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Constructions of Lyapunov Functions for Large-scale Networked Control Systems with Packet-based Communication

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Abstract*— In this tutorial presentation we focus on the construction of Lyapunov or storage functions for large-scale networked control systems (NCSs) in which sensors, controllers and actuators are connected via multiple (local) communication networks which operate asynchronously and independently of each other. Within each packet-based communication network only one node can communicate at a given transmission time (requiring communication protocols) and the transmission intervals and delays may vary over time. These artefacts cause network-induced communication errors in the overal closed-loop system and can be detrimental for stability and performance. For these NCSs we provide explicit constructions of Lyapunov functions by modelling the large-scale NCS as an interconnection of a finite or even an infinite number of hybrid subsystems, and combining 'local' Lyapunov functions for the controlled dynamics (including network-induced errors) and the protocols in a systematic manner. These constructions lead to the numerical computation of maximum allowable transmission intervals (MATIs) and maximum allowable delays (MADs) for each of the individual networks. The availability of the Lyapunov or storage functions guarantee properties such as global asymptotic or exponential stability, input-to-state stability (ISS) and $\mathcal{L}_p$-stability for the large-scale NCS. Interestingly, the control performance expressed in terms of ISS and $\mathcal{L}_p$-gains can be traded with the network parameters (MATIs and MADs). Hence, tradeoffs can be made between the quality-of-control of the overal hybrid system and the required quality-of-service of the underlying communication infrastructure. Also event-triggered communication schemes will be shortly discussed. The results are illustrated with an example of vehicle platooning.

I. Introduction

In networked control systems (NCSs), sensor and actuator data is transmitted via shared (wired or wireless) communication networks [1]. This offers several advantages over conventional control systems, in which sensor and actuation data is transmitted using dedicated point-to-point wired links, including reduced installation costs, better maintainability

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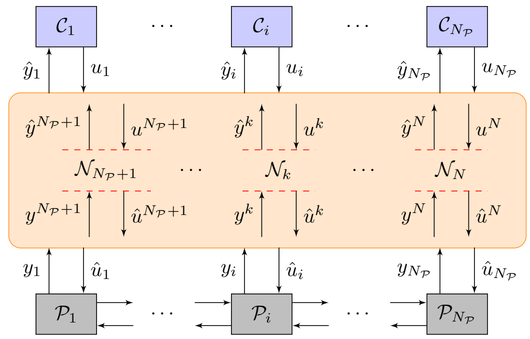

Fig. 1. NCS with multiple communication networks connecting plant(s) and controller(s).

and greater flexibility. On the other hand, shared packet-based communication networks also introduce communication errors as a result of network imperfections such as varying transmission intervals and delays, and quantisation errors. Additionally, since a network is usually shared by multiple sensor, controller and actuator nodes, there is a need for a medium access (MAC) protocol that governs the access of the nodes to the network in order to prevent packet losses as much as possible. As a result, one needs to design the communication networks and controllers in such a way that the NCS displays desirable behaviour in terms of stability and performance that is robust to these network-induced phenomena.

In contrast with many existing works in which it is typically assumed that all sensor and actuation data is transmitted over *one* single communication network, in this tutorial presentation we consider large-scale NCSs with *multiple local* communication networks operating independently and asynchronously. The latter setting is often much more reasonable than assuming the presence of only one global network. For example, in the control of large-scale systems it is often more natural and cost-efficient to use a local controller for each subsystem than one global controller for the whole system. Indeed, in this setup it is more natural to close the local control loops over several local communication networks (some wired, others maybe wireless), instead of assuming the presence of one global communication network. See Figure 1.

The general objective of this tutorial presentation is to discuss an analysis and design framework for these large-

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scale NCSs that provides quantitative information that relates the network parameters and the control performance:

- The network performance ("*quality-of-service*") expressed in terms of the network parameters such as the maximum allowable transmission intervals (MATIs), the maximum allowable delays (MADs), the network reliability, etc.
- The control performance ("*quality-of-control*") in terms of global asymptotic or exponential stability, convergence rates, input-to-state stability (ISS) gains, $\mathcal{L}_p$-stability, etc.

To keep the analysis tractable in view of the large scale of the NCSs, it is desirable to provide methodologies that are based on the local dynamics of the plants to be controlled, the local controllers, the local MAC protocols, and the interconnection structure. Indeed, techniques using conditions based on global monolithic models might be too complex for any practical use for situations where the size of the NCS is large. In other words, the objective is to provide a general framework for the stability analysis of large-scale NCSs with multiple local communication networks, and to provide, based on local conditions, network parameters for each local communication network such that specific stability or performance properties of the overall system are guaranteed.

II. Interconnections of hybrid systems

The main results are based on modelling the large-scale NCS as an interconnection hybrid subsystems inspired by the setups in [2]–[6] in which only one global communication network was used. The resulting models adopt the hybrid system formalism as advocated in [7]. For instance, the setup of Figure 1 is rewritten into the interconnection as in Figure 2 in which each block $\mathcal{G}_1, \ldots, \mathcal{G}_{N_\mathcal{P}}, \mathcal{E}_{N_{\mathcal{P}+1}}, \ldots, \mathcal{E}_N$ corresponds to a hybrid submodel $\mathcal{H}_i$ (with local state x_i) of the form

$$\mathcal{H}_i : \begin{cases} \dot{x}_i \in F_i(x, w), & (x, w) \in \mathcal{F}_i, \\ x_i^+ \in G_i(x, w), & (x, w) \in \mathcal{J}_i, \end{cases} \tag{1}$$

where $w \in \mathbb{R}^{n_w}$ is a disturbance, $x_i \in \mathcal{X}_i \subseteq \mathbb{R}^{n_{x_i}}$ is the state of subsystem $\mathcal{H}_i$, and $x = (x_1, x_2, \ldots, x_N) \in \mathcal{X} := \mathcal{X}_1 \times \mathcal{X}_2 \times \cdots \times \mathcal{X}_N \subseteq \mathbb{R}^{n_x}$ is the state of the overall system, with $n_x = \sum_{i=1}^{N} n_{x_i}$. Here, $F_i : \mathcal{X} \times \mathbb{R}^{n_w} \rightrightarrows \mathbb{R}^{n_{x_i}}$ is the flow map, $G_i : \mathcal{X} \times \mathbb{R}^{n_w} \rightrightarrows \mathcal{X}_i$ is the jump map, and $\mathcal{F}_i, \mathcal{J}_i \subseteq \mathcal{X} \times \mathbb{R}^{n_w}$ are the flow set and jump set of subsystem $\mathcal{H}_i$, $i \in \{1, 2, \ldots, N\}$. Note that the dynamics of (1) is dependent on the full state x, indicating the coupling of the subsystems, and that the disturbance w is the same for all subsystems, see [8] for more details.

III. Main approach

Based on perceiving the large-scale NCS as interconnections of hybrid submodels, 'local' Lyapunov-based conditions can be formulated for the local closed-loop dynamics and the local MAC protocols that will lead to guarantees on stability, input-to-state stability (ISS) or $\mathcal{L}_p$-stability if the

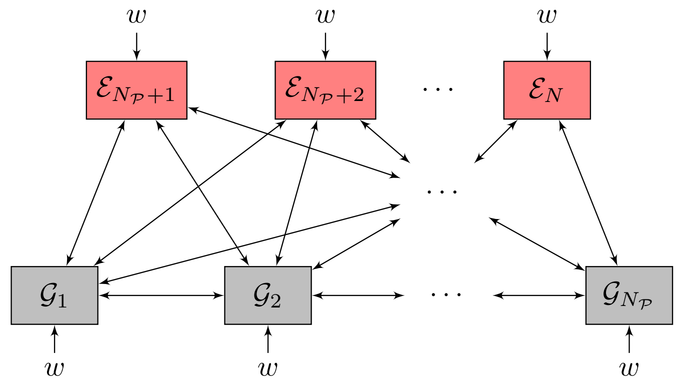
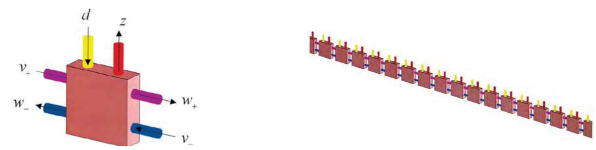


Fig. 2. The networked control setup of Figure 1, viewed as an interconnection of $N_\mathcal{P}$ controlled subsystems $\mathcal{G}_i$, $i \in \{1, 2, \ldots, N_\mathcal{P}\}$, and $N_\mathcal{N}$ network-induced error systems $\mathcal{E}_k$, $k \in \{N_\mathcal{P}+1, N_\mathcal{P}+2, \ldots, N\}$.



(a) Basic building block. (b) Infinite interconnection.

Fig. 3. Spatially invariant interconnected systems in one spatial dimension from [24].

network parameters (MATI and MAD) are chosen appropriately for each individual communication network. These results will be obtained by combining these 'local' conditions with information on the interconnection structure, which will be captured using two well-known formalism in the literature:

- Small-gain conditions that combine the interactions between the hybrid submodels via specific (ISS) gains thereby leading to compact and insightful small gain conditions. These results draw inspiration from the theory of interconnections of (hybrid) ISS systems that is already well-developed (see, e.g., [9]–[19]).
- Exploiting spatial invariance of the (possibly infinite number of) subsystems and build upon particular interconnection structures as in [20]–[24] (which assumed *ideal* communications between the subsystems), see also Figure 3. In this setup the systems consist of interconnections of similar dynamical units or subsystems (Figure 3(a)) that interact with their nearest neighbours (Figure 3(b)) or their controllers through packet-based networks. Several configurations are possible such as the one depicted in Figure 3(b), but also periodic interconnections and finite interconnections with boundary conditions can be accommodated in our framework.

The results are constructive in nature in the sense that *global* Lyapunov and storage functions are constructed for the overall hybrid system based on the mentioned local conditions and the interconnection structure (including possibly small gain conditions). Since the results are formulated in terms of local conditions on the subsystem dynamics, combined with a small-gain condition and/or specific interconnection structures, the proposed framework is numerically tractable and therefore indeed suitable for the stability and

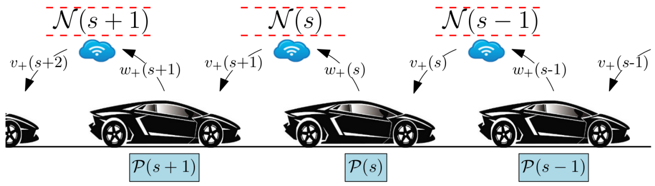


Fig. 4. Platooning application with packet-based communication between the vehicles.

performance analysis of large-scale interconnected systems with packet-based communication. Tradeoffs between the quality-of-service (in terms of MATI and MAD) of different networks, tradeoffs between the quality-of-control of the overall system and the guaranteed quality-of-service of the individual networks, and tradeoffs between MATI and MAD within a communication network follow naturally from our framework.

IV. Final words

The main results in this tutorial presentation are based on the results of our recent papers. For the case of a finite number of subsystems and communication networks building upon small-gain arguments, see [8], [25], [26]. For the situation of an (in)finite number of spatially invariant systems interconnected via networked communication, see [27]. More details on the framework can be found in these references. In this presentation we will also highlight that the time-based triggering of the communication (bounded by MATI and MAD) can also be changed into event-triggered communication using the work [28], [29]. Also for this case our results will be constructive in nature and explicit expression for the Lyapunov and storage functions for the global NCS possibly will be given based on local conditions. The results will be illustrated through an example on vehicle platooning (Figure 4).

References

[1] J. P. Hespanha, P. Naghshtabrizi, and Y. Xu, "A survey of recent results in networked control systems," *Proc. IEEE*, vol. 95, no. 1, pp. 138–162, 2007.

[2] W. P. M. H. Heemels, A. R. Teel, N. Van de Wouw, and D. Nešić, "Networked control systems with communication constraints: Tradeoffs between transmission intervals, delays and performance," *IEEE Trans. Automat. Contr.*, vol. 55, no. 8, pp. 1781–1796, 2010.

[3] D. Nešić and A. R. Teel, "Input-output stability properties of networked control systems," *IEEE Trans. Automat. Contr.*, vol. 49, no. 10, pp. 1650–1667, 2004.

[4] ——, "Input-to-state stability of networked control systems," *Automatica*, vol. 40, no. 12, pp. 2121–2128, 2004.

[5] G. C. Walsh, O. Beldiman, and L. G. Bushnell, "Asymptotic behavior of nonlinear networked control systems," *IEEE Trans. Automat. Contr.*, vol. 46, no. 7, pp. 1093–1097, 2001.

[6] D. Carnevale, A. R. Teel, and D. Nešić, "A Lyapunov proof of an improved maximum allowable transfer interval for networked control systems," *IEEE Trans. Automat. Contr.*, vol. 52, no. 5, pp. 892–897, 2007.

[7] R. Goebel, R. G. Sanfelice, and A. R. Teel, *Hybrid Dynamical Systems: Modeling, Stability, and Robustness*. Princeton University Press, 2012.

[8] D. P. Borgers, R. Geiselhart, and W. P. M. H. Heemels, "Tradeoffs between quality-of-control and quality-of-service in large-scale nonlinear networked control systems," accepted in "Nonlinear Analysis: Hybrid Systems".

[9] Z.-P. Jiang, I. M. Y. Mareels, and Y. Wang, "A Lyapunov formulation of the nonlinear small-gain theorem for interconnected ISS systems," *Automatica*, vol. 32, no. 8, pp. 1211–1215, 1996.

[10] S. N. Dashkovskiy, B. S. Rüffer, and F. R. Wirth, "An ISS small gain theorem for general networks," *Math. Control, Signals Systems*, vol. 19, no. 2, pp. 93–122, 2007.

[11] D. Nešić and A. Teel, "A Lyapunov-based small-gain theorem for hybrid ISS systems," in *47th IEEE Conf. Decision and Control*, 2008, pp. 3380–3385.

[12] S. N. Dashkovskiy, B. S. Rüffer, and F. R. Wirth, "Small gain theorems for large scale systems and construction of ISS Lyapunov functions," *SIAM J. Control Optim.*, vol. 48, no. 6, pp. 4089–4118, 2010.

[13] T. Liu, D. J. Hill, and Z.-P. Jiang, "Lyapunov formulation of ISS cyclic-small-gain in continuous-time dynamical networks," *Automatica*, vol. 47, no. 9, pp. 2088–2093, 2011.

[14] R. Sanfelice, "Results on input-to-output and input-output-to-state stability for hybrid systems and their interconnections," in *49th IEEE Conf. Decision and Control*, 2010, pp. 2396–2401.

[15] ——, "Interconnections of hybrid systems: Some challenges and recent results," *J. Nonlinear Systems and Applications*, vol. 2, no. 1-2, pp. 111–121, 2011.

[16] T. Liu, Z.-P. Jiang, and D. J. Hill, "Lyapunov formulation of the ISS cyclic-small-gain theorem for hybrid dynamical networks," *Nonlinear Analysis: Hybrid Systems*, vol. 6, no. 4, pp. 988–1001, 2012.

[17] S. N. Dashkovskiy and M. Kosmykov, "Input-to-state stability of interconnected hybrid systems," *Automatica*, vol. 49, no. 4, pp. 1068–1074, 2013.

[18] D. Liberzon, D. Nešić, and A. Teel, "Lyapunov-based small-gain theorems for hybrid systems," *IEEE Trans. Automat. Contr.*, vol. 59, no. 6, pp. 1395–1410, 2014.

[19] R. Geiselhart and F. R. Wirth, "On maximal gains guaranteeing a small-gain condition," *SIAM J. Control Optim.*, vol. 53, no. 1, pp. 262–286, 2015.

[20] C. Langbort and R. D'Andrea, "Imposing boundary conditions for a class of spatially-interconnected systems," in *Proc. American Control Conference*, vol. 1, 2003, pp. 107–112.

[21] R. D'Andrea, C. Langbort, and R. Chandra, "A state space approach to control of interconnected systems," in *Mathematical Systems Theory in Biology, Communication, Computation, and Finance*, ser. IMA Book Series, J. Rosenthal Edition. New York: Springer-Verslag, 2003.

[22] C. Langbort and R. D'Andrea, "Distributed control of spatially reversible interconnected systems with boundary conditions," *SIAM J. Control Optim.*, vol. 44, no. 1, pp. 1–28, 2005.

[23] B. Bamieh, F. Paganini, and M. A. Dahleh, "Distributed control of spatially invariant systems," *IEEE Trans. Automat. Contr.*, vol. 47, no. 7, pp. 1091–1107, 2002.

[24] R. D'Andrea and G. E. Dullerud, "Distributed control design for spatially interconnected systems," *IEEE Trans. Automat. Contr.*, vol. 48, no. 9, pp. 1478–1495, 2003.

[25] D. P. Borgers and W. P. M. H. Heemels, "Stability analysis of large-scale networked control systems with local networks: A hybrid small-gain approach," *Hybrid Systems: Computation and Control*, pp. 103–112, 2014.

[26] W. P. M. H. Heemels, D. P. Borgers, N. van de Wouw, D. Nešić, and A. R. Teel, "Stability analysis of nonlinear networked control systems with asynchronous communication: A small-gain approach," in *52nd IEEE Conf. Decision and Control*, 2013, pp. 4631–4637.

[27] S. H. J. Heijmans, D. P. Borgers, and W. P. M. H. Heemels, "Stability analysis of spatially invariant interconnected systems with networked communication," *5th IFAC Workshop on Distributed Estimation and Control in Networked Systems, IFAC PapersOnline, Volume 48, Issue 22, 2015, Pages 221-226*, 2015.

[28] V. S. Dolk, D. P. Borgers, and W. P. M. H. Heemels, "Dynamic event-triggered control: Tradeoffs between transmission intervals and performance," pp. 2764–2769, 2014.

[29] ——, "Output-based and decentralized dynamic event-triggered control with guaranteed $\mathcal{L}_p$-gain performance and zeno-freeness," *Accepted in IEEE Trans. Automat. Contr.. DOI: 10.1109/TAC.2016.2536707*, 2016.