(t)$ of $\mathcal{H}$ with $d = 0$ satisfy

$$\|(x(t), e(t))\|_{\ell_2} \le \beta\left(\|(x(0), e(0))\|_{\ell_2}, t\right)$$

for all $t \in \mathbb{R}_{\ge 0}$. Moreover, if $\beta \in \exp\text{-}\mathcal{KL}$, the set $\mathcal{E}$ (9) is said to be uniformly globally exponentially stable (UGES).

In order to guarantee UGAS, first the following conditions on the local scheduling protocol is assumed to hold.

Assumption 1. For the local scheduling protocol h there exist the functions $\underline{\alpha}_W, \overline{\alpha}_W \in \mathcal{K}_\infty$ and $W : \mathbb{N} \times \mathbb{R}^{m_+ + m_-} \to \mathbb{R}_{\ge 0}$ that is locally Lipschitz in its second argument, and the constant $\lambda \in (0,1)$ such that for all $\kappa(s) \in \mathbb{N}$, and $e(s) \in \mathbb{R}^{m_+ + m_-}$, $s \in \mathbb{Z}$, it holds that

$$\underline{\alpha}_W(|e(s)|) \le W(\kappa(s), e(s)) \le \overline{\alpha}_W(|e(s)|) \quad (10a)$$

$$W(\kappa(s)+1, h(\kappa(s), e(s))) \le \lambda W(\kappa(s), e(s)). \quad (10b)$$

Now, based on extensions of the stability results for finite-dimensional networked control systems, see, e.g., Nešić and Teel (2004) or Carnevale et al. (2007), novel conditions under which the set $\mathcal{E}$ of (9) is UGAS can be obtained when $d = 0$. For notational convenience, we write $f(x, e, 0)$ as $f(x, e)$ and $g(x, e, 0)$ as $g(x, e)$.

Theorem 1. Consider the overall system $\mathcal{H}$ composed of the identical subsystems $\mathcal{H}(s)$, $s \in \mathbb{Z}$. Assume that Assumption 1 holds and that there exist a locally Lipschitz function $V : \ell_2^{1,m_0} \to \mathbb{R}_{\ge 0}$, a continuous function $H : \mathbb{R}^{m_0} \times \mathbb{R}^{m_+ + m_-} \to \mathbb{R}$, real numbers $\gamma > 0$, $L \ge 0$, functions $\underline{\alpha}_V, \overline{\alpha}_V \in \mathcal{K}_\infty$, and a continuous, positive definite function ϱ such that for all $\kappa(s) \in \mathbb{N}$, $x(s) \in \mathbb{R}^{m_0}$ $v(s) \in \mathbb{R}^{m_+ + m_-}$, and almost all $e(s) \in \mathbb{R}^{m_+ + m_-}$ it holds that

$$\left\langle \frac{\partial W(\kappa(s), e(s))}{\partial e(s)}, g(x, e)(s) \right\rangle \le L W(\kappa(s), e(s)) + H(x(s), v(s)), \quad (11)$$

with the function W from Assumption 1 and for all $e \in \ell_2^{1,m_+ + m_-}$, all $\kappa \in \ell^{1,1}$, and almost all $x \in \ell_2^{1,m_0}$

$$\underline{\alpha}_V\left(\|x\|_{\ell_2}\right) \le V(x) \le \overline{\alpha}_V\left(\|x\|_{\ell_2}\right)$$

$$\langle \nabla V(x), f(x, e)\rangle_{\ell_2} \le \sum_{s \in \mathbb{Z}} \left\{ -\varrho(W(\kappa(s), e(s))) - H^2(x(s), v(s)) + \gamma^2 W^2(\kappa(s), e(s)) \right\} - \varrho(\|x\|_{\ell_2}). \quad (12)$$

If now τ_{mati} satisfies

$$\tau_{mati} \le \begin{cases} \dfrac{1}{Lr} \arctan\left(\dfrac{r(1-\lambda)}{2\frac{\lambda}{1+\lambda}\left(\frac{\gamma}{L}\right) + 1 + \lambda} \right), & \gamma > L \\ \dfrac{1-\lambda}{L(1+\gamma)}, & \gamma = L \\ \dfrac{1}{Lr} \operatorname{arctanh}\left(\dfrac{r(1-\lambda)}{2\frac{\lambda}{1+\lambda}\left(\frac{\gamma}{L}\right) + 1 + \lambda} \right), & \gamma < L \\ \dfrac{1}{\gamma} \arctan\left(\dfrac{(1+\lambda)(1-\lambda)}{2\lambda} \right), & L = 0 \end{cases}$$

for $r = \sqrt{\left|(\gamma/L)^2 - 1\right|}$ then the set $\mathcal{E}$ of (9) is UGAS.

If, in addition, there exist strictly positive real numbers $\underline{\alpha}_W^c$, $\overline{\alpha}_W^c$, $\underline{\alpha}_V^c$, $\overline{\alpha}_V^c$, and ε such that for all $e(s) \in \mathbb{R}^{m_+ + m_-}$ and $\kappa(s) \in \mathbb{N}$, $\underline{\alpha}_W^c |e(s)| \le W(\kappa(s), e(s)) \le \overline{\alpha}_W^c |e(s)|$, and for all $x \in \ell_2^{1,m_0}$, $\underline{\alpha}_V^c \|x\|_{\ell_2}^2 \le V(x) \le \overline{\alpha}_V^c \|x\|_{\ell_2}^2$ and $\varrho(\cdot) \ge \varepsilon^2(\cdot)^2$, then the set $\mathcal{E}$ is UGES. The proof is given in Heijmans (2015).

Remark 2. For UGAS for the network-free, ideal communication case, i.e., where $e(t, s) = 0$ for all $t \in \mathbb{R}_{\ge 0}$ and all $s \in \mathbb{Z}$, only condition (12) is relevant, which reduces to

$$\langle \nabla V(x), f(x, 0, 0)\rangle_{\ell_2} \le -\varrho(\|x\|_{\ell_2}).$$

Remark 3. Various scheduling protocols exist which satisfy the conditions (10) including try-once-discard (TOD), round-robin (RR), and sampled-data (SD) as shown in Nešić and Teel (2004). Indeed, we can have $\underline{\alpha}_W^c |e(s)| \le W(\kappa(s), e(s)) \le \overline{\alpha}_W^c |e(s)|$, $\underline{\alpha}_W^c, \overline{\alpha}_W^c \in \mathbb{R}_{\ge 0}$ for $\underline{\alpha}_{W,TOD}^c = \underline{\alpha}_{W,SD}^c = \underline{\alpha}_{W,RR}^c = 1$, $\overline{\alpha}_{W,TOD}^c = \overline{\alpha}_{W,SD}^c = 1$, and $\overline{\alpha}_{W,RR}^c = \sqrt{l}$, and also (10b) is satisfied for $\lambda_{TOD} = \lambda_{RR} = \sqrt{(l-1)/l}$ or λ_{SD} arbitrarily small with l the number of nodes in each network.

It should be noted that $V : \ell_2^{1,m_0} \to \mathbb{R}_{\ge 0}$ is a *global* Lyapunov function, making it, due to the infinite-dimensional character of the system, very hard and often intractable to construct such a function. Therefore, it is important to exploit the interconnection structure such that local conditions result. In the following sections, this will be shown for the case of linear systems.

5. THE LINEAR CASE

In this section, again the model described by (1) is considered, however now for the case that the plant $\mathcal{P}(s)$ is a linear time-invariant system, as studied in D'Andrea and Dullerud (2003) for the case without communication networks.

5.1 System description

By using the spatial shift operator of (4) and the interconnection condition (5), the interconnected system can be expressed in the linear case for every fixed $s \in \mathbb{Z}$ as

$$\mathcal{P}(s) : \begin{bmatrix} \dot{x}(s) \\ w(s) \\ z(s) \end{bmatrix} = \begin{bmatrix} A_{\mathbf{TT}} & A_{\mathbf{TS}} & B_{\mathbf{T}} \\ A_{\mathbf{ST}} & A_{\mathbf{SS}} & B_{\mathbf{S}} \\ C_{\mathbf{T}} & C_{\mathbf{S}} & D \end{bmatrix} \begin{bmatrix} x(s) \\ (\Delta_{\mathbf{S},m}\hat{w})(s) \\ d(s) \end{bmatrix}. \quad (13)$$

Based on the operators $A_{\mathbf{TT}}$, $A_{\mathbf{TS}}$, etc. of (13), which map $\mathbb{R}^\bullet$ to $\mathbb{R}^\bullet$, we also define the operators $\tilde{A}_{\mathbf{TT}}$, $\tilde{A}_{\mathbf{TS}}$, etc., which map $\ell_2^{1,\bullet}$ to $\ell_2^{1,\bullet}$, as $(\tilde{A}_{\mathbf{TT}}x)(s) = A_{\mathbf{TT}}x(s)$, $(\tilde{A}_{\mathbf{TS}}v)(s) = A_{\mathbf{TS}}v(s)$, etc., for $s \in \mathbb{Z}$.

Remark 4. To preserve generality and the connection to D'Andrea and Dullerud (2003), contrary to (1), in (13) a direct feed-through term $A_{\mathbf{SS}}$ and a term $B_{\mathbf{S}}$ are present. However, as a result of Remark 1, $A_{\mathbf{SS}} = 0$ as well as $B_{\mathbf{S}} = 0$ in all the considered equations.

Based on (3), (4), (7), and (13), it follows that for the interconnection variables we have that

$$w(s) = \left(\tilde{I}_{m_+ + m_-} - \tilde{A}_{\mathbf{SS}}\Delta_{\mathbf{S},m}\right)^{-1} \left(\tilde{A}_{\mathbf{ST}}x + \tilde{B}_{\mathbf{S}}d + \tilde{A}_{\mathbf{SS}}\Delta_{\mathbf{S},m}e\right)(s). \quad (14)$$

Thus, by eliminating the interconnection variables in the state and error dynamics, the hybrid model $\mathcal{H}(s)$ in the format of (8) is given by

$$\begin{bmatrix} A_{\mathbf{TT}}^\top X_{\mathbf{T}} + X_{\mathbf{T}} A_{\mathbf{TT}} + \varepsilon^2 I_{m_0} + M^2 A_{\mathbf{TT}}^\top \widehat{A}_{\mathbf{ST}} A_{\mathbf{TT}} + A_{\mathbf{ST}}^\Delta & A_{\mathbf{ST},\mathbf{SS},e}^\Delta & X_{\mathbf{T}} A_{\mathbf{TS}} + M^2 A_{\mathbf{TT}}^\top \widehat{A}_{\mathbf{ST}} A_{\mathbf{TS}} + A_{\mathbf{ST},\mathbf{SS},v}^\Delta \\ (A_{\mathbf{ST},\mathbf{SS},e}^\Delta)^\top & \underline{\alpha}_W^c{}^2[\varepsilon^2 - \gamma^2] I_{m_+ + m_-} + A_{\mathbf{SS},e}^\Delta & A_{\mathbf{SS},ev}^\Delta \\ (A_{\mathbf{TS}})^\top X_{\mathbf{T}} + M^2 A_{\mathbf{TS}}^\top \widehat{A}_{\mathbf{ST}} A_{\mathbf{TT}} + (A_{\mathbf{ST},\mathbf{SS},v}^\Delta)^\top & (A_{\mathbf{SS},ev}^\Delta)^\top & M^2 A_{\mathbf{TS}}^\top \widehat{A}_{\mathbf{ST}} A_{\mathbf{TS}} + A_{\mathbf{SS},v}^\Delta \end{bmatrix} \preceq 0. \quad (18)$$

$$\mathcal{H}(s): \begin{cases} \left.\begin{aligned} \dot{x}(s) &= (\mathbf{A}x)(s) + (\mathbf{B}d)(s) \\ &\quad + (\mathbf{E}e)(s) \\ \dot{e}(s) &= -A_{\mathbf{ST}}\dot{x}(s) - B_{\mathbf{S}}\dot{d}(s) \\ \dot{\tau}(s) &= 1 \\ \dot{\kappa}(s) &= 0 \end{aligned}\right\} & \text{when } \tau(s) \in [0, \tau_{mati}] \\ \left.\begin{aligned} x^+(s) &= x(s) \\ e^+(s) &= h(\kappa(s), e(s)) \\ \tau^+(s) &= 0 \\ \kappa^+(s) &= \kappa(s) + 1 \end{aligned}\right\} & \text{when } \tau(s) \in [\delta, \tau_{mati}] \end{cases} \quad (15)$$

with the output equation

$$z(s) = (\mathbf{C}x)(s) + (\mathbf{D}d)(s) + (\mathbf{H}e)(s)$$

where the ℓ_2-operators are defined as

$$\begin{aligned} \mathbf{A} &= \tilde{A}_{\mathbf{TT}} + \tilde{A}_{\mathbf{TS}} \left(\Delta_{\mathbf{S},m}^{-1} - \tilde{A}_{\mathbf{SS}}\right)^{-1} \tilde{A}_{\mathbf{ST}}, \\ \mathbf{B} &= \tilde{B}_{\mathbf{T}} + \tilde{A}_{\mathbf{TS}} \left(\Delta_{\mathbf{S},m}^{-1} - \tilde{A}_{\mathbf{SS}}\right)^{-1} \tilde{B}_{\mathbf{S}}, \\ \mathbf{C} &= \tilde{C}_{\mathbf{T}} + \tilde{C}_{\mathbf{S}} \left(\Delta_{\mathbf{S},m}^{-1} - \tilde{A}_{\mathbf{SS}}\right)^{-1} \tilde{A}_{\mathbf{ST}}, \\ \mathbf{D} &= \tilde{D} + \tilde{C}_{\mathbf{S}} \left(\Delta_{\mathbf{S},m}^{-1} - \tilde{A}_{\mathbf{SS}}\right)^{-1} \tilde{B}_{\mathbf{S}} \\ \mathbf{E} &= \tilde{A}_{\mathbf{TS}} \left(\Delta_{\mathbf{S},m}^{-1} - \tilde{A}_{\mathbf{SS}}\right)^{-1} \\ \mathbf{H} &= \tilde{C}_{\mathbf{S}} \left(\Delta_{\mathbf{S},m}^{-1} - \tilde{A}_{\mathbf{SS}}\right)^{-1}. \end{aligned} \quad (16)$$

Remark 5. When perfect communication is assumed, i.e., $\hat{w} = w$, we have that $e = 0$, in which case (15) recovers the setup described in D'Andrea and Dullerud (2003).

Remark 6. In (16) and (14), the inverses of $\left(\Delta_{\mathbf{S},m}^{-1} - \tilde{A}_{\mathbf{SS}}\right)$ and $\left(\tilde{I}_{m_+ + m_-} - \tilde{A}_{\mathbf{SS}}\Delta_{\mathbf{S},m}\right)$, respectively, exist as a result of $A_{\mathbf{SS}} = 0$ from Remark 4.

5.2 LMI condition for UGES

To systematically verify the conditions of Theorem 1, we will now provide local LMI-based reformulations. These reformulations are based on extensions of the ideal communication case treated in D'Andrea and Dullerud (2003).

Hereto, we take Theorem 1 as a starting point and consider the conditions (10), (11), and (12), where we assume that (10a) is satisfied in the sense that there exist strictly positive constants $\underline{\alpha}_W^c$ and $\overline{\alpha}_W^c$ such that $\underline{\alpha}_W^c |e(s)| \leq W(\kappa(s), e(s)) \leq \overline{\alpha}_W^c |e(s)|$, $s \in \mathbb{Z}$, see also Remark 3.

Let us now consider the condition (11). We assume that for almost all $e(s) \in \mathbb{R}^{m_+ + m_-}$ and all $\kappa(s) \in \mathbb{N}$, $s \in \mathbb{Z}$, it holds that

$$\left| \frac{\partial W(\kappa(s), e(s))}{\partial e(s)} \right| \leq M \quad (17)$$

for some constant $M > 0$. For the considered protocols in Remark 3, (17) is satisfied for $M_{RR} = \sqrt{l}$ and $M_{TOD} = M_{SD} = 1$. When now using (13) and (15), it follows that $|\dot{e}(s)| = \left|A_{\mathbf{ST}}\left(A_{\mathbf{TT}}x(s) + B_{\mathbf{T}}d(s) + A_{\mathbf{TS}}v(s)\right) + B_{\mathbf{S}}\dot{d}(s)\right|$, from which we directly obtain that, in combination with (17) and for the case $d = 0$, in (11) $L = 0$ and

$$H(x(s), v(s)) = M \left| A_{\mathbf{ST}} A_{\mathbf{TT}} x(s) + A_{\mathbf{ST}} A_{\mathbf{TS}} v(s) \right|.$$

Based on these observations we can now compose LMI conditions which guarantee UGES of the set $\mathcal{E}$ of (9) in the case $d = 0$. To do so, (13) needs to be further partitioned to reflect the structure of $\Delta_{\mathbf{S},m}$ of (4), i.e.,

$$A_{\mathbf{SS}} = \begin{bmatrix} A_{\mathbf{SS}}^{++} & A_{\mathbf{SS}}^{+-} \\ A_{\mathbf{SS}}^{-+} & A_{\mathbf{SS}}^{--} \end{bmatrix} = \mathbf{0},\ A_{\mathbf{ST}} = \begin{bmatrix} A_{\mathbf{ST}}^{++} \\ A_{\mathbf{ST}}^{--} \end{bmatrix},\ A_{\mathbf{TS}} = \begin{bmatrix} A_{\mathbf{TS}}^{++} & A_{\mathbf{TS}}^{--} \end{bmatrix}.$$

Based on this partitioning, we introduce

$$\begin{aligned} &A_{\mathbf{SS},v}^{+} = \begin{bmatrix} A_{\mathbf{SS}}^{++} & A_{\mathbf{SS}}^{+-} \\ 0 & I_{m_-} \end{bmatrix} \quad A_{\mathbf{SS},e}^{+} = \begin{bmatrix} I_{m_+} & 0 \\ 0 & 0 \end{bmatrix} \quad A_{\mathbf{ST}}^{+} = \begin{bmatrix} A_{\mathbf{ST}}^{++} \\ 0 \end{bmatrix} \\ &A_{\mathbf{SS},v}^{-} = \begin{bmatrix} I_{m_+} & 0 \\ A_{\mathbf{SS}}^{-+} & A_{\mathbf{SS}}^{--} \end{bmatrix} \quad A_{\mathbf{SS},e}^{-} = \begin{bmatrix} 0 & 0 \\ 0 & I_{m_-} \end{bmatrix} \quad A_{\mathbf{ST}}^{-} = \begin{bmatrix} 0 \\ A_{\mathbf{ST}}^{--} \end{bmatrix}. \\ &A_{\mathbf{TS}}^{+} = \begin{bmatrix} A_{\mathbf{TS}}^{++} & 0 \end{bmatrix} \qquad A_{\mathbf{TS}}^{-} = \begin{bmatrix} 0 & A_{\mathbf{TS}}^{--} \end{bmatrix}. \end{aligned}$$

Consider now the space $\mathcal{X}_{\mathbf{T}} := \left\{X_{\mathbf{T}} \in \mathbb{R}_{\mathbf{S}}^{m_0 \times m_0} \mid X_{\mathbf{T}} \succ 0\right\}$, and based on Theorem 1 from D'Andrea and Dullerud (2003), the following theorem is composed.

Theorem 2. Consider the overall system $\mathcal{H}$, composed of the identical subsystems $\mathcal{H}(s)$ of (15), $s \in \mathbb{Z}$, with $d = 0$, $A_{\mathbf{SS}} = 0$, and $B_{\mathbf{S}} = 0$. Assume there exist a function $W : \mathbb{N} \times \mathbb{R}^{m_+ + m_-} \to \mathbb{R}_{\geq 0}$ that is locally Lipschitz in its second argument, a matrix $X_{\mathbf{T}} \in \mathcal{X}_{\mathbf{T}}$, a matrix $X_{\mathbf{S}} \in \mathbb{R}_{\mathbf{S}}^{(m_+ + m_-) \times (m_+ + m_-)}$, and the constants $\underline{\alpha}_W^c, \overline{\alpha}_W^c \in \mathbb{R}_{\geq 0}$, $M > 0$, $0 < \varepsilon < \gamma$, and $\lambda \in (0, 1)$ such that (10) holds with $\underline{\alpha}_W(r) = \underline{\alpha}_W^c r$ and $\overline{\alpha}_W(r) = \overline{\alpha}_W^c r$, (17) holds, and the LMI (18) holds with $\widehat{A}_{\mathbf{ST}} := A_{\mathbf{ST}}^\top A_{\mathbf{ST}}$ and

$$\begin{aligned} A_{\mathbf{ST}}^\Delta &:= (A_{\mathbf{ST}}^+)^\top X_{\mathbf{S}} A_{\mathbf{ST}}^+ - (A_{\mathbf{ST}}^-)^\top X_{\mathbf{S}} A_{\mathbf{ST}}^- \\ A_{\mathbf{SS},ev}^\Delta &:= \left(A_{\mathbf{SS},e}^+\right)^\top X_{\mathbf{S}} A_{\mathbf{SS},v}^+ - \left(A_{\mathbf{SS},e}^-\right)^\top X_{\mathbf{S}} A_{\mathbf{SS},v}^- \\ A_{\mathbf{SS},v}^\Delta &:= (A_{\mathbf{SS},v}^+)^\top X_{\mathbf{S}} A_{\mathbf{SS},v}^+ - (A_{\mathbf{SS},v}^-)^\top X_{\mathbf{S}} A_{\mathbf{SS},v}^- \\ A_{\mathbf{SS},e}^\Delta &:= (A_{\mathbf{SS},e}^+)^\top X_{\mathbf{S}} A_{\mathbf{SS},e}^+ - (A_{\mathbf{SS},e}^-)^\top X_{\mathbf{S}} A_{\mathbf{SS},e}^- \\ A_{\mathbf{ST},\mathbf{SS},v}^\Delta &:= (A_{\mathbf{ST}}^+)^\top X_{\mathbf{S}} A_{\mathbf{SS},v}^+ - (A_{\mathbf{ST}}^-)^\top X_{\mathbf{S}} A_{\mathbf{SS},v}^- \\ A_{\mathbf{ST},\mathbf{SS},e}^\Delta &:= (A_{\mathbf{ST}}^+)^\top X_{\mathbf{S}} A_{\mathbf{SS},e}^+ - (A_{\mathbf{ST}}^-)^\top X_{\mathbf{S}} A_{\mathbf{SS},e}^-. \end{aligned}$$

If now τ_{mati} satisfies $\tau_{mati} \leq \frac{1}{\gamma} \arctan\left(\frac{(1+\lambda)(1-\lambda)}{2\lambda}\right)$, then the set $\mathcal{E}$ of (9) is UGES.

The proof can be found in Heijmans (2015). Note that (18) is a *local* LMI that can easily be verified to conclude stability of the *infinite-dimensional* system $\mathcal{H}$. Since γ is the only free variable for the computation of the bound $\frac{1}{\gamma} \arctan\left(\frac{(1+\lambda)(1-\lambda)}{2\lambda}\right)$ for τ_{mati} as λ follows from the local scheduling protocol, τ_{mati} can be maximized by means of minimizing γ subject to the LMI of (18).

6. NUMERICAL EXAMPLE

In this section we show the effectiveness of Theorem 2 by applying it to a platoon of vehicles with wireless communication between the vehicles. For a detailed description we refer to Heijmans (2015) or Firooznia (2012). Consider an infinite string of spatially invariant vehicles each with a length $L_v \in \mathbb{R}_{\geq 0}$, as shown in Fig. 3. The absolute position of the rear bumper of the vehicle at position s is denoted by $r(s) \in \mathbb{R}$, from which it follows that the distance $d_v(s) \in \mathbb{R}_{\geq 0}$ between the vehicle at s and the preceding vehicle at $s-1$, called the headway, is defined as $d_v(s) = r(s-1) - r(s) - L_v(s)$.

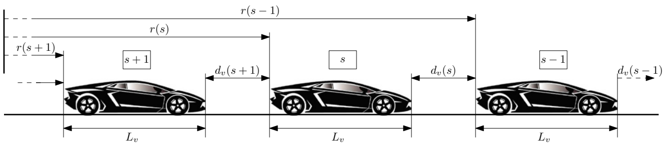


Fig. 3. A string of vehicles.

The distance error $\varepsilon(s) \in \mathbb{R}$ is defined as

$$\varepsilon(s) = d_v(s) - d_r(s), \quad \text{for every fixed } s \in \mathbb{Z},$$

where $d_r(s)$ is the desired headway, given by $d_r(s) = d_{ss}(s) + hv_v(s)$, where $d_{ss}(s)$ is the standstill distance and h the constant time headway. Following now a similar analysis as described in Firooznia (2012), a closed loop model for every vehicle $\mathcal{P}(s)$, can be obtained, given by

$$\begin{bmatrix} \dot{\varepsilon}(s) \\ \dot{v}_v(s) \\ \dot{a}(s) \\ \dot{\jmath}(s) \end{bmatrix} = \begin{bmatrix} 0 & -1 & -h & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2h & -8h & -\dfrac{1+0.8h}{0.1h} & -\dfrac{h+0.1}{0.1h} \end{bmatrix} \begin{bmatrix} \varepsilon(s) \\ v_v(s) \\ a(s) \\ \jmath(s) \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 8h & 10h & \dfrac{1}{h} \end{bmatrix} \begin{bmatrix} \varepsilon(s-1) \\ v_v(s-1) \\ a(s-1) \\ \jmath(s-1) \end{bmatrix}, \tag{19}$$

with $v_v(s)$, $a(s)$, and $\jmath$ the vehicle's velocity, acceleration, and jerk. The model (19) for every vehicle can be reformulated such that the structure as described in (13) is obtained. However, in contrast to Firooznia (2012), the values of $v_v(s-1)$ and $0.1\jmath(s-1)+a(s-1)$ are now received by the vehicle $\mathcal{P}(s)$ from the vehicle $\mathcal{P}(s-1)$ by means of networked communication. Hence, by communicating $w_-(s) = (v_v(s), 0.1\jmath(s) + a(s))$, from the structure of (13) we find the spatially invariant subsystem (15), with $m_0 = 4$, $m_+ = 0$, $m_- = 2$, $x(s) = (\varepsilon(s), v_v(s), a(s), \jmath(s))$,

$$A_{\mathbf{TS}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 8h & 10h \end{bmatrix}, \quad A_{\mathbf{TT}} = \begin{bmatrix} 0 & -1 & -h & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2h & -8h & -\frac{1+0.8h}{0.1h} & -\frac{h+0.1}{0.1h} \end{bmatrix},$$

$$A_{\mathbf{ST}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.1 \end{bmatrix},$$

and where $A_{\mathbf{SS}}$, $B_{\mathbf{S}}$, $B_{\mathbf{T}}$, $C_{\mathbf{S}}$, $C_{\mathbf{T}}$, and D are all zero matrices. Now by using Theorem 2 the τ_{mati} bound can be computed, such that UGES of the infinite string is guaranteed. Hereto, for the scheduling protocol the SD-protocol is taken with $\lambda = 1 \cdot 10^{-3}$. The values for τ_{mati} guaranteeing stability are plotted in Fig. 4 for various time headways.

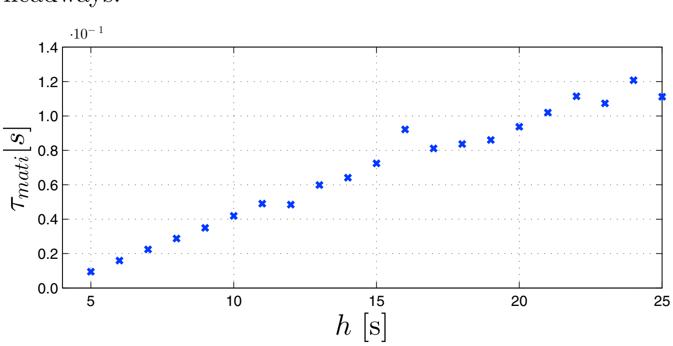


Fig. 4. Values of τ_{mati} guaranteeing UGES for the infinite vehicle string for various time headways h.

7. CONCLUSION AND FUTURE WORK

In this paper a novel approach was presented for the stability analysis of systems consisting of an infinite number of spatially invariant subsystems where packet-based communication networks are part of the system configuration. Based on a model consisting of an infinite interconnection of spatially invariant hybrid systems, bounds on the maximally allowable transmission interval were derived for the communication networks such that uniform global asymptotic (or exponential) stability of the overall system is guaranteed. When particularized to the case of linear subsystems the conditions were reformulated in terms of local linear matrix inequalities (LMIs) using only local information on the subsystem, the interconnection structure and the adopted scheduling protocol. As a consequence, UGES for the overall infinite-dimensional system can be guaranteed based on local properties leading to tractable and easy-to-verify conditions. An illustrative example of a platoon of vehicles showed how the novel results can be applied in a systematic manner. The results presented in this paper have many natural extensions and applications as mentioned in Heijmans (2015), which provide the possibilities for new and more extensive research in this direction of networked and spatially invariant control systems.

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