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Model-based periodic event-triggered control for linear systems[☆]

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ABSTRACT

Periodic event-triggered control (PETC) is a control strategy that combines ideas from conventional periodic sampled-data control and event-triggered control. By communicating periodically sampled sensor and controller data only when needed to guarantee stability or performance properties, PETC is capable of reducing the number of transmissions significantly, while still retaining a satisfactory closed-loop behavior. In this paper, we will study observer-based controllers for linear systems and propose advanced event-triggering mechanisms (ETMs) that will reduce communication in both the sensor-to-controller channels and the controller-to-actuator channels. By exploiting model-based computations, the new classes of ETMs will outperform existing ETMs in the literature. To model and analyze the proposed classes of ETMs, we present two frameworks based on perturbed linear and piecewise linear systems, leading to conditions for global exponential stability and ℓ_2-gain performance of the resulting closed-loop systems in terms of linear matrix inequalities. The proposed analysis frameworks can be used to make tradeoffs between the network utilization on the one hand and the performance in terms of ℓ_2-gains on the other. In addition, we will show that the closed-loop performance realized by an observer-based controller, implemented in a conventional periodic time-triggered fashion, can be recovered arbitrarily closely by a PETC implementation. This provides a justification for emulation-based design. Next to centralized model-based ETMs, we will also provide a decentralized setup suitable for large-scale systems, where sensors and actuators are physically distributed over a wide area. The improvements realized by the proposed model-based ETMs will be demonstrated using numerical examples.

1. Introduction

In most digital control applications, sampling of the outputs of the plant, and computing and implementing new actuator signals are executed periodically. Although periodic sampling might be preferable from an analysis and design point of view, it is sometimes less preferable from a resource utilization point of view. Executing the control task at times when no disturbances are acting on the system and the system is operating desirably is clearly a waste of communication resources. To mitigate the unnecessary waste of communication resources, it is of interest to consider an alternative control paradigm, namely event-triggered control (ETC), which started to attract quite some interest in the late 1990s (Arzén, 1999; Åström & Bernhardsson, 1999; Heemels et al., 1999; Hendricks, Jensen, Chevalier, & Vesterholm, 1994), although mainly for different reasons including the reduction of computational resources and dealing with the event-based nature of the plants' sensors. Also some earlier attempts towards event-triggered sampling and ETC, going back to the 1960s (Draper, Wrigley, & Hovorka, 1960), can be found, see, e.g., the overview paper (Åström, 2008). ETC is a control strategy in which the control task is executed after the occurrence of an event, generated by some well-designed event-triggering condition, rather than the elapse of a certain fixed period of time, as in conventional periodic sampled-data control. In this way, ETC is capable of significantly reducing the number of control task executions, while retaining a satisfactory closed-loop performance, as simulation and experimental results show in, e.g., Arzén (1999), Åström and Bernhardsson (1999), Heemels et al. (1999), Hendricks et al. (1994), Henningsson and Cervin (2009), Kwon, Kim, Lee, and Paek (1999), Lehmann and Lunze (2010) and Yook, Tilbury, and Soparkar (2002). For these appealing reasons, ETC received ample attention in the last decade (Åström & Bernhardsson, 2002; Donkers & Heemels, 2012; Gawthrop & Wang, 2009; Heemels, Sandee, & van den Bosch, 2008; Henningsson, Johannesson, & Cervin, 2008; Kofman & Braslavsky, 2006; Lunze & Lehmann, 2010; Miskowicz, 2006; Otanez, Moyne, & Tilbury, 2002; Tabuada, 2007; Wang & Lemmon, 2009). In most of these ETC schemes the event-triggering condition has to be

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verified continuously, and therefore they are sometimes referred to as "continuous event-triggered control" (CETC) schemes.

Recently, interest was shown in a class of event-triggered control algorithms that aim at integrating ideas from conventional periodic time-triggered control and ETC paradigms. This results in so-called *periodic event-triggered control* (PETC) (Arzén, 1999; Heemels, Donkers, & Teel, 2011, 2013; Heemels et al., 2008; Henningsson et al., 2008; Yook et al., 2002), in which the event-triggering condition is verified periodically, and every sampling time it is decided whether or not to transmit new measurement and control values. The network resources are used only when needed to guarantee stability or certain performance properties. The resulting PETC laws can be considered in a continuous-time framework, see, e.g., Heemels et al. (2011, 2013) or can be studied as event-triggered controllers for discrete-time systems (disregarding the intersample behavior) as in, e.g., Cogill (2009), Eqtami, Dimarogonas, and Kyriakopoulos (2010), Lehmann (2011, Section 4.5), Li and Lemmon (2011) and Yook et al. (2002). In this paper we study them from the latter perspective.

The objective of this paper is to develop new and improved classes of event-triggering mechanisms (ETMs). Besides some notable exceptions (Garcia & Antsaklis, 2011; Lehmann & Lunze, 2011; Li & Lemmon, 2011; Lunze & Lehmann, 2010; Yook et al., 2002), which will be discussed in more detail below, the ETMs currently available in the literature are rather basic for both CETC and PETC. Indeed, often sensors transmit their information to the controller when the difference between the current measured output and the most recently transmitted output value exceeds a certain absolute threshold value or a relative bound with respect to the measured output value, see, for instance, Åström and Bernhardsson (2002), Gawthrop and Wang (2009), Heemels et al. (2008), Henningsson et al. (2008), Miskowicz (2006), Otanez et al. (2002), Tabuada (2007) and Wang and Lemmon (2009) for the case of state-feedback and Donkers and Heemels (2012) and Kofman and Braslavsky (2006) for the case of output-feedback controllers. This is a rather basic strategy for the ETMs, which we will refer to as the *baseline strategy*. In this paper, we will propose model-based ETMs for both the sensor-to-controller (S–C) and the controller-to-actuator (C–A) channels that result in significant reductions of the communication traffic if compared to both time-triggered periodic implementations as well as ETMs using this baseline strategy, while preserving desirable stability and performance properties. In this paper, the focus will be on PETC strategies, although we believe that the main ideas also apply in the context of CETC.

The main idea of the proposed S–C ETMs is the use of a Luenberger observer at the sensor system, and the use of a model-based *predictor* that runs both at the sensor and the controller system. This observer and this predictor both produce an estimate of the state of the plant. The observer will generally produce better estimates than the predictor, as it has access to all the measured outputs, while the predictor has not. Due to the fact that the sensor system runs a copy of the predictor, it is aware of the estimate the controller system has. The S–C ETM triggers a transmission of the estimated state of the observer to the controller system, only if the estimate of the controller system deviates too much from the (better) estimate the observer has.

The main principle behind the C–A ETMs proposed in this paper, is to utilize predictive techniques as used for networked control systems (NCSs) with unreliable channels exhibiting delays and packet loss, see, e.g., Bemporad (1998), Chaillet and Bicchi (2008) and Hu, Liu, and Rees (2007) and the references therein. In the mentioned references, the predictive control ideas are used for *counteracting* packet loss and transmission delays. In this paper, we will exploit the idea of predictive control and buffering future control values for a different purpose. Namely, we will exploit it for *reducing* the number of transmissions. In fact, both the S–C and the C–A ETMs can result in significant reductions of the number of transmissions, by introducing more computations. In this sense "computations are traded for bandwidth" (Yook et al., 2002). In addition, if computation is cheap compared to communication in terms of power usage (which is often the case), the proposed PETC strategies can also result in considerable reductions in the overall energy usage of battery-powered wireless sensors, controllers and actuators, thereby having positive effects on their battery lives.

In the literature, related model-based ETMs were presented for the S–C channels, see Garcia and Antsaklis (2011), Lehmann and Lunze (2011), Li and Lemmon (2011), Lunze and Lehmann (2010) and Yook et al. (2002). There are, however, some essential differences. First of all, (Lehmann & Lunze, 2011; Lunze & Lehmann, 2010; Yook et al., 2002) use ETMs based on absolute thresholds resulting in ultimate boundedness and BIBO stability types of conditions, while in this paper we are interested in developing ETMs that achieve exponential stability and guaranteed (finite) ℓ_2-gains. This does not only require a different type of ETMs, but also different mathematical analysis frameworks. Secondly, the works (Garcia & Antsaklis, 2011; Lehmann & Lunze, 2011; Lunze & Lehmann, 2010) are formulated in the context of CETC, while in this paper we focus on PETC implementations. Thirdly, in this paper we will also consider communication saving measures in the C–A channels by using advanced model-based ETMs, as opposed to Garcia and Antsaklis (2011), Lehmann and Lunze (2011), Lunze and Lehmann (2010), and Yook et al. (2002), which assume controllers and actuators to be collocated or hard-wired, and to Li and Lemmon (2011), which does not use predictive techniques in the C–A channel. Fourthly, in Garcia and Antsaklis (2011) and Lunze and Lehmann (2010) state-feedback control laws are considered, while here the focus is on output-based dynamic controllers. Fifthly, in Garcia and Antsaklis (2011), Lehmann and Lunze (2011), Li and Lemmon (2011) and Lunze and Lehmann (2010) the ETMs and controllers are implemented in a centralized setting, while in the present paper we will also provide decentralized implementations of both the controller and the ETMs based on simple low-order *local* models. Our decentralized strategy does not require running a *global* high-order estimator at each local node as in Yook et al. (2002).

As such, the main contributions of this paper are the proposition and the formalization of the mentioned S–C and C–A ETMs leading to model-based PETC strategies for discrete-time linear plants subject to disturbances, not requiring the full state to be available for feedback. We provide two general modeling and analysis frameworks based on perturbed linear (PL) systems and piecewise linear (PWL) systems, and derive linear matrix inequality (LMI)-based conditions for global exponential stability and guaranteed ℓ_2-gains of the closed-loop systems. In fact, these frameworks for modeling and analysis form another distinctive difference with the works (Garcia & Antsaklis, 2011; Lehmann & Lunze, 2011; Li & Lemmon, 2011; Lunze & Lehmann, 2010; Yook et al., 2002). In these works analysis methods are adopted that are in nature closer to the PL system approach, and they do not consider the PWL system approach. This latter approach can be shown to be never outperformed by the PL system approach and in most cases actually provides (much) better guarantees (see Section 4.3 below). The PL system approach, on the other hand, will be used to show that the performance of any observer-based controller, implemented in a conventional time-triggered fashion, can be recovered by a PETC implementation. This result forms a strong justification for so-called *emulation-based design*, which is a two-stage design procedure. In the first design stage the controller parameters are chosen using standard discrete-time ($\mathcal{H}_\infty/\ell_2$) synthesis tools (assuming a periodic time-triggered implementation and thus ignoring the event-triggered implementation). In the second stage, the parameters of the ETMs are chosen by making appropriate tradeoffs between the network utilization on the one hand and the performance in terms of ℓ_2-gains on the other hand. This justification for the emulation-based design forms an important motivation for presenting the PL system approach, next to the facts that it is closer to the existing literature, see, e.g., Garcia and Antsaklis (2011), Lehmann and Lunze (2011), Li and Lemmon (2011), Lunze and Lehmann (2010), Tabuada (2007) and Yook et al. (2002), and that it results in simpler $\mathcal{H}_\infty$-norm conditions that guarantee closed-loop stability when compared to the PWL system approach. The PWL system approach is of importance as it generically results in less conservative results, as already mentioned above.

The results mentioned above will be presented for a model-based PETC setup with a centralized controller and centralized ETMs in the first part of the paper. In the second part, we will indicate how these ideas and methodologies can be applied in the context of decentralized observer-based control laws using small local observers and predictors. The latter setup is highly relevant for large-scale plants in which sensors, actuators and controllers are physically distributed over a wide area. In fact, centralized ETMs can be prohibitive in this case, as the conditions that generate events would need access to all the plant or controller outputs at every sampling time, which can be an unrealistic assumption for large-scale systems. In addition, the size of the complete plant model also might be too large for usage in the ETMs for computational reasons. The solution proposed in the second part will overcome both issues.

Notations. For a vector $x \in \mathbb{R}^n$, we denote by $\|x\| := \sqrt{x^\top x}$ its 2-norm. For a symmetric matrix $A \in \mathbb{R}^{n\times n}$, $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximum and minimum eigenvalue of A, respectively. For a matrix $A \in \mathbb{R}^{n\times m}$, we denote by $A^\top \in \mathbb{R}^{m\times n}$ the transposed of A, and by $\|A\| := \sqrt{\lambda_{\max}(A^\top A)}$ its induced 2-norm. For vectors $x^i \in \mathbb{R}^{n_i}$, $i \in \{1,\dots,N\}$, the vector $x = \mathrm{col}(x^1, x^2, \dots, x^N) \in \mathbb{R}^n$, where $n = \sum_{i=1}^N n_i$, is given by $[(x^1)^\top\ (x^2)^\top\ \cdots\ (x^N)^\top]^\top$. By $\mathrm{diag}(A_1,\dots,A_N)$, we denote a block-diagonal matrix with the matrices $A_1,\dots,A_N$ on the diagonal, and for the sake of brevity we sometimes write symmetric matrices of the form $\begin{bmatrix} A & B \\ B^\top & C\end{bmatrix}$ as $\begin{bmatrix} A & B \\ \star & C\end{bmatrix}$. We call a matrix $P \in \mathbb{R}^{n\times n}$ positive definite (positive semidefinite) and write $P \succ 0$ ($P \succeq 0$), if P is symmetric and $x^\top P x > 0$ ($x^\top P x \geqslant 0$) for all $x \neq 0$. A matrix $A \in \mathbb{R}^{n\times n}$ is called Schur, if all its eigenvalues are within the open unit circle of the complex plane. The set ℓ_2^n consists of all infinite sequences $w = \{w_k\}_{k\in\mathbb{N}}$ with $w_k \in \mathbb{R}^n$, $k \in \mathbb{N}$, and $\sum_{k\in\mathbb{N}} \|w_k\|^2 < \infty$. For each $w \in \ell_2^n$, we write $\|w\|_{\ell_2} = \sqrt{\sum_{k\in\mathbb{N}} \|w_k\|^2}$.

2. Problem formulation

In this paper, we will study the networked control configuration as shown in Fig. 1, in which the plant is given by a discrete-time linear time-invariant (LTI) model of the form

$$\mathcal{P} : \begin{cases} x_{k+1} = Ax_k + Bu_k + Ew_k \\ y_k = Cx_k, \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^{n_x}$, $u_k \in \mathbb{R}^{n_u}$, $w_k \in \mathbb{R}^{n_w}$ and $y_k \in \mathbb{R}^{n_y}$ denote the state, control input, disturbance and measured output, respectively, at discrete time instant $k \in \mathbb{N}$. The sensors of the plant transmit their measurement information to the controller, and the controller transmits the control data to the actuators over a shared and possibly wireless network, for which communication resources and/or energy sources, e.g., the batteries for the wireless devices, are limited. For this reason, it is desirable to reduce the transmissions over the sensor-to-controller (S–C) and controller-to-actuator (C–A) channels as much as possible, while still guaranteeing desirable closed-loop behavior. This leads us to the problem of designing smart sensor, controller and actuator systems for the setup in Fig. 1 such that this objective is accomplished.

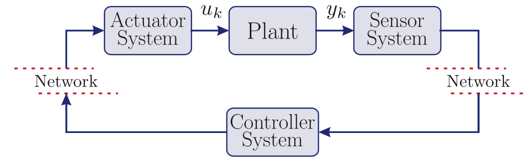


Fig. 1. Networked control configuration.

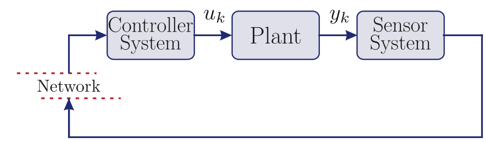


Fig. 2. Control configuration with the shared network only in S–C channel.

In solving this problem, we will start by proposing a model-based PETC strategy for the configuration in Fig. 2, in which only communication-saving measurements in the S–C channel are desirable. This configuration is of independent interest in various situations including the case in which the controller and actuators of the plant are collocated or connected through dedicated (wired) point-to-point connections and only the S–C channel has limited resources. After proposing and analyzing effective PETC strategies for this setup in Sections 3 and 4, we treat the setup of Fig. 1 in Sections 5 and 6 and extensions towards decentralized implementations in Section 7.

3. Model-based PETC strategy for S–C channels

In this section, we propose a solution for the problem formulated in Section 2 in the context of Fig. 2.

3.1. Proposed solution

In the solution we propose, the smart sensor system in Fig. 2 will consist of a Luenberger observer $\mathcal{O}$, a predictor $\mathcal{P}r$ and an event-triggering mechanism ETM^s that determines when information should be transmitted to the controller system, see Fig. 3. The Luenberger observer will be of the form

$$\mathcal{O} : x^s_{k+1} = Ax^s_k + Bu_k + L(y_k - Cx^s_k) \tag{2}$$

in which x^s_k denotes the estimated state at the sensor system at time $k \in \mathbb{N}$, and the matrix L is a suitably designed observer gain. The predictor $\mathcal{P}r$ is given by

$$\mathcal{P}r : x^c_{k+1} = \begin{cases} Ax^c_k + Bu_k, & \text{when } x^s_k \text{ is not sent} \\ Ax^s_k + Bu_k, & \text{when } x^s_k \text{ is sent.} \end{cases} \tag{3}$$

Finally, the S–C ETM is given at time $k \in \mathbb{N}$ by

$$\mathrm{ETM}^s : x^s_k \text{ is sent} \Leftrightarrow \|x^s_k - x^c_k\| > \sigma_s \|x^s_k\|, \tag{4}$$

where $\sigma_s \geqslant 0$ is a suitably chosen design parameter. Before explaining the functioning of $\mathcal{P}r$ and ETM^s in more detail, it is convenient to also introduce the controller system. The controller system consists of the same predictor $\mathcal{P}r$, and a controller gain K, see Fig. 3. In fact, the control signal is given by

$$u_k = \begin{cases} Kx^c_k, & \text{when } x^s_k \text{ is not sent} \\ Kx^s_k, & \text{when } x^s_k \text{ is sent.} \end{cases} \tag{5}$$

As the sensor system also runs a copy of the predictor $\mathcal{P}r$ (both initialized at the same initial estimate), the sensor system is aware of the estimate x^c_k the controller system has, and, consequently, can determine u_k to compute the state estimates x^s_k according to (2).

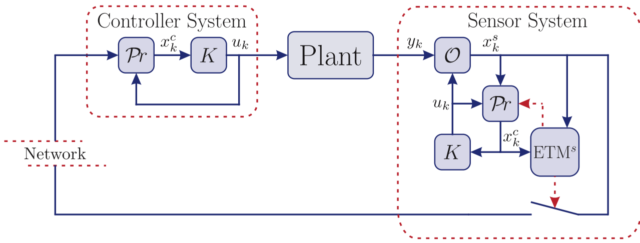


Fig. 3. Model-based PETC strategy with only S–C ETM.

Remark 3.1. (Approximate) disturbance models $\hat{w}_{k+1} = S\hat{w}_k$ with $\hat{w}_k$ an estimate of w_k in (1), $k \in \mathbb{N}$, and S a matrix of suitable dimensions, can be included in the observer and predictor structures. Indeed, the observer (2) is then extended to

$$\begin{pmatrix} x^s_{k+1} \\ \hat{w}^s_{k+1} \end{pmatrix} = \bar{A}\begin{pmatrix} x^s_k \\ \hat{w}^s_k \end{pmatrix} + \bar{B}u_k + \bar{L}(y_k - Cx^s_k), \tag{6}$$

where $\bar{A} = \begin{pmatrix} A & E \\ 0 & S \end{pmatrix}$, $\bar{B} = \begin{pmatrix} B \\ 0 \end{pmatrix}$ and $\bar{L}$ is a well designed observer gain. The predictor then becomes

$$\begin{pmatrix} x^c_{k+1} \\ \hat{w}^c_{k+1} \end{pmatrix} = \begin{cases} \bar{A}\begin{pmatrix} x^c_k \\ \hat{w}^c_k \end{pmatrix} + \bar{B}u_k, & \text{when } \begin{pmatrix} x^s_k \\ \hat{w}^s_k \end{pmatrix} \text{ is not sent} \\ \bar{A}\begin{pmatrix} x^s_k \\ \hat{w}^s_k \end{pmatrix} + \bar{B}u_k, & \text{when } \begin{pmatrix} x^s_k \\ \hat{w}^s_k \end{pmatrix} \text{ is sent.} \end{cases} \tag{7}$$

The triggering condition that determines when $\begin{pmatrix} x^s_k \\ \hat{w}^s_k \end{pmatrix}$ is sent can be left unchanged and can thus be chosen as (4). This inclusion of disturbance models can further enhance the communication savings between sensors and controllers. The stability and performance analysis presented in this paper can be applied *mutatis mutandis* to this setup as well.

3.2. Rationale of the proposed scheme

To explain the rationale behind the proposed scheme, observe that the Luenberger observer produces reliable estimates x^s_k of the state x_k as it has full access to all the output measurements. The predictor $\mathcal{P}r$, as in (3), located in the controller system also produces an estimate x^c_k for the plant state x_k. In principle, the predictor $\mathcal{P}r$ generates these estimates using only predictions based on the model (1) without any innovations (the first case in (3)), and the control value u_k is updated based on this prediction (the first case in (5)). As long as the controller does not receive better state estimates x^s_k from the sensor system, the controller keeps on operating in this manner.

However, the sensor system runs a copy of the predictor $\mathcal{P}r$ and is, therefore, aware of the estimate x^c_k the controller has of the state. If the sensor system detects at $k \in \mathbb{N}$ that the estimate x^s_k of the Luenberger observer (2) deviates significantly from the estimate x^c_k, i.e., $\|x^s_k - x^c_k\| > \sigma_s\|x^s_k\|$, the estimate x^s_k is transmitted to the controller, and corresponding updates of the estimate x^c_{k+1} (cf. the second case in (3)) and the control signal u_k as in (5) are made. Hence, as long as $\|x^s_k - x^c_k\|$ is sufficiently small, no transmissions between sensor and controller systems are needed. Interestingly, in case of absence of disturbances ($w = 0$), it holds that if $x^s_{k_0} = xk_0$ for some $k_0 \in \mathbb{N}$, then at most *one* transmission is needed between sensor and controller for $k \geqslant k_0$. In case a *deadbeat observer* is used, i.e., $A - LC$ has only eigenvalues equal to zero, and no disturbances are present, it follows that if for a $k_0 \geqslant n_x$ a transmission of x^s_k occurs (which is then necessarily equal to x_k), no further transmissions are needed because then $x^s_k = x^c_k = x_k$ for all $k > k_0$.

We will show that this new observer-based PETC strategy can provide the same stability and ℓ_2-gain performance guarantees, while requiring significantly less transmissions when compared to both a standard periodic time-triggered implementation of the observer-based controller and a PETC implementation along the lines of Donkers (2011), Donkers and Heemels (2012), Heemels et al. (2011, 2013), Tabuada (2007) and Wang and Lemmon (2009). In fact, the latter will be used as a *baseline strategy*, to which the newly proposed class of ETMs will be compared.

3.3. A baseline strategy

In this section, we briefly introduce the basic PETC implementations of output-based controllers using ideas presented in Donkers (2011), Donkers and Heemels (2012), Heemels et al. (2011, 2013), Tabuada (2007), and Wang and Lemmon (2009). The ideas in these papers would lead in the case of observer-based controllers to the PETC strategy given by

$$x^c_{k+1} = Ax^c_k + Bu_k + L(\hat{y}_k - Cx^c_k), \tag{8a}$$

a control law

$$u_k = Kx^c_k, \tag{8b}$$

and an ETM operating as

$$\hat{y}_k = \begin{cases} y_k, & \text{when } \|\hat{y}_{k-1} - y_k\| > \sigma_s\|y_k\| \\ \hat{y}_{k-1}, & \text{when } \|\hat{y}_{k-1} - y_k\| \leqslant \sigma_s\|y_k\|. \end{cases} \tag{9}$$

Hence, in this setup a sensor reading is transmitted to the controller only when the difference between the latest transmitted value and the current sensor reading is large compared to the current value of the reading. In addition, a very basic "hold strategy", i.e., $\hat{y}_k = \hat{y}_{k-1}$, is used when no new output measurement is transmitted. As such, the observer in the controller operates as a normal observer using the latest received value $\hat{y}_k$ to innovate its estimates.

3.4. Closed-loop modeling

In this section, we derive for the proposed solution based on (2)–(5) a suitable closed-loop model in the form of a piecewise linear (PWL) system. In addition, an alternative perturbed linear (PL) system model will be presented describing how the event-triggered implementation perturbs the standard periodic-time triggered implementation of the observer-based controller.

3.4.1. Piecewise linear model

To obtain the closed-loop PWL model, we will use the definitions $e^s_k := x^s_k - x_k$ and $e^c_k := x^c_k - x_k$, and will adopt the state variable $\xi_k = \mathrm{col}(x_k, e^s_k, e^c_k) \in \mathbb{R}^{n_\xi}$ with $n_\xi := 3n_x$. Some manipulations lead to the closed-loop model

$$\xi_{k+1} = \begin{cases} \mathcal{A}_1\xi_k + \mathcal{E}w_k, & \xi_k^\top Q_s\xi_k > 0 \\ \mathcal{A}_2\xi_k + \mathcal{E}w_k, & \xi_k^\top Q_s\xi_k \leqslant 0 \end{cases} \tag{10}$$

with

$$\mathcal{A}_1 = \begin{bmatrix} A+BK & BK & 0 \\ 0 & A-LC & 0 \\ 0 & A & 0 \end{bmatrix}, \quad \mathcal{A}_2 = \begin{bmatrix} A+BK & 0 & BK \\ 0 & A-LC & 0 \\ 0 & 0 & A \end{bmatrix}, \tag{11a}$$

$$\mathcal{E} = \begin{bmatrix} E \\ -E \\ -E \end{bmatrix}, \qquad Q_s = \begin{bmatrix} -\sigma_s^2 I & -\sigma_s^2 I & 0 \\ -\sigma_s^2 I & (1-\sigma_s^2)I & -I \\ 0 & -I & I \end{bmatrix}. \tag{11b}$$

Note that with the choice of Q_s as in (11) $\xi_k^\top Q_s\xi_k > 0$ is equivalent to $\|e^s_k - e^c_k\| > \sigma_s\|x_k + e^s_k\|$, which is the same as the event-triggering condition $\|x^s_k - x^c_k\| > \sigma_s\|x^s_k\|$ in (4). As such, the discrete-time PWL system (10) describes the overall closed-loop system consisting of the plant $\mathcal{P}$, the sensor and the controller systems given by (1)–(5).

3.4.2. Perturbed linear system

For the PL model, we will use the state variable $\chi_k = \mathrm{col}(x_k, e_k^s)$, $k \in \mathbb{N}$. Note that information on e_k^c (and thus also on x_k^c) is removed from the state variable. This leads to the PL system

$$\begin{cases} \chi_{k+1} = A_p\chi_k + E_p w_k + V_p v_k \\ s_k = x_k + e_k^s = S_p\chi_k \end{cases} \tag{12}$$

with

$$A_p = \begin{bmatrix} A+BK & BK \\ 0 & A-LC \end{bmatrix}, \qquad E_p = \begin{bmatrix} E \\ -E \end{bmatrix}, \\ V_p = \begin{bmatrix} -BK \\ 0 \end{bmatrix}, \qquad S_p = \begin{bmatrix} I & I \end{bmatrix}, \tag{13}$$

and s_k an auxiliary output variable. Here, v_k is an event-triggering-induced perturbation at time $k \in \mathbb{N}$ and, in fact, it is given by

$$v_k := \begin{cases} 0, & \text{if } \|e_k^s - e_k^c\| > \sigma_s\|s_k\| \\ e_k^s - e_k^c, & \text{if } \|e_k^s - e_k^c\| \leqslant \sigma_s\|s_k\|, \end{cases} \tag{14}$$

which provides the connection to (10). Note that in case we set $v_k = 0$, $k \in \mathbb{N}$, the system (12) reduces to

$$\begin{cases} \chi_{k+1} = A_p\chi_k + E_p w_k \\ s_k = S_p\xi_k, \end{cases} \tag{15}$$

which is the closed-loop system when x_k^s is transmitted to the controller for all $k \in \mathbb{N}$ and, hence, this would reduce the system to the standard periodic time-triggered implementation of the Luenberger observer $\mathcal{O}$ and the state feedback law $u_k = Kx_k^s$, $k \in \mathbb{N}$. The PL system (12) expresses how this standard time-triggered system is perturbed by introducing the ETM in the loop. Interestingly, we can immediately obtain a bound

$$\|v_k\| \leqslant \sigma_s\|s_k\|, \quad k \in \mathbb{N} \tag{16}$$

from (14), which will be useful in deriving simple and insightful conditions on σ_s, K and L guaranteeing stability and a certain ℓ_2-gain. Note though that the PL model (12) with bound (16) forms a less precise model of the closed-loop PETC system than the PWL model, as it does not describe the ETM explicitly, but only uses specific bounds (16) induced by it.

4. Stability and ℓ_2-gain analysis

In this section, we define the notions of performance and stability used in this paper and we will analyze stability and performance of the closed-loop ETC system based on both the PWL model (10) and the PL model (12). To do so, let us introduce the performance output z given for $k \in \mathbb{N}$ by

$$z_k = C_{zx}x_k + D_z w_k. \tag{17}$$

Definition 4.1. The system (10) with $w = 0$ is said to be *globally exponentially stable* (GES), if there exist constants $c \in \mathbb{R}_{\geqslant 0}$ and $\rho \in [0, 1)$ such that for all initial states $\xi_0 \in \mathbb{R}^{n_\xi}$ and $w_k = 0$, $k \in \mathbb{N}$, the corresponding solution to (10) satisfies for all $k \in \mathbb{N}$ that

$$\|\xi_k\| \leqslant c\rho^k\|\xi_0\|. \tag{18}$$

Definition 4.2. The system given by (10) and (17) is said to have an ℓ_2-gain from w to z smaller than or equal to $\gamma \in \mathbb{R}_{\geqslant 0}$, if there is a function $\beta : \mathbb{R}^{n_\xi} \to \mathbb{R}$ such that for all initial states $\xi_0 \in \mathbb{R}^{n_\xi}$ and all inputs $w \in \ell_2^{n_w}$, the corresponding output z satisfies for all $k \in \mathbb{N}$ that

$$\|z\|_{\ell_2} \leqslant \beta(\xi_0) + \gamma\|w\|_{\ell_2}. \tag{19}$$

The ℓ_2-gain from w to z is defined as the infimum of all values γ that satisfy the above property.

Remark 4.3. In case the plant model (1) is obtained as a discretization of a continuous-time linear plant, it is also of interest to study stability and performance of the interconnection of the PETC law and the continuous-time plant in a sampled-data setting (including intersample behavior). In fact, it is not hard to show that GES of the closed-loop PETC system given by (10) implies also GES of the corresponding continuous-time closed-loop system, see Heemels et al. (2011, 2013). $\mathcal{L}_2$-gain analysis (including intersample behavior) for such continuous-time closed-loop systems can be found in the works (Heemels et al., 2011, 2013) as well.

4.1. Piecewise linear system approach

The first stability and performance results of this paper will be presented below based on the PWL model (10). To arrive at such results, we will employ a piecewise quadratic (PWQ) Lyapunov function of the form

$$V(\xi) = \begin{cases} \xi^\top P_1\xi, & \text{when } \xi_k^\top Q_s\xi_k > 0, \\ \xi^\top P_2\xi, & \text{when } \xi_k^\top Q_s\xi_k \leqslant 0, \end{cases} \tag{20}$$

we can guarantee an ℓ_2-gain of the PETC system (10) with (17), according to the following result.

Theorem 4.4. *The closed-loop PETC system given by (10) and (17) has an ℓ_2-gain smaller than or equal to $\gamma \in \mathbb{R}_{\geqslant 0}$, and (10) with $w = 0$ is GES, if there exist matrices P_1, P_2 and scalars $\varepsilon \in \mathbb{R}_{>0}$, $\alpha_{ij} \in \mathbb{R}_{\geqslant 0}$, $\beta_{ij} \in \mathbb{R}_{\geqslant 0}$ and $\kappa_i \in \mathbb{R}_{\geqslant 0}$, $i, j \in \{1, 2\}$, satisfying*

$$\begin{bmatrix} \Omega_{ij}^{\xi\xi} & \Omega_{ij}^{\xi w} & C_{z\xi}^\top \\ \star & \Omega_{ij}^{ww} & D_z^\top \\ \star & \star & I \end{bmatrix} \succeq 0, \quad i, j \in \{1, 2\} \tag{21a}$$

and

$$P_i + (-1)^i\kappa_i Q_s \succ 0, \quad i \in \{1, 2\}, \tag{21b}$$

where $C_{z\xi} = [C_{zx}\ 0\ 0]$, and for $i, j \in \{1, 2\}$

$$\Omega_{ij}^{\xi\xi} = P_i - \mathcal{A}_i^\top P_j\mathcal{A}_i - \varepsilon I + (-1)^i\alpha_{ij}Q_s + (-1)^j\beta_{ij}\mathcal{A}_i^\top Q_s\mathcal{A}_i \tag{22a}$$

$$\Omega_{ij}^{\xi w} = -\mathcal{A}_i^\top P_j\mathcal{E} + (-1)^j\beta_{ij}\mathcal{A}_i^\top Q_s\mathcal{E} \tag{22b}$$

$$\Omega_{ij}^{ww} = \gamma^2 I - \mathcal{E}^\top P_j\mathcal{E} + (-1)^j\beta_{ij}\mathcal{E}^\top Q_s\mathcal{E}. \tag{22c}$$

Proof. From (20), it follows that $V(\xi) = \xi^\top P_1\xi \geqslant \lambda_{\min}(P_1 - \kappa_1 Q_s)\|\xi\|^2 + \kappa_1\xi^\top Q_s\xi \geqslant \lambda_{\min}(P_1 - \kappa_1 Q_s)\|\xi\|^2$ for all ξ satisfying $\xi^\top Q_s\xi > 0$ and that $V(\xi) = \xi^\top P_2\xi \geqslant \lambda_{\min}(P_2 + \kappa_2 Q_s)\|\xi\|^2 - \kappa_2\xi^\top Q_s\xi \geqslant \lambda_{\min}(P_2 + \kappa_2 Q_s)\|\xi\|^2$ for all ξ satisfying $\xi^\top Q_s\xi \leqslant 0$. Due to (21b), this proves that for the candidate storage function (20), there exist a $c_1 = \min\{\lambda_{\min}(P_1 - \kappa_1 Q_s), \lambda_{\min}(P_2 + \kappa_2 Q_s)\} > 0$ and some $c_2 \geqslant c_1$ such that $c_1\|\xi\|^2 \leqslant V(\xi) \leqslant c_2\|\xi\|^2$ for all $\xi \in \mathbb{R}^{n_\xi}$. Hence, V is a positive definite storage function.

The positive definiteness of V in combination with the satisfaction of the dissipation inequality

$$V(\xi_{k+1}) - V(\xi_k) \leqslant -\varepsilon\|\xi_k\|^2 - \|z_k\|^2 + \gamma^2\|w_k\|^2 \tag{23}$$

would now be sufficient to guarantee a finite ℓ_2-gain that is smaller than or equal to γ and in case $w = 0$ GES of (10). To prove the satisfaction of the dissipation inequality (23), note that if $V(\xi) = \xi^\top P_i\xi$, it holds that $(-1)^i\xi^\top Q_s\xi \leqslant 0$, and if $V(\mathcal{A}_i\xi + \mathcal{E}w) = (\mathcal{A}_i\xi + \mathcal{E}w)^\top P_j(\mathcal{A}_i\xi + \mathcal{E}w)$, then $(-1)^j(\xi^\top\mathcal{A}_i^\top + w^\top\mathcal{E}^\top)Q_s(\mathcal{A}_i\xi + \mathcal{E}w) \leqslant 0$, for $i, j \in \{1, 2\}$ (where we omitted the subscript k for brevity).

Using these observations and applying a Schur complement to the LMIs (21a), we obtain

$$V(\xi_{k+1})-V(\xi_k)+\varepsilon\|\xi_k\|^2+\|z_k\|^2-\gamma^2\|w_k\|^2=\begin{bmatrix}\xi_k\\ w_k^\top\end{bmatrix}^\top \times\begin{bmatrix}-P_i+\mathcal{A}_i^\top P_j\mathcal{A}_i+\varepsilon I+C_{z\xi}^\top C_{z\xi} & \mathcal{A}_i^\top P_j\mathcal{E}+C_{z\xi}^\top D_z\\ \star & \mathcal{E}^\top P_j\mathcal{E}+D_z^\top D_z-\gamma^2 I\end{bmatrix}\begin{bmatrix}\xi_k\\ w_k^\top\end{bmatrix}$$
$$\leqslant\begin{bmatrix}\xi_k\\ w_k^\top\end{bmatrix}^\top\begin{bmatrix}(-1)^i\alpha_{ij}Q_s+(-1)^j\beta_{ij}\mathcal{A}_i^\top Q_s\mathcal{A}_i & (-1)^j\beta_{ij}\mathcal{A}_i^\top Q_s\mathcal{E}\\ \star & +(-1)^j\beta_{ij}\mathcal{E}^\top Q_s\mathcal{E}\end{bmatrix}\begin{bmatrix}\xi_k\\ w_k^\top\end{bmatrix}$$
$$=(-1)^i\alpha_{ij}\xi_k^\top Q_s\xi_k+(-1)^j\beta_{ij}(\xi_k^\top\mathcal{A}_i^\top+w_k^\top\mathcal{E}^\top)Q_s(\mathcal{A}_i\xi_k+\mathcal{E}w_k),$$

which is non-positive. This completes the proof. □

Based on Theorem 4.4, one can minimize the upper bound γ on the ℓ_2-gain subject to the LMIs (21) given a value of σ_s. This leads to tradeoff curves between σ_s and γ, which can be used to find the largest minimal inter-transmission times (for fixed K and L) while still guaranteeing a certain ℓ_2-gain and/or GES in the case $w=0$.

4.2. Perturbed linear system approach

We will now present stability and performance results that are based on the PL system model. These results can be seen as a consequence of a small gain theorem combining the ℓ_2-gain σ_s from s to v in (16) with an upper bound on the ℓ_2-gain of (12) from v to s. Instrumental for formalizing this idea are the concepts of dissipativity, storage functions and supply rates, see, e.g., Boyd, El Ghaoui, Feron, and Balakrishnan (1994), and Willems (1972).

Theorem 4.5. *Suppose that there exists a positive definite matrix P such that the storage function $V(\chi)=\chi^\top P\chi$ satisfies along the solutions of (12) and (17) that*

$$V(\chi_{k+1})-V(\chi_k)\leqslant-\varepsilon\|\chi_k\|^2-\|s_k\|^2+\theta^2\|v_k\|^2 -\alpha\|z_k\|^2+\alpha\gamma^2\|w_k\|^2 \quad (24)$$

for some $\theta\in\mathbb{R}_{\geqslant0}$, $\gamma\in\mathbb{R}_{\geqslant0}$ and $\varepsilon,\alpha\in\mathbb{R}_{>0}$. Then for all $0\leqslant\sigma_s\leqslant\theta^{-1}$ the closed-loop PETC system (1)–(5) with (17) captured in (10) and (17) has an ℓ_2-gain from w to z smaller than or equal to γ, and (10) with $w=0$ is GES.

Proof. Substituting (16) in (24) gives

$$V(\chi_{k+1})-V(\chi_k)\leqslant-\varepsilon\|\chi_k\|^2-(1-\theta^2\sigma_s^2)\|s_k\|^2 -\alpha\|z_k\|^2+\alpha\gamma^2\|w_k\|^2. \quad (25)$$

Hence, if $0\leqslant\sigma_s\leqslant\theta^{-1}$, this yields

$$V(\chi_{k+1})-V(\chi_k)\leqslant-\varepsilon\|\chi_k\|^2-\alpha\|z_k\|^2+\alpha\gamma^2\|w_k\|^2, \quad (26)$$

which by standard ℓ_2-gain arguments, see, e.g., van der Schaft (2000), proves that the ℓ_2-gain from w to z is smaller than or equal to γ. In addition, for $w=0$, we obtain from (26) that $V(\chi_{k+1})-V(\chi_k)\leqslant-\varepsilon\|\chi_k\|^2$, $k\in\mathbb{N}$. Since there exist $c_1,c_2\in\mathbb{R}_{>0}$ such that $c_1\|\chi\|^2\leqslant V(\chi)\leqslant c_2\|\chi\|^2$ for all χ, it follows that there are $c\in\mathbb{R}_{\geqslant0}$ and $\rho\in[0,1)$ such that for all $k\in\mathbb{N}$ $\|\chi_k\|\leqslant c\rho^k\|\chi_0\|$. To prove GES of (10) for $w=0$, we have to show the existence of $\bar c\in\mathbb{R}_{\geqslant0}$ and $\bar\rho\in[0,1)$ with $\|\xi_k\|\leqslant\bar c\bar\rho^k\|\xi_0\|$ along all trajectories of (10) with $w=0$. Since $\xi_k=\text{col}(\chi_k,e_k^c)$, it is only left to show that there exist exponentially decaying bounds on the norm of e_k^c.

To show the latter, we take a $k\in\mathbb{N}$ and we distinguish two cases, namely x_k^s is sent ($\xi_k^\top Q_s\xi_k>0$) and x_k^s is not sent ($\xi_k^\top Q_s\xi_k\leqslant0$). In the first case, we have according to (10) with $w=0$ that $\|e_{k+1}^c\|=\|Ae_k^s\|\leqslant g\|\chi_k\|$ for some $g\in\mathbb{R}_{\geqslant0}$. In the second case, we have that $\|e_{k+1}^c\|=\|Ae_k^c\|\leqslant\|Ae_k^s\|+\|A(e_k^c-e_k^s)\|$. Using now that $\xi_k^\top Q_s\xi_k\leqslant0$ is equivalent to $\|e_k^c-e_k^s\|\leqslant\sigma_s\|x_k+e_k^s\|$ it follows that $\|e_{k+1}^c\|\leqslant\bar g\|\chi_k\|$ for some $\bar g$. Combining the two cases with the exponentially decaying bounds on the norm of χ_k leads now to GES of (10) for $w=0$. □

Guaranteeing the validity of (24) and positive definiteness of V can be done by showing feasibility of the LMIs

$$\begin{bmatrix}\Xi(P,\alpha,\varepsilon) & A_p^\top PE_p & A_p^\top PV_p & \alpha C_{z\chi}^\top & S_p^\top\\ \star & \alpha\gamma^2 I-E_p^\top PE_p & E_p^\top PV_p & \alpha D_z^\top & 0\\ \star & \star & \theta^2 I-V_p^\top PV_p & 0 & 0\\ \star & \star & \star & \alpha I & 0\\ \star & \star & \star & \star & I\end{bmatrix}\succeq 0 \quad (27a)$$

with $\Xi(P,\alpha,\varepsilon)=P-\varepsilon I-A_p^\top PA_p$ and $C_{z\chi}=[C_{zx}\ 0]$, and

$$P\succ 0. \quad (27b)$$

In fact, if $A+BK$ and $A-LC$ are Schur, then positive γ and θ satisfying these LMIs can always be found. In general, there will be a Pareto optimal curve of (θ,γ) for which the LMIs (27) or equivalently, (24) holds for $P\succ0$. Typically, a small γ (good performance) leads to a large θ resulting in small values of σ_s and, consequently, more transmissions. The Pareto optimal curve can be obtained by minimizing $\theta\in\mathbb{R}_{\geqslant0}$ subject to the LMI constraints (27) (for a range of fixed values of γ).

In a similar manner as above, we can also obtain a stability result for the case that $w=0$.

Theorem 4.6. *Suppose that $A-LC$ and $A+BK$ are Schur. Then there exists a $\sigma_0>0$, such that for all $0\leqslant\sigma\leqslant\sigma_0$, it holds that the closed-loop PETC system (10) with $w=0$ is GES.*

Proof. Since $A-LC$ and $A+BK$ being Schur implies that A_p is Schur, it is known that (12) has a finite ℓ_2-gain from v to s. In fact, there is a $P\succ0$ such that for $V(\chi)=\chi^\top P\chi$ along the solutions of (12) it holds that

$$V(\chi_{k+1})-V(\chi_k)\leqslant-\varepsilon\|\chi_k\|^2-\|s_k\|^2+\theta^2\|v_k\|^2 \quad (28)$$

for some $\theta\in\mathbb{R}_{\geqslant0}$ and some small $\varepsilon\in\mathbb{R}_{>0}$. Due to (16), it is obvious that

$$V(\chi_{k+1})-V(\chi_k)\leqslant-\varepsilon\|\chi_k\|^2-(1-\theta^2\sigma_s^2)\|s_k\|^2. \quad (29)$$

Hence, using similar arguments as in the proof of Theorem 4.5, we obtain GES when $1-\theta^2\sigma_s^2\geqslant0$, which shows that indeed $0\leqslant\sigma_s\leqslant\sigma_0:=\theta^{-1}$ is a sufficient condition for GES of (10) when $w=0$. □

To obtain the largest (minimum) inter-transmission times, while still guaranteeing GES using Theorem 4.6, it follows from its proof that σ_s should be as large as possible and thus that θ should be minimized, while satisfying (28). This corresponds to determining the true ℓ_2-gain θ^* of (12) from v to s (with $w=0$). Since the model (12) is LTI, this amounts to computing its $\mathcal{H}_\infty$-norm given by $\sup_{\omega\in[0,\pi]}\|S_p(e^{j\omega}I-A_p)^{-1}V_p\|$. Hence, when $A+BK$ and $A-LC$ are Schur, it can be shown that GES of the closed-loop PETC system (10) with $w=0$ is guaranteed for any $0\leqslant\sigma_s<\theta^*=1/(\sup_{\omega\in[0,\pi]}\|S_p(e^{j\omega}I-A_p)^{-1}V_p\|)$. Consequently, a standard $\mathcal{H}_\infty$-norm calculation for a linear system provides stability bounds in terms of σ_s for the PETC closed-loop system.

4.3. Discussion

Following Heemels et al. (2011, 2013), it can be shown that if Theorem 4.5 guarantees GES ($w=0$) and a certain upper bound on the ℓ_2-gain for some value of σ_s, then Theorem 4.4 also guarantees these properties. Hence, the PWL system approach is never outperformed by the PL system approach, and, in fact, in most cases results in improved ETMs, as we will see also in the numerical examples in Section 8. One reason is that the PWL system approach exactly models the closed-loop PETC system, while the PL system approach does not. A second reason is that Theorem 4.5 exploits *common* quadratic Lyapunov functions, which can be a source of conservatism, while Theorem 4.4 allows for piecewise quadratic Lyapunov functions. Based on this observation, one might wonder why the PL system approach is still of interest. The reason is threefold.

First of all, the PL system approach provides a justification for so-called *emulation-based design*. Such a justification cannot be obtained in a straightforward manner from the PWL system approach. Indeed, Theorem 4.5 reveals that if the original *time-triggered* system given by (15) and (17) is GES for $w = 0$, and has a certain ℓ_2-gain γ^* from w to z (equal to the $\mathcal{H}_\infty$-norm $\sup_{\omega\in[0,\pi]} \|C_{z\chi}(e^{j\omega}I - A_p)^{-1}E_p + D_z\|$), then for any $\gamma > \gamma^*$, there are ε, α and θ (the former one possibly very small and the latter one possibly very large) such that (24) holds for some $P \succ 0$. This shows that if the standard time-triggered implementation of the observer-based controller has certain performance properties in terms of an ℓ_2-gain γ^*, the PETC implementation can be designed in order to preserve these properties as closely as desired by a proper choice of $0 < \sigma_s \leqslant \theta^{-1}$. This forms indeed a strong justification for emulation-based design. In an emulation-based design, first the controller parameters are chosen using standard discrete-time synthesis tools (assuming a periodic time-triggered implementation and thus ignoring the event-triggered implementation) such that closed-loop stability and a desirable $\mathcal{H}_\infty/\ell_2$-gain from w to z are obtained. Subsequently, the parameters of the ETMs are chosen by trading some of the original ℓ_2-gain γ^* (larger γ) for less communication (smaller θ and, thus, larger σ_s). The analysis results for the PL system approach show that there is no "discontinuity" in the ℓ_2-gain as a function of σ_s in 0 when changing from the time-triggered implementation to the event-triggered setup, and hence a gradual increase in ℓ_2-gain occurs when increasing σ_s.

Secondly, although the PWL and the PL system approaches both result in LMIs, for the PWL system this is only the case after the parameter σ_s of the ETM is fixed, and, hence, a line search in σ_s is necessary to find the largest value of σ_s for which GES ($w = 0$) and/or a certain upper bound γ on the ℓ_2-gain is guaranteed. For the PL system approach (27) is directly an LMI in the free parameters and θ can be minimized for a given value of γ. Besides in case of GES only, a simple $\mathcal{H}_\infty$-norm calculation already provides direct bounds $0 \leqslant \sigma_s < \theta^* = 1/(\sup_{\omega\in[0,\pi]} \|S_p(e^{j\omega}I - A_p)^{-1}V_p\|)$ guaranteeing closed-loop stability. Note that if the goal is to find the Pareto optimal curve (σ_s, γ), based on the PWL system approach one can vary the σ_s and then minimize γ subject to the LMIs (21) (for fixed value of σ_s). This procedure is of a similar computational nature as computing such curves for the PL system approach, although for the latter the LMIs are of smaller size and have less free variables. Hence, overall the PL system approach is (somewhat) computationally friendlier than the PWL system approach.

Finally, note that the PL system approach is related to the existing literature, see, e.g., Eqtami et al. (2010), Lehmann and Lunze (2011), Lunze and Lehmann (2010), Tabuada (2007), and Yook et al. (2002), which study the closed-loop system based on perturbations of the periodic time-triggered controller implementation instead of on direct models as the PWL model, and it is appropriate to establish this connection.

5. Including controller-to-actuator ETMs

In this section, we will consider the network configuration as shown in Fig. 1, in which there is also a shared communication network between the controller and actuator system for which the communication and/or energy resources are limited. For this case, we propose to combine the ideas of Section 3 for the S–C ETM with a new class of ETMs for the C–A channel. The C–A ETMs will be based on the transmission of model-based predictions of future control values to a buffer located in the actuator system. A copy of this buffer will be situated at the controller system. As long as the controller system considers the buffered control values to be adequate, no updates are being transmitted. In this way, a C–A ETM can be introduced based on the (relative) difference between the buffered control values at the actuator system and the newly computed control values at the controller system. The controller system transmits updated current and future control values to the actuator system, only in the case this difference becomes too large.

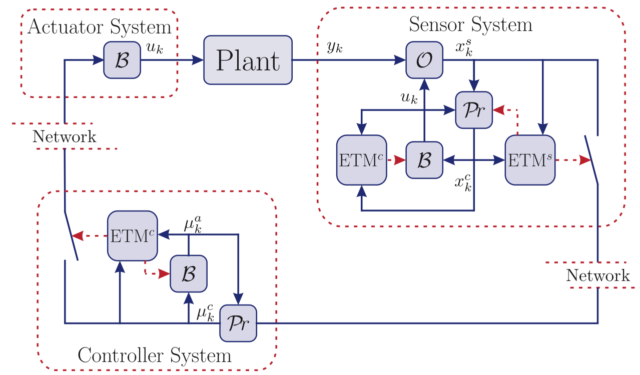


Fig. 4. Model-based PETC with S–C and C–A ETMs.

Remark 5.1. In case the actuators have sufficient computational resources themselves, one might consider using the scheme proposed in Section 3 collocating the controller and the actuators, thereby essentially removing the controller-to-actuator channel.

5.1. Proposed solution

To formalize the described strategy, consider Fig. 4. The operation of the sensor and controller system is exactly the same as in Section 3 with the only difference that the "downstream" modifications that influence the values of u_k are copied in the "upstream" blocks, i.e., the control and sensor systems. In fact, $\mathcal{O}$ and $\mathcal{P}r$ remain the same as in (2) and (3), respectively. To model the idea of sending predicted future control values, we introduce the variable $\mu_k^a \in \mathbb{R}^{Nn_u}$, which reflects the control values being stored in the buffer $\mathcal{B}$ in the actuator system. If no updated information is received at times $k, k+1, \ldots, k+N-1$, μ_k^a contains the N control values that will be implemented as the control values $u_k, u_{k+1}, \ldots, u_{k+N-1}$ at time $k, k+1, \ldots, k+N-1$, respectively. However, if at time k the controller system sends updated future control information, a data packet containing $\mu_k^c \in \mathbb{R}^{Nn_u}$ is received by the actuator system, which contains a new value for the current control signal u_k and $N-1$ predictions of future control values for times $k+1, k+2, \ldots, k+N-1$. Using the matrices $\Pi \in \mathbb{R}^{n_u\times Nn_u}$ and $\Gamma \in \mathbb{R}^{Nn_u\times Nn_u}$ given by

$$\Pi = [I \;\; 0 \;\; \cdots \;\; 0], \qquad \Gamma = \begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ 0 & 0 & I & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & I \\ 0 & 0 & \cdots & 0 & I \end{bmatrix} \tag{30}$$

with each matrix 0 and I of size $n_u \times n_u$, we can describe the above setup for $k \in \mathbb{N}$ by

$$\mathcal{B} : u_k = \begin{cases} \Pi\mu_k^c, & \text{when } \mu_k^c \text{ is sent} \\ \Pi\mu_k^a, & \text{when } \mu_k^c \text{ is not sent} \end{cases} \tag{31a}$$

$$\mu_{k+1}^a = \begin{cases} \Gamma\mu_k^c, & \text{when } \mu_k^c \text{ is sent} \\ \Gamma\mu_k^a, & \text{when } \mu_k^c \text{ is not sent.} \end{cases} \tag{31b}$$

Remark 5.2. Note that we opted here for a "hold strategy" beyond the prediction horizon $N-1$ in the buffer, see the last block row of Γ in (30). In case it is preferable to use a "zero strategy" after the prediction horizon, i.e., $u_{k+N}=0$ when the last transmission of control values was done at time $k\in\mathbb{N}$, the last block row of Γ should be replaced by the zero block row and the analysis and design techniques presented below can still be applied without any further modifications.

To obtain μ_k^c, the controller system produces at each discrete time $k\in\mathbb{N}$ the best estimate μ_k^c of current and future control values using the model of the plant (1) (assuming that $w=0$) and its best available estimate of the state of the plant x_k, which is x_k^c in case no transmission of x_k^s occurs and is x_k^s in case such a transmission does occur. This is summarized as

$$\mu_k^c=\begin{cases}\Phi x_k^s, & \text{when } x_k^s \text{ is sent}\\ \Phi x_k^c, & \text{when } x_k^s \text{ is not sent}\end{cases} \tag{32}$$

with $\Phi:=[K^\top\ (K(A+BK))^\top\ \cdots\ (K(A+BK)^{N-1})^\top]^\top$. The update x_{k+1}^c of the estimated state x_k^c is given in (3).

To complete the model-based PETC strategy, the only items missing are the ETMs in the sensor-to-controller and the controller-to-actuator channels. The S–C ETM is chosen to be the same as in Section 3 and, thus, is given by (4). The C–A ETM at time $k\in\mathbb{N}$ is given by

$$\text{ETM}^c:\ \mu_k^c \text{ is sent } \Leftrightarrow \|\Pi\mu_k^c-\Pi\mu_k^a\|>\sigma_c\|\Pi\mu_k^c\|, \tag{33}$$

indicating that the control packet containing current and predicted future control values is transmitted only when the currently computed control value $\Pi\mu_k^c$ deviates too much from the current control value $\Pi\mu_k^a$ in the buffer.

Remark 5.3. In case we select $N=1$ indicating that there are no predictions of future control values, (31b) reduces to $\mu_{k+1}^a=u_k$, and the ETM (33) reduces to

$$\mu_k^c \text{ is sent } \Leftrightarrow \|\mu_k^c-u_{k-1}\|>\sigma_c\|\mu_k^c\|, \tag{34}$$

which is similar to the baseline strategy as discussed in Section 3.3, but now applied in the C–A channel. The baseline C–A ETM as in (34) was used before in Donkers and Heemels (2012), Heemels et al. (2011), and Li and Lemmon (2011), and is thus a special case of what is proposed here by taking $N=1$.

Remark 5.4. In Bernardini and Bemporad (2012) a "*reverse*" communication-saving ETM was introduced in the S–C channel for model predictive controllers (MPC), which is related to the C–A ETM as proposed in this paper. In particular, Bernardini and Bemporad (2012) proposes to install buffers containing future predicted *outputs* at the sensors, and only transmit current sensor readings if there is a mismatch between the predicted values stored in the buffers and the current sensor readings. The current paper provides new perspectives and analysis frameworks for the setup in Bernardini and Bemporad (2012) as well.

5.2. Closed-loop modeling

We will now derive a closed-loop model for the proposed solution based on (1)–(3), and (31)–(33) in terms of a PWL system model. As before, we will also provide a PL system description.

5.2.1. Piecewise linear model

To obtain a closed-loop model of the described strategy, we will use the state variable $\zeta_k:=\operatorname{col}(x_k,e_k^s,e_k^c,\mu_k^a)$ in which $e_k^s:=x_k^s-x_k$ and $e_k^c:=x_k^c-x_k$, as before, for $k\in\mathbb{N}$. This leads after some careful derivations to

$$\zeta_{k+1}=\begin{cases}\mathcal{A}_{11}\zeta_k+\mathcal{E}w_k, & \text{if both } x_k^s \text{ and } \mu_k^c \text{ are sent}\\ \mathcal{A}_{12}\zeta_k+\mathcal{E}w_k, & \text{if } x_k^s \text{ is sent and } \mu_k^c \text{ is not sent}\\ \mathcal{A}_{21}\zeta_k+\mathcal{E}w_k, & \text{if } x_k^s \text{ is not sent and } \mu_k^c \text{ is sent}\\ \mathcal{A}_{22}\zeta_k+\mathcal{E}w_k, & \text{if both } x_k^s \text{ and } \mu_k^c \text{ are not sent}\end{cases} \tag{35}$$

with

$$\mathcal{A}_{11}=\begin{bmatrix}A+BK & BK & 0 & 0\\ 0 & A-LC & 0 & 0\\ 0 & A & 0 & 0\\ \Gamma\Phi & \Gamma\Phi & 0 & 0\end{bmatrix},\quad \mathcal{A}_{12}=\begin{bmatrix}A & 0 & 0 & B\Pi\\ 0 & A-LC & 0 & 0\\ 0 & A & 0 & 0\\ 0 & 0 & 0 & \Gamma\end{bmatrix}, \tag{36a}$$

$$\mathcal{A}_{21}=\begin{bmatrix}A+BK & 0 & BK & 0\\ 0 & A-LC & 0 & 0\\ 0 & 0 & A & 0\\ \Gamma\Phi & 0 & \Gamma\Phi & 0\end{bmatrix},\quad \mathcal{A}_{22}=\begin{bmatrix}A & 0 & 0 & B\Pi\\ 0 & A-LC & 0 & 0\\ 0 & 0 & A & 0\\ 0 & 0 & 0 & \Gamma\end{bmatrix}, \tag{36b}$$

$$\mathcal{E}=[E^\top\ -E^\top\ -E^\top\ 0]^\top. \tag{36c}$$

To obtain the complete closed-loop PWL model, we include the regional information for each linear submodel based on the ETMs. While the ETC in (4) can be straightforwardly represented as

$$x_k^s \text{ is sent } \Leftrightarrow \|x_k^s-x_k^c\|>\sigma_s\|x_k^s\| \Leftrightarrow \zeta_k^\top Q_s\zeta_k>0 \tag{37}$$

$$\text{with } Q_s=\begin{bmatrix}-\sigma_s^2 I & -\sigma_s^2 I & 0 & 0\\ -\sigma_s^2 I & (1-\sigma_s^2)I & -I & 0\\ 0 & -I & I & 0\\ 0 & 0 & 0 & 0\end{bmatrix}, \tag{38}$$

the ETM (33) has to be considered for two different cases. Namely, for either $\zeta_k^\top Q_s\zeta_k>0$ (implying that $\Pi\mu_k^c=Kx_k^s=Kx_k+Ke_k^s$), or $\zeta_k^\top Q_s\zeta_k\leqslant 0$ (implying that $\Pi\mu_k^c=Kx_k^c=Kx_k+Ke_k^c$). By using the definitions

$$Q_c=\begin{bmatrix}(1-\sigma_c^2)K^\top K & (1-\sigma_c^2)K^\top K & 0 & -K^\top\Pi\\ (1-\sigma_c^2)K^\top K & (1-\sigma_c^2)K^\top K & 0 & -K^\top\Pi\\ 0 & 0 & 0 & 0\\ -\Pi^\top K & -\Pi^\top K & 0 & \Pi^\top\Pi\end{bmatrix} \tag{39a}$$

$$\bar{Q}_c=\begin{bmatrix}(1-\sigma_c^2)K^\top K & 0 & (1-\sigma_c^2)K^\top K & -K^\top\Pi\\ 0 & 0 & 0 & 0\\ (1-\sigma_c^2)K^\top K & 0 & (1-\sigma_c^2)K^\top K & -K^\top\Pi\\ -\Pi^\top K & 0 & -\Pi^\top K & \Pi^\top\Pi\end{bmatrix}, \tag{39b}$$

we can observe that according to (33), we have that

$$\mu_k^c \text{ is sent } \Leftrightarrow \zeta_k^\top Q_c\zeta_k>0 \tag{40}$$

in case that $\zeta_k^\top Q_s\zeta_k>0$, and that we have that

$$\mu_k^c \text{ is sent } \Leftrightarrow \zeta_k^\top\bar{Q}_c\zeta_k>0 \tag{41}$$

in case that $\zeta_k^\top Q_s\zeta_k\leqslant 0$. This leads to the following PWL system describing the closed-loop PETC system:

$$\zeta_{k+1}=\begin{cases}\mathcal{A}_{11}\zeta_k+\mathcal{E}w_k, & \text{if } \zeta_k^\top Q_s\zeta_k^\top>0,\ \zeta_k^\top Q_c\zeta_k>0\\ \mathcal{A}_{12}\zeta_k+\mathcal{E}w_k, & \text{if } \zeta_k^\top Q_s\zeta_k^\top>0,\ \zeta_k^\top Q_c\zeta_k\leqslant 0\\ \mathcal{A}_{21}\zeta_k+\mathcal{E}w_k, & \text{if } \zeta_k^\top Q_s\zeta_k^\top\leqslant 0,\ \zeta_k^\top\bar{Q}_c\zeta_k>0\\ \mathcal{A}_{22}\zeta_k+\mathcal{E}w_k, & \text{if } \zeta_k^\top Q_s\zeta_k^\top\leqslant 0,\ \zeta_k^\top\bar{Q}_c\zeta_k\leqslant 0.\end{cases} \tag{42}$$

5.2.2. Perturbed linear model

As we have done in Section 3.4, we will also derive a PL model. This model will be based on the state variable $\chi_k = \text{col}(x_k, e_k^s)$, $k \in \mathbb{N}$, and can be obtained from the PWL system (42). The resulting PL model is given by

$$\begin{cases} \chi_{k+1} = A_p\chi_k + V_p^{sc}v_k^{sc} + V_p^{ca}v_k^{ca} + E_pw_k \\ s_k = S_p\chi_k \end{cases} \quad (43)$$

with A_p, E_p and S_p as in (13), $V_p^{sc} = \left[-(BK)^\top \; 0\right]^\top$, $V_p^{ca} = \left[-B^\top \; 0\right]^\top$, and v_k^{sc} and v_k^{ca}, $k \in \mathbb{N}$, are considered as perturbations in (43). In fact, these are given as

$$v_k^{sc} = \begin{cases} e_k^s - e_k^c, & \text{if } \|x_k^s - x_k^c\| \leqslant \sigma_s\|x_k^s\| \\ 0, & \text{otherwise,} \end{cases} \quad (44)$$

$$v_k^{ca} = \begin{cases} \Pi\mu_k^c - \Pi\mu_k^a, & \text{if } \|\Pi\mu_k^c - \Pi\mu_k^a\| \leqslant \sigma_c\|\Pi\mu_k^c\| \\ 0, & \text{otherwise,} \end{cases} \quad (45)$$

which establishes the connection to (42).

As in Section 4.2, when we take $v_k^{sc} = 0$ and $v_k^{ca} = 0$, $k \in \mathbb{N}$, in (43), we obtain the time-triggered loop (without buffer), in which at each discrete-time step $k \in \mathbb{N}$ both x_k^s and μ_k^c are transmitted. As such, this situation corresponds to the system (1) being controlled by an observer-based controller consisting of $\mathcal{O}$ and $u_k = Kx_k^s$, $k \in \mathbb{N}$, in a conventional periodic time-triggered implementation. The PL model (43) indicates how the standard time-triggered system is perturbed by introducing the *S*–*C* and *C*–*A* ETMs in the loop.

6. Analysis of PETC with *S*–*C* and *C*–*A* ETMs

In this section, we carry out the stability and performance analysis of the PETC strategies with both *S*–*C* and *C*–*A* ETMs using the PWL and the PL system approaches.

6.1. PWL system approach

To obtain compact LMI-based stability conditions for stability and ℓ_2-gain guarantees based on the PWL system approach, we will adopt the definition $\Delta Q_c := \bar{Q}_c - Q_c$.

Theorem 6.1. *The closed-loop PETC system as captured in (42) and (17) has an ℓ_2-gain smaller than or equal to $\gamma \in \mathbb{R}_{\geqslant 0}$, and (42) with $w = 0$ is GES, if there exist matrices P_{ij}, $i, j \in \{1, 2\}$, scalars $\varepsilon \in \mathbb{R}_{>0}$, α_{ijlm}, β_{ijlm}, $\bar{\alpha}_{ijlm}$, $\bar{\beta}_{ijlm} \in \mathbb{R}_{\geqslant 0}$, $i, j, l, m \in \{1, 2\}$, and κ_{ij}, $\bar{\kappa}_{ij} \in \mathbb{R}_{\geqslant 0}$, $i, j \in \{1, 2\}$, satisfying for $i, j, l, m \in \{1, 2\}$*

$$\begin{bmatrix} \Omega_{ijlm}^{\zeta\zeta} & \Omega_{ijlm}^{\zeta w} & C_{z\zeta}^\top \\ \star & \Omega_{ijlm}^{ww} & D_z^\top \\ \star & \star & I \end{bmatrix} \succeq 0, \quad (46a)$$

and

$$P_{ij} + \Psi(\kappa_{ij}, \bar{\kappa}_{ij}, i, j) \succ 0, \quad (46b)$$

where $C_{z\zeta} = [C_{zx} \; 0 \; 0 \; 0]$, and for $i, j, l, m \in \{1, 2\}$

$$\begin{aligned} \Omega_{ijlm}^{\zeta\zeta} &= P_{ij} - \mathcal{A}_{ij}^\top P_{lm}\mathcal{A}_{ij} - \varepsilon I + \Psi(\alpha_{ijlm}, \bar{\alpha}_{ijlm}, i, j) \\ &\quad + \mathcal{A}_{ij}^\top \Psi(\beta_{ijlm}, \bar{\beta}_{ijlm}, l, m)\mathcal{A}_{ij} \end{aligned} \quad (47a)$$

$$\Omega_{ijlm}^{\zeta w} = -\mathcal{A}_{ij}^\top P_{lm}\mathcal{E} + \mathcal{A}_{ij}^\top \Psi(\beta_{ijlm}, \bar{\beta}_{ijlm}, l, m)\mathcal{E} \quad (47b)$$

$$\Omega_{ijlm}^{ww} = \gamma^2 I - \mathcal{E}^\top P_{lm}\mathcal{E} + \mathcal{E}^\top \Psi(\beta_{ijlm}, \bar{\beta}_{ijlm}, l, m)\mathcal{E} \quad (47c)$$

in which

$$\Psi(\alpha, \bar{\alpha}, i, j) = \alpha(-1)^i Q_s + \bar{\alpha}(-1)^j \left(Q_c + \frac{1}{2}((-1)^i + 1)\Delta Q_c\right).$$

Proof. The proof is similar to the proof of Theorem 4.4 and is, therefore, omitted. □

6.2. Perturbed linear system approach

Based on the PL system model, we can obtain the following stability and performance result.

Theorem 6.2. *Suppose that there exists a positive definite matrix P such that the storage function $V(\chi) = \chi^\top P\chi$ satisfies along the solutions of (43) and (17) that*

$$\begin{aligned} V(\chi_{k+1}) - V(\chi_k) &\leqslant -\varepsilon\|\chi_k\|^2 - \|s_k\|^2 + \theta_s^2\|v_k^{sc}\|^2 \\ &\quad + \theta_c^2\|v_k^{ca}\|^2 - \alpha\|z_k\|^2 + \alpha\gamma^2\|w_k\|^2 \end{aligned} \quad (48)$$

for some $\theta_s, \theta_c \in \mathbb{R}_{\geqslant 0}$, $\gamma \in \mathbb{R}_{\geqslant 0}$ and $\alpha, \varepsilon \in \mathbb{R}_{>0}$. Then for all σ_s, $\sigma_c \in \mathbb{R}_{\geqslant 0}$ satisfying

$$\theta_c^2(\sigma_c\|K\|(1 + \sigma_s))^2 + \theta_s^2\sigma_s^2 \leqslant 1 \quad (49)$$

the closed-loop PETC system captured in (42) with (17) has an ℓ_2-gain from w to z smaller than or equal to γ and (42) with $w = 0$ is GES.

Proof. We observe first that

$$\|v_k^{sc}\| \leqslant \sigma_s\|x_k^s\| = \sigma_s\|x_k + e_k^s\| = \sigma_s\|s_k\|. \quad (50)$$

In addition, as $\Pi\Phi = K$, we have in case that x_k^s is sent, i.e., $\|x_k^s - x_k^c\| > \sigma_s\|x_k^s\|$, that

$$\|v_k^{ca}\| \leqslant \sigma_c\|Kx_k + Ke_k^s\| \leqslant \sigma_c\|K\|\|s_k\| \quad (51)$$

and in case that x_k^s is not sent, i.e., $\|x_k^s - x_k^c\| \leqslant \sigma_s\|x_k^s\|$

$$\begin{aligned} \|v_k^{ca}\| &\leqslant \sigma_c\|Kx_k + Ke_k^c\| \\ &\leqslant \sigma_c\|Kx_k + Ke_k^s\| + \sigma_c\|Ke_k^c - Ke_k^s\| \\ &\leqslant \sigma_c\|K\|\|s_k\| + \sigma_c\|Kx_k^c - Kx_k^s\| \\ &\leqslant \sigma_c\|K\|\|s_k\| + \sigma_c\sigma_s\|K\|\|x_k^s\| \\ &= \sigma_c\|K\|(1 + \sigma_s)\|s_k\|, \end{aligned} \quad (52)$$

where we used that $e_k^c - e_k^s = x_k^c - x_k^s$ in the third inequality. Combining (51) and (52), we obtain

$$\|v_k^{ca}\| \leqslant \sigma_c\|K\|(1 + \sigma_s)\|s_k\|. \quad (53)$$

Substituting (50) and (53) in (48) and using (49) lead to an ℓ_2-gain smaller than or equal to γ by standard ℓ_2-gain arguments. In addition, for $w = 0$, it follows similarly that $V(\chi_{k+1}) - V(\chi_k) \leqslant -\varepsilon\|\chi_k\|^2$, $k \in \mathbb{N}$. Since there exist $c_1, c_2 \in \mathbb{R}_{>0}$ such that $c_1\|\chi\|^2 \leqslant V(\chi) \leqslant c_2\|\chi\|^2$ for all χ, it follows that there are $c \in \mathbb{R}_{\geqslant 0}$ and $\rho \in [0, 1)$ such that $\|\chi_k\| \leqslant c\rho^k\|\chi_0\|$ along all trajectories of (42) with $w = 0$. To prove GES of (42) with $w = 0$, we have to show the existence of $\bar{c} \in \mathbb{R}_{\geqslant 0}$ and $\bar{\rho} \in [0, 1)$ such that $\|\zeta_k\| \leqslant \bar{c}\bar{\rho}^k\|\zeta_0\|$, $k \in \mathbb{N}$. Since $\zeta_k = \text{col}(\chi_k, e_k^c, \mu_k^a)$, only the exponential decay of e_k^c and μ_k^a is left to show. In fact, proving exponential decay of e_k^c can be obtained in a similar manner as in the proof of Theorem 4.5, which showed that there is a $\hat{g} \in \mathbb{R}_{\geqslant 0}$ such that

$$\|e_{k+1}^c\| \leqslant \hat{g}\|\chi_k\|, \quad k \in \mathbb{N}. \quad (54)$$

Hence, to complete the proof, it remains to show that μ_k^a decays exponentially.

To do so, consider $k \in \mathbb{N}_{\geqslant N}$ and distinguish two cases. The first case corresponds to the situation in which for all $l \in \{k-N+1, k-N+2, \ldots, k\}$ μ_l^c is not sent. In this case, due to (33) it holds that $\|\Pi\mu_l^c - \Pi\mu_l^a\| \leqslant \sigma_c\|\Pi\mu_l^c\|$, $l \in \{k-N+1, k-N+2, \ldots, k\}$. Hence, for $l \in \{k-N+1, k-N+2, \ldots, k\}$

$$\begin{aligned} \|\Pi\mu_l^a\| &\leqslant \|\Pi\mu_l^c\| + \|\Pi\mu_l^c - \Pi\mu_l^a\| \leqslant (1 + \sigma_c)\|\Pi\mu_l^c\| \\ &\overset{(32),(54)}{\leqslant} g\max\{\|\chi_{l-1}\|, \|\chi_l\|\} \leqslant gc\rho^{l-1}\|\chi_0\| \end{aligned} \quad (55)$$

for some $g \in \mathbb{R}_{\geqslant 0}$. Since for $l \in \{k-N+1, k-N+2, \ldots, k\}\mu_l^c$ is not sent, it holds due to the second case in (31b) that $\mu_{k+1}^a = \text{col}(\Pi\mu_k^a, \Pi\mu_k^a, \ldots, \Pi\mu_k^a)$. Using now (55) for $l = k$ gives

$$\|\mu_{k+1}^a\| = \sqrt{N}\|\Pi\mu_k^a\| \leqslant \sqrt{N}gc\rho^{k-1}\|\chi_0\|. \tag{56}$$

Let us consider the second case in which for some $l \in \{k-N+1, k-N+2, \ldots, k\}\mu_l^c$ is sent to the actuator system. Let l^* be the largest value in $\{k-N+1, k-N+2, \ldots, k\}$ for which μ_l^c is sent to the actuator system. Then it holds for some $\bar{g} \in \mathbb{R}_{\geqslant 0}$ (only depending on Γ and Φ) that

$$\begin{aligned}\|\mu_{l^*+1}^a\| &\overset{(31b)}{=} \|\Gamma\mu_{l^*}^c\| \overset{(32),(54)}{\leqslant} \bar{g}\max\{\|\chi_{l^*-1}\|, \|\chi_{l^*}\|\} \\ &\leqslant \quad \bar{g}c\rho^{l^*-1}\|\chi_0\|.\end{aligned} \tag{57}$$

From this, it follows that for some $\tilde{g} \in \mathbb{R}_{\geqslant 0}$

$$\|\mu_{k+1}^a\| \overset{(31b)}{=} \|\Gamma^{l^*-k}\mu_{l^*+1}^a\| \overset{(57)}{\leqslant} \tilde{g}\rho^{l^*-1}\|\chi_0\| \leqslant \tilde{g}\rho^{k-N}\|\chi_0\|. \tag{58}$$

Hence, combining (56) and (58) shows the required exponential decay of μ_k^a, thereby completing the proof. □

In case $A+BK$ and $A-LC$ are both Schur, it is clear that also the matrix A_p is Schur. In this case, there always exist finite θ_s and θ_c satisfying the hypothesis of the theorem above, which provide reasonable conditions on the feasibility of (48). Note that these conditions can easily be translated into LMIs. From the expression (48), it can also be deduced that in case the standard time-triggered implementation of the observer-based controller (modeled by taking $v_k^{sc} = 0$ and $v_k^{ca} = 0$, $k \in \mathbb{N}$, in (43)) has a certain ℓ_2-gain γ^* from w to z, and $A+BK$ and $A-LC$ are Schur, then for any $\gamma > \gamma^*$ there exist values of $\theta_s, \theta_c \in \mathbb{R}_{\geqslant 0}$, and $\alpha, \varepsilon \in \mathbb{R}_{>0}$ such that (48) holds. In other words, as in Section 4.3, also in this context it follows that the PETC implementation can always be chosen to arbitrarily closely resemble the performance properties of the standard time-triggered periodic implementation of the observer-based controller. In addition, an emulation-based design can be applied. Once the observer and feedback gains are synthesized assuming a standard periodic time-triggered implementation, one can tune the ETMs (σ_s and σ_c) by selecting larger values for $\gamma > \gamma^*$ leading to smaller values of $\theta_s, \theta_c \in \mathbb{R}_{\geqslant 0}$ for which (48) is feasible, and thus larger values for $\sigma_s, \sigma_c \in \mathbb{R}_{\geqslant 0}$ resulting in less utilization of the communication resources. These important tradeoffs will be illustrated using numerical examples in Section 8.

Remark 6.3. Obviously, the feasibility of the PL system approach as in Theorem 6.2 does not depend on the length N of the buffer. Although larger values of N do not lead to better performance guarantees (at least not when using the PL system approach), they potentially lead to a further reduction of the number of transmissions. Hence, the larger the buffer can be chosen (as far as practical constraints in terms of computation and communication allow), the larger the reduction of resource usage can be expected.

7. Decentralized PETC and ETMs

In this section, we will show how the ideas presented in the previous sections can be implemented in a decentralized fashion. As already mentioned in the introduction, this setup is particularly relevant for large-scale plants in which sensors, actuators and controllers are physically distributed over a wide area. In fact, centralized ETMs and controllers can be prohibitive in this case, as the conditions that generate events and the controllers would need access to all the plant or controller outputs at every sampling time. Also to avoid putting a large computational burden on the ETMs, it might be undesirable to use the complete plant model

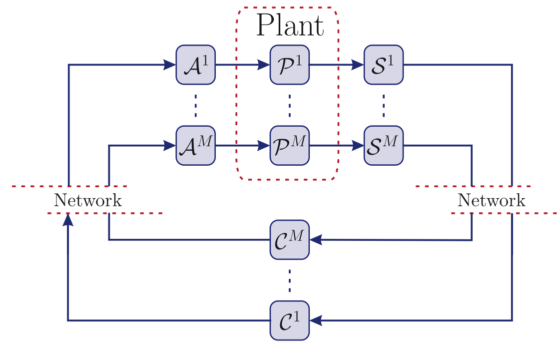


Fig. 5. Setup of decentralized PETC with ETM in sensor-to-controller and controller-to-actuator channel.

in these ETMs. To provide a decentralized solution to both issues, let us consider a large-scale plant that consists of M interacting subsystems given for $i \in \{1, \ldots, M\}$ by

$$\mathcal{P}^i : \begin{cases} x_{k+1}^i = A_i x_k^i + B_i u_k^i + \sum_{j\neq i} A_{ij} x_k^j + E_i w_k \\ y_k^i = C_i x_k^i \end{cases} \tag{59}$$

with $x_k^i \in \mathbb{R}^{n_{x,i}}$, $u_k^i \in \mathbb{R}^{n_{u,i}}$, and $y_k^i \in \mathbb{R}^{n_{y,i}}$ the state, control input and measured output, corresponding to subsystem $\mathcal{P}^i$, and $w_k \in \mathbb{R}^{n_w}$ is the (global) disturbance at time $k \in \mathbb{N}$. Note that in the dynamics of (59) the coupling terms $\sum_{j\neq i} A_{ij} x_k^j$ are present, representing the influence of subsystems $\mathcal{P}^j, j \neq i$, on subsystem $\mathcal{P}^i$.

We assume that each subsystem $\mathcal{P}^i$ has its own control loop consisting of a sensor system $\mathcal{S}^i$, a controller system $\mathcal{C}^i$, and an actuator system $\mathcal{A}^i$, $i \in \{1, \ldots, M\}$, as in Fig. 5, which will be chosen in line with Section 5. In particular, each sensor system $\mathcal{S}^i$ comprises a local observer

$$\mathcal{O}^i : x_{k+1}^{s,i} = A_i x_k^{s,i} + B_i u_k^i + L_i(y_k^i - C_i x_k^{s,i}). \tag{60}$$

This observer has a Luenberger structure based on the model (59) for $\mathcal{P}^i$ in which the coupling terms $\sum_{j\neq i} A_{ij} x_k^j$ are ignored.

Following the setup in Section 5, we install, for each subsystem, a buffer $\mathcal{B}^i$ as part of the i-th actuator system $\mathcal{A}^i$ with copies of $\mathcal{B}^i$ at the i-th control system $\mathcal{C}^i$ and the sensor system $\mathcal{S}^i$, $i \in \{1, \ldots, M\}$. Besides a copy of the buffer $\mathcal{B}^i$, the i-th control system $\mathcal{C}^i$ is also equipped with a predictor $\mathcal{P}r^i$ that provides an estimate $x_k^{c,i}$ of the state of subsystem i based on the information it receives from the sensor system $\mathcal{S}^i$. The predictor $\mathcal{P}r^i$ has the form

$$\mathcal{P}r^i : x_{k+1}^{c,i} = \begin{cases} A_i x_k^{c,i} + B_i u_k^i, & \text{when } x_k^{s,i} \text{ is not sent} \\ A_i x_k^{s,i} + B_i u_k^i, & \text{when } x_k^{s,i} \text{ is sent} \end{cases} \tag{61}$$

similar to (3). The i-th control system also computes current and $N_i - 1$ future control values collected in $\mu_k^{c,i} \in \mathbb{R}^{n_{u,i}N_i}$ based on the estimate $x_k^{c,i}$, in case $x_k^{s,i}$ is not sent, or on the basis of $x_k^{s,i}$ otherwise, i.e.,

$$\mu_k^{c,i} = \begin{cases} \Phi_i x_k^{s,i}, & \text{when } x_k^{s,i} \text{ is sent} \\ \Phi_i x_k^{c,i}, & \text{when } x_k^{s,i} \text{ is not sent} \end{cases} \tag{62}$$

with $\Phi_i := \begin{bmatrix} K_i^\top & (K_i(A_i + B_iK_i))^\top & \cdots & (K_i(A_i + B_iK_i)^{N_i-1})^\top \end{bmatrix}^\top$, in which $N_i \in \mathbb{N}_{\geqslant 1}$ denotes the prediction horizon of the future control values for the i-th subsystem. Finally, the update of the buffers is described by using the matrices $\Pi_i \in \mathbb{R}^{n_{u,i} \times N_i n_{u,i}}$ and

$\Gamma_i \in \mathbb{R}^{N_i n_{u,i} \times N_i n_{u,i}}$ given by

$$\Pi_i = [I \;\; 0 \;\; \cdots \;\; 0], \qquad \Gamma = \begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ 0 & 0 & I & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & I \\ 0 & 0 & \cdots & 0 & I \end{bmatrix}, \tag{63}$$

where each matrix I and 0 is now of dimension $n_{u,i} \times n_{u,i}$. Using these matrices, we can describe the above setup for $k \in \mathbb{N}$ by

$$\mathcal{B}^i : u_k^i = \begin{cases} \Pi_i \mu_k^{c,i}, & \text{when } \mu_k^{c,i} \text{ is sent} \\ \Pi_i \mu_k^{a,i}, & \text{when } \mu_k^{c,i} \text{ is not sent} \end{cases} \tag{64a}$$

$$\mu_{k+1}^{a,i} = \begin{cases} \Gamma_i \mu_k^{c,i}, & \text{when } \mu_k^{c,i} \text{ is sent} \\ \Gamma_i \mu_k^{a,i}, & \text{when } \mu_k^{c,i} \text{ is not sent.} \end{cases} \tag{64b}$$

To complete the setup, we define the ETMs in the i-th S–C channel and the i-th C–A channel by

$$\text{ETM}^{s,i} : x_k^{s,i} \text{ is sent} \Leftrightarrow \|x_k^{s,i} - x_k^{c,i}\| > \sigma_s^i \|x_k^{s,i}\|, \tag{65}$$

where $\text{ETM}^{s,i}$ uses a copy of $\mathcal{P}r^i$ at the i-th sensor system $\mathcal{S}^i$ to have $x_k^{c,i}$ available, and by

$$\text{ETM}^{c,i} : \mu_k^{c,i} \text{ is sent} \;\Leftrightarrow\; \|\Pi_i \mu_k^{c,i} - \Pi_i \mu_k^{a,i}\| > \sigma_c^i \|\Pi_i \mu_k^{c,i}\|, \tag{66}$$

in which $\mu_k^{a,i}$ is known at $\text{ETM}^{c,i}$ since the i-th controller system $\mathcal{C}^i$ runs a copy of $\mathcal{B}^i$.

Although a PWL system model can be derived as well and corresponding analysis can be carried out along the same lines as in Section 5, for space reasons we only provide the PL system model and corresponding stability and ℓ_2-gain analysis. To do so, we introduce the following matrices:

$$A_d = \text{diag}(A_1, \ldots, A_M), \qquad B = \text{diag}(B_1, \ldots, B_M);$$
$$C = \text{diag}(C_1, \ldots, C_M), \qquad E = [E_1^\top \cdots E_M^\top]^\top,$$
$$A = \begin{bmatrix} A_1 & A_{12} & \cdots & A_{1M} \\ A_{21} & A_2 & & A_{2M} \\ \vdots & & \ddots & \vdots \\ A_{M1} & A_{M2} & \cdots & A_M \end{bmatrix}, \qquad A_c := A - A_d.$$

Now using again the variable $\chi_k = \text{col}(x_k, e_k^s)$ with $x_k = \text{col}(x_k^1, \ldots, x_k^M)$, $x_k^s = \text{col}(x_k^{s,1}, \ldots, x_k^{s,M})$, $e_k^s = x_k^s - x_k$, $K = \text{diag}(K_1, \ldots, K_M)$ and $L = \text{diag}(L_1, \ldots, L_M)$, we obtain the PL system model given by

$$\begin{cases} \chi_{k+1} = A_p \chi_k + V_p^{sc} v_k^{sc} + V_p^{ca} v_k^{ca} + E_p w_k \\ s_k = S_p \chi_k \end{cases} \tag{67}$$

with

$$A_p = \begin{bmatrix} A + BK & BK \\ -A_c & A_d - LC \end{bmatrix}, \quad V_p^{sc} = \begin{bmatrix} -BK \\ 0 \end{bmatrix}, \quad V_p^{ca} = \begin{bmatrix} -B \\ 0 \end{bmatrix} \tag{68a}$$

$$E_p = \begin{bmatrix} E^\top & -E^\top \end{bmatrix}^\top, \qquad S_p = \begin{bmatrix} I & I \end{bmatrix}^\top \tag{68b}$$

and

$$v_k^{sc} = \text{col}(v_k^{sc,1}, \ldots, v_k^{sc,M}), \qquad v_k^{ca} = \text{col}(v_k^{ca,1}, \ldots, v_k^{ca,M}), \tag{69}$$

in which

$$v_k^{sc,i} = \begin{cases} e_k^{s,i} - e_k^{c,i}, & \text{if } \|x_k^{s,i} - x_k^{c,i}\| \leqslant \sigma_{s,i} \|x_k^{s,i}\| \\ 0, & \text{otherwise} \end{cases} \tag{70a}$$

$$v_k^{ca,i} = \begin{cases} \Pi_i \mu_k^{c,i} - \Pi_i \mu_k^{a,i}, \\ \quad \text{if } \|\Pi_i \mu_k^{c,i} - \Pi_i \mu_k^{a,i}\| \leqslant \sigma_{c,i} \|\Pi_i \mu_k^{c,i}\| \\ 0, \quad \text{otherwise,} \end{cases} \tag{70b}$$

where $e_k^{c,i} = x_k^{c,i} - x_k^i$, $k \in \mathbb{N}$, $i \in \{1, \ldots, M\}$.

The next result can be obtained by extending the reasoning in Theorem 4.5.

Theorem 7.1. *Suppose that there exists a positive definite matrix P such that the storage function $V(\chi) = \chi^\top P \chi$ satisfies along the solutions of (67) and (17) that*

$$V(\chi_{k+1}) - V(\chi_k) \leqslant -\varepsilon \|\chi_k\|^2 - \|s_k\|^2 + \sum_{i=1}^M \theta_{s,i}^2 \|v_k^{sc,i}\|^2 + \sum_{i=1}^M \theta_{c,i}^2 \|v_k^{ca,i}\|^2 - \alpha \|z_k\|^2 + \alpha \gamma^2 \|w_k\|^2 \tag{71}$$

for some $\gamma \in \mathbb{R}_{\geqslant 0}$, $\varepsilon, \alpha \in \mathbb{R}_{>0}$, and $\theta_{s,i}, \theta_{c,i} \in \mathbb{R}_{\geqslant 0}$, $i = 1, \ldots, M$. Then, for all $\sigma_{s,i}, \sigma_{c,i} \in \mathbb{R}_{\geqslant 0}$, $i \in \{1, \ldots, M\}$, satisfying

$$\sum_{i=1}^M \theta_{c,i}^2 (\sigma_{c,i} \|K_i\| (1 + \sigma_{s,i}))^2 + \theta_{s,i}^2 \sigma_{s,i}^2 < 1, \tag{72}$$

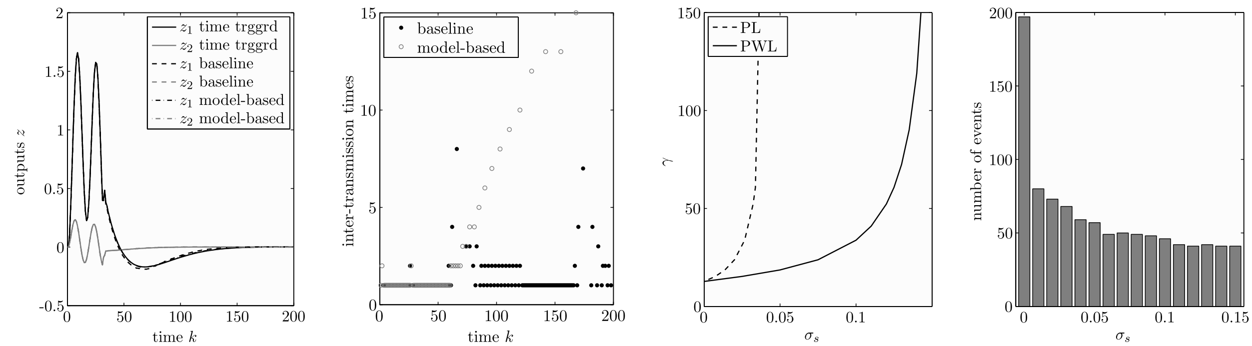
the closed-loop PETC system has an ℓ_2-gain from w to z smaller than or equal to γ and for $w = 0$ it is GES.

From this result, it follows that if the matrix $A_p = \begin{bmatrix} A + BK & BK \\ -A_c & A_d - LC \end{bmatrix}$ in (70) is Schur, then there always exist positive values $\sigma_{s,i}$ and $\sigma_{c,i}$ for which GES (for $w = 0$) and a finite ℓ_2-gain from w to z can be obtained. Interestingly, the matrix A_p being Schur is equivalent to the observer-based decentralized controller consisting of $\mathcal{O}^i$ and $u_k^i = K_i x_k^{s,i}$, $k \in \mathbb{N}$, $i \in \{1, \ldots, M\}$, stabilizing the overall plant consisting of (59), $i \in \{1, \ldots, M\}$, when implemented in a standard periodic time-triggered fashion (without ETMs). In case the decomposition of the plant as given by the subsystems (59), $i \in \{1, \ldots, M\}$ is "weakly coupled" in the sense that the matrices $A_{ij}, i \neq j$ are relatively small, it is more likely that this condition can be met. Ideas from Bauer, Donkers, van de Wouw, and Heemels (submitted) can be used to synthesize appropriate decentralized observer and state-feedback gains such that the matrix A_p is rendered Schur. Note that, as before, the ℓ_2-gain from w to z for this time-triggered periodic closed-loop system can be approached arbitrarily close by a corresponding PETC implementation by a suitable choice of $\sigma_{s,i}, \sigma_{c,i} \in \mathbb{R}_{>0}$, $i \in \{1, \ldots, M\}$.

8. Illustrative examples

In this section, we illustrate the newly proposed model-based PETC strategies for both the case with only a S–C ETM as well as the case with both S–C and C–A ETMs. For the former case, we will compare the newly proposed model-based PETC scheme with a periodic time-triggered controller and with the *baseline* PETC strategy as discussed in Section 3.3. For the latter case, we will discuss the influence of the parameters σ_s, σ_c and N on the upper bound on the ℓ_2-gain and the number of transmissions.

In both examples, we consider a well-known benchmark example in the networked control system literature, see, e.g., Donkers, Heemels, van de Wouw, and Hetel (2011), Heemels, Teel, van de Wouw, and Nešić (2010), Nešić and Teel (2004) and Walsh and Ye (2001), consisting of a model of a batch reactor. To arrive at a model of the form (1), we discretize the continuous-time plant model adopted in the aforementioned works using a zero-order hold with sampling period $h = 0.15$ and add a disturbance input



(a) The evolution of the outputs as function of time k for the time-triggered (trggrd), the baseline PETC and the model-based PETC strategy.

(b) The inter-transmission times as function of time k for the baseline and the model-based PETC strategy.

(c) The upper bound on the ℓ_2-gain as function of σ_s for the model-based PETC strategy.

(d) The number of transmissions as function of σ_s for the model-based PETC strategy.

Fig. 6. PETC with S–C ETM.

matrix, resulting in the matrices as in

$$A = \begin{bmatrix} 1.2943 & 0.0163 & 0.6935 & -0.5260 \\ -0.0740 & 0.5459 & -0.0217 & 0.0868 \\ 0.0986 & 0.4490 & 0.4520 & 0.4733 \\ -0.0049 & 0.4488 & 0.1104 & 0.8062 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.0146 & -0.1827 \\ 0.6413 & 0.0038 \\ 0.3782 & -0.3143 \\ 0.3777 & -0.0320 \end{bmatrix}, \tag{73}$$

$$C = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \qquad E = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \\ 0.2 & 0 \\ 0 & 0.1 \end{bmatrix},$$

For the PETC strategies discussed in this section, we use the observer and state feedback gains K and L as in (2) and (5), respectively, with the numerical values

$$K = \begin{bmatrix} 0.1780 & -0.3056 & 0.2677 & -0.2446 \\ 1.2729 & 0.3096 & 1.7253 & -1.4448 \end{bmatrix},$$

$$L = \begin{bmatrix} 1.5328 & -0.0649 & 0.2007 & 0.0665 \\ 0.0515 & 0.6414 & 0.6099 & 0.3476 \end{bmatrix}^\top. \tag{74}$$

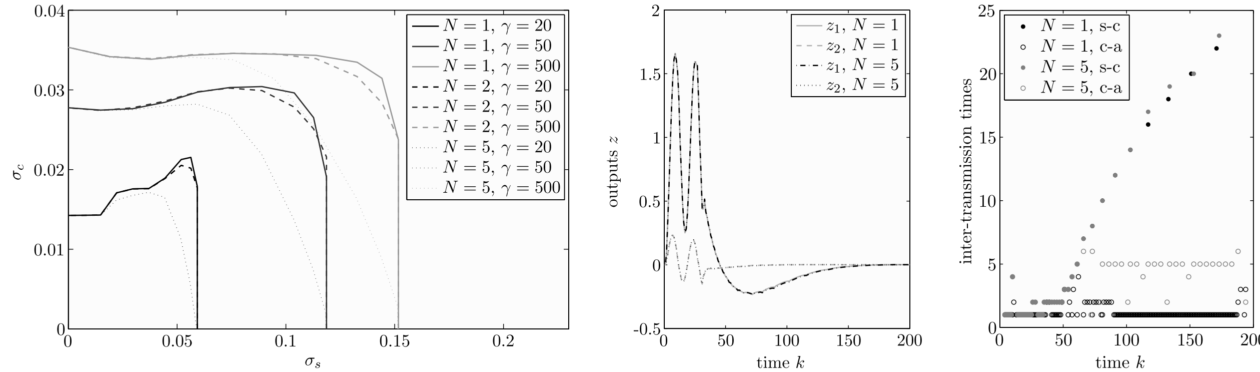
These values are the result of applying the design techniques presented in Bauer et al. (submitted). For the ℓ_2-gain analysis, we choose the performance output z as in (17), where $C_{zx} = C$ with C given in (73), and $D_z = 0$.

Example 1. Let us first consider the case in which only an ETM is present in the S–C channel and we compare the newly proposed model-based PETC of Section 3.1, with both the baseline PETC strategy of Section 3.3 and periodic time-triggered control. To make a fair comparison between the model-based and the baseline PETC strategies, we select σ_s for both cases such that the guaranteed upper bound γ on the ℓ_2-gain of the resulting closed-loop system satisfies $\gamma = 100$ and use the PWL system approach of Section 4.1 to construct the corresponding values for σ_s. This results in $\sigma_s = 0.135$ for the model-based PETC strategy. Using similar techniques for the baseline PETC strategy gives $\sigma_s = 0.0343$. Note that the corresponding periodic time-triggered control strategy results in an ℓ_2-gain of $\gamma^* = 12.75$.

The response of the performance output z to the initial condition $x_0 = 0$ and the disturbance satisfying $w_k = \sin \frac{3\pi \cdot k}{25} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ for $0 \leqslant k \leqslant 30$ and $w_k = 0$ for $k > 30$, for the three strategies is shown in Fig. 6a. We can conclude that all three control strategies (i.e., periodic time-triggered control, the baseline PETC and the model-based PETC) show almost indistinguishable responses. However, the number of transmissions that are needed is 200 for periodic time-triggered control, 148 for baseline PETC and only 41 for the model-based PETC in the first 200 units of time. Hence, the newly proposed model-based PETC needs significantly fewer transmissions than the other two approaches to realize similar responses, albeit at the price of more computations. This is also further illustrated in Fig. 6b showing the inter-transmission times. Note though that, obviously, the reduction in the number of transmissions depends on the disturbance characteristics. In case of temporary disturbances the proposed schemes are especially effective.

We will now study more closely the influence of the parameter σ_s on the upper bound γ on the ℓ_2-gain of the model-based PETC system and the number of transmissions that are generated for the aforementioned initial condition and disturbance, see Fig. 6c and d, respectively. The analysis of the influence of the parameter σ_s on γ also allows us to compare the PL system approach of Section 4.2 and the PWL system approach of Section 4.1. Fig. 6c shows that for both approaches the obtained upper bound on the ℓ_2-gain increases as σ_s increases, indicating that closed-loop performances degrades as σ_s increases. This figure also shows that the PWL system approach is less conservative than the PL system approach, as already mentioned in Section 4.3. From Fig. 6d, it can be shown that the increase of the guaranteed ℓ_2-gain, through an increased σ_s, leads to fewer transmissions, which demonstrates the tradeoff that has to be made between the closed-loop performance and the number of transmissions. Note that for σ_s approximating zero, the upper bound of the ℓ_2 gain for the model-based PETC strategy approaches $\gamma^* = 12.75$, which is the ℓ_2-gain of the corresponding periodic time-triggered control strategy. This is in accordance with the discussion in Section 4.3 showing that the ℓ_2-gain of the PETC strategy can approach the ℓ_2-gain of the periodic time-triggered implementation arbitrarily closely. Interestingly, even for a small σ_s, which only leads to a minor degradation of the closed-loop performance in terms of the ℓ_2-gain, the amount of data transmitted over the network, is already significantly reduced. For instance, starting from a periodic time-triggered observer-based controller, we can set $\sigma_s = 0.01$, which leads to an upper bound on the ℓ_2-gain of the corresponding PETC strategy of 13.77 as guaranteed by the PWL approach, see Fig. 6c, indicating an 8% performance degradation, while the number of transmissions reduces from 200 to 80, see Fig. 6d, which is a reduction of 60%.

Example 2. Let us now consider the case where the PETC system has both S–C and C–A ETMs. In this case, it is of interest to illustrate the influence of the parameters σ_s, σ_c, and N on the ℓ_2-gain and the number of transmissions of the resulting PETC system. In Fig. 7a, it is shown how these parameters influence the guaranteed upper



(a) The upper bound on the ℓ_2-gain as function of σ_s and σ_c for several buffer lengths N.

(b) The evolution of the outputs as function of time k.

(c) The inter-transmission times as function of time k for different buffer lengths.

Fig. 7. PETC with S–C and C–A ETMs.

bound on the performance γ, using the PWL system approach. Similar to the previous example, we can observe that in order to achieve a smaller ℓ_2-gain, the parameters σ_s and σ_c should be chosen smaller. Since also here the parameters σ_s and σ_c influence the number of transmissions, we again see that tradeoffs can be made between the closed-loop performance and the number of transmissions. The influence of N on the ℓ_2-gain is rather marginal.

When we now choose $N = 1$ and $N = 5$, and select $\sigma_s = 0.083$ for both cases, and $\sigma_c = 0.033$ for the $N = 1$-case and $\sigma_c = 0.030$ for the $N = 5$-case, we have a guaranteed upper bound on the ℓ_2-gain from w to z of $\gamma = 100$ for both cases. Note that $N = 1$ corresponds to a baseline C–A ETM as discussed in Remark 5.3. We now simulate the corresponding closed-loop PETC systems for the same initial condition and the same disturbance as before, and we obtain the responses for z as shown in Fig. 7b. We can observe that both cases (i.e., $N = 1$ and $N = 5$) show almost indistinguishable responses. Moreover, Fig. 7c shows that the inter-transmission times as a function of time for the S–C channel are almost identical for both cases, as well. For this disturbance input, both cases result in 48 transmissions in the S–C channel. However, the number of transmissions in the C–A channel is 167 for $N = 1$ and 68 for $N = 5$, see also the inter-transmission times of the C–A channel in Fig. 7c. We can, thus, conclude that a longer buffer, which allows more future control inputs to be stored, yields fewer transmissions in the C–A channel, particularly for time $k > 50$, i.e., when disturbances are absent, without sacrificing performance guarantees and obtain almost identical responses. This shows that the proposed model-based PETC strategy is able to significantly reduce communication, when computing more predictions of future control inputs.

9. Conclusions

In this paper, we proposed and formalized new model-based PETC strategies for both the sensor-to-controller (S–C) and the controller-to-actuator (C–A) channels. The results apply to discrete-time linear plants subject to disturbances, and do not require the full state to be available for feedback. The S–C event-triggering mechanisms (ETMs) are based on exploiting a Luenberger observer in the sensor system and two identical predictors in both the controller and sensor system. The sensor system only transmits the estimated state produced by the Luenberger observer to the controller, if the estimate of the predictor deviates too much from the observer's estimate. The C–A ETMs are based on predictive control techniques storing model-based predictions of future control values in buffers close to the actuators. Improved current and future control values are transmitted by the controller system, only when these stored values are no longer adequate.

To analyze these new model-based PETC strategies, we provided two general modeling and analysis frameworks based on perturbed linear (PL) and piecewise linear (PWL) systems, and derived linear matrix inequality (LMI)-based conditions for global exponential stability and guaranteed ℓ_2-gains. The PL system approach provided the valuable insight that the performance of any observer-based controller, implemented in a conventional periodic time-triggered fashion, can be recovered by a PETC implementation, thereby providing a justification for emulation-based design. An additional motivation for the PL system approach is that it is computationally (somewhat) simpler than the PWL system approach as, for instance, it leads to stability guarantees in terms of simple $\mathcal{H}_\infty$-norm calculations. The PWL system approach is of importance as it can be shown to be less conservative than the PL system approach. Both the PL and PWL approach can be used to obtain tradeoffs between the utilization of the communication resources on the one hand and performance guarantees in terms of ℓ_2-gains on the other hand. Besides presenting the model-based PETC setup for a centralized controller, we also demonstrated how the main ideas apply in the context of decentralized observer-based control laws for large-scale plants using small local observers and predictors.

Via numerical examples we illustrated the significant reductions that can be accomplished compared to both standard periodic time-triggered implementations and currently available PETC laws using baseline strategies for the ETMs. These examples demonstrate that model-based PETC provides a powerful strategy for the reduction of the number of transmissions, and, in case computation is cheaper than communication in terms of energy usage, also the increase in the battery life of wireless devices.

References

Arzén, K. (1999). A simple event-based PID controller. In *Proceedings of IFAC world conference. Vol. 18* (pp. 423–428).

Åström, K. J. (2008). Event based control. In *Analysis and design of nonlinear control systems* (pp. 127–148). Springer.

Åström, K.J., & Bernhardsson, B.M. (1999). Comparison of periodic and event based sampling for first order stochastic systems. In *Proceedings of IFAC world conference* (pp. 301–306).

Åström, K.J., & Bernhardsson, B.M. (2002). Comparison of Riemann and Lebesgue sampling for first order stochastic systems. In *Proceedings of conference on decision & control* (pp. 2011–2016).

Bauer, N.W., Donkers, M.C.F., van de Wouw, N., & Heemels, W.P.M.H. Decentralized observer-based control via networked communication. *Automatica* (submitted).

Bemporad, A. (1998). Predictive control of teleoperated constrained systems with unbounded communication delays. In *Proceedings of IEEE conference on decision & control* (pp. 2133–2138).

Bernardini, D., & Bemporad, A. (2012). Energy-aware robust model predictive control based on noisy wireless sensors. *Automatica*, *48*(1), 36–44.
Boyd, S., El Ghaoui, L., Feron, E., & Balakrishnan, V. (1994). *Linear matrix inequalities in system and control theory*. SIAM.
Chaillet, A., & Bicchi, A. (2008). Delay compensation in packet-switching networked controlled systems. In *Proceedings of IEEE conference on decision & control* (pp. 3620–3625).
Cogill, R. (2009). Event-based control using quadratic approximate value functions. In *Joint IEEE conference on decision & control and Chinese control conference* (pp. 5883–5888).
Donkers, M.C.F. (2011). Networked and event-triggered control, Ph.D. Thesis Eindhoven University of Technology.
Donkers, M. C. F., & Heemels, W. P. M. H. (2012). Output-based event-triggered control with guaranteed $\mathcal{L}_\infty$-gain and improved and decentralised event-triggering. *IEEE Transactions on Automatic Control*, *57*(6), 1362–1376.
Donkers, M. C. F., Heemels, W. P. M. H., van de Wouw, N., & Hetel, L. (2011). Stability analysis of networked control systems using a switched linear systems approach. *IEEE Transactions on Automatic Control*, *56*(9), 2101–2115.
Draper, S. S., Wrigley, W., & Hovorka, J. (1960). *Inertial guidance*. Oxford: Pergamon Press.
Eqtami, A., Dimarogonas, V., & Kyriakopoulos, K.J. (2010). Event-triggered control for discrete-time systems. In *Proceedings of American control conference* (pp. 4719–4724).
Garcia, E., & Antsaklis, P. (2011). Model-based event-triggered control with time-varying network delays. In *proceedings of joint IEEE conference on decision & control and European control conference* (pp. 1650–1655).
Gawthrop, P. J., & Wang, L. B. (2009). Event-driven intermittent control. *International Journal of Control*, *82*, 2235–2248.
Heemels, W.P.M.H., Donkers, M.C.F., & Teel, A.R. (2011). Periodic event-triggered control based on state feedback. In *Proceedings of joint IEEE conference on decision & control and European control conference* (pp. 2571–2576).
Heemels, W. P. M. H., Donkers, M. C. F., & Teel, A. R. (2013). Periodic event-triggered control for linear systems. *IEEE Transactions on Automatic Control*, *58*(4).
Heemels, W. P. M. H., Gorter, R. J. A., van Zijl, A., van den Bosch, P. P. J., Weiland, S., Hendrix, W. H. A., et al. (1999). Asynchronous measurement and control: a case study on motor synchronization. *Control Engineering Practice*, *7*, 1467–1482.
Heemels, W. P. M. H., Sandee, J. H., & van den Bosch, P. P. J. (2008). Analysis of event-driven controllers for linear systems. *International Journal of Control*, *81*, 571–590.
Heemels, W. P. M. H., Teel, A. R., van de Wouw, N., & Nešić, D. (2010). Networked control systems with communication constraints: tradeoffs between transmission intervals, delays and performance. *IEEE Transactions on Automatic Control*, *55*, 1781–1796.
Hendricks, E., Jensen, M., Chevalier, A., & Vesterholm, T. (1994). Problems in event based engine control. In *Proceedings of American control conference. Vol. 2* (pp. 1585–1587).
Henningsson, T., & Cervin, A. (2009). Comparison of LTI and event-based control for a moving cart with quantized position measurements. In *Proceedings of European control conference*.
Henningsson, T., Johannesson, E., & Cervin, A. (2008). Sporadic event-based control of first-order linear stochastic systems. *Automatica*, *44*, 2890–2895.
Hu, W., Liu, G., & Rees, D. (2007). Event-driven networked predictive control. *IEEE Transactions on Industrial Electronics*, *54*(3), 1603–1613.
Kofman, E., & Braslavsky, J.H. Level crossing sampling in feedback stabilization under data-rate constraints. In *Proceedings of IEEE conference on decision & control* (pp. 4423–4428).
Kwon, W. H., Kim, Y. H., Lee, S. J., & Paek, K. (1999). Event-based modeling and control for the burnthrough point in sintering processes. *IEEE Transactions on Control Systems Technology*, *7*, 31–41.
Lehmann, D. (2011). *Event-based state-feedback control*. Berlin: Logos Verlag.
Lehmann, D., & Lunze, J. (2010). Extension and experimental evaluation of an event-based state-feedback approach. *Control Engineering Practice*, *19*, 101–112.
Lehmann, D., & Lunze, J. (2011). Event-based output–feedback control. In *Proceedings of mediterranean conference on control & automation*.
Li, L., & Lemmon, M. (2011). Weakly coupled event triggered output feedback system in wireless networked control systems. In *Allerton conference on communication, control and computing*. Urbana-Champaign, USA.
Lunze, J., & Lehmann, D. (2010). A state-feedback approach to event-based control. *Automatica*, *46*, 211–215.
Miskowicz, M. (2006). Send-on-delta concept: an event-based data-reporting strategy. *Sensors*, *6*, 49–63.
Nešić, D., & Teel, A. R. (2004). Input–output stability properties of networked control systems. *IEEE Transactions on Automatic Control*, *49*(10), 1650–1667.
Otanez, P.G., Moyne, J.R., & Tilbury, D.M. (2002). Using deadbands to reduce communication in networked control systems. In *Proceedings of American control conference* (pp. 3015–3020).
Tabuada, P. (2007). Event-triggered real-time scheduling of stabilizing control tasks. *IEEE Transactions on Automatic Control*, *52*, 1680–1685.
van der Schaft, A. (2000). *$\mathcal{L}_2$-gain and passivity techniques in nonlinear control*. Springer.
Walsh, G. C., & Ye, H. (2001). Scheduling of networked control systems. *IEEE Control Systems Magazine*, *21*(1), 57–65.
Wang, X., & Lemmon, M.D. (2009). Event-triggering in distributed networked systems with data dropouts and delays. In *Proceedings of conference on hybrid systems: computation & control* (pp. 366–380).
Willems, J. C. (1972). Dissipative dynamical systems part I: general theory. *Archive for Rational Mechanics and Analysis*, *45*, 321–351.
Yook, J. K., Tilbury, D. M., & Soparkar, N. R. (2002). Trading computation for bandwidth: reducing communication in distributed control systems using state estimators. *IEEE Transactions on Control Systems Technology*, *10*(4), 503–518.

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