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Resource-aware MPC for constrained linear systems: Two rollout approaches

T.M.P. Gommans*, T.A.F. Theunisse, D.J. Antunes, W.P.M.H. Heemels

Eindhoven University of Technology, Department of Mechanical Engineering, P.O. Box 513, 5600 MB Eindhoven, The Netherlands

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ABSTRACT

In systems with resource constraints, such as actuation limitations in sparse control applications or limited bandwidth in networked control systems, it is desirable to use control signals that are either sparse or sporadically changing in time. Motivated by these applications, in this paper we propose two resource-aware MPC schemes for discrete-time linear systems subject to state and input constraints. The two MPC schemes exploit ideas from rollout strategies to determine simultaneously the new (continuous) control inputs and the (discrete) time instants at which the control actions are updated. The first scheme provides performance guarantees by design, in the sense that it allows the user to select a desired suboptimal level of performance, where the degree of suboptimality provides a trade-off between the guaranteed closed-loop control performance on the one hand and the utilization of (communication/actuation) resources on the other hand. The second scheme provides a guaranteed (average) resource utilization, while cleverly allocating these resources in order to maximize the control performance. By means of numerical examples, we demonstrate the effectiveness of the proposed strategies.

1. Introduction

In more and more applications, it becomes essential to address resource constraints explicitly in the design and implementation of the control law. One important domain in which this is apparent is the field of sparse control, where certain control input profiles are preferable from a resource point of view. The use of, for instance, sparse or sporadically changing actuation signals can have several benefits, such as improved fuel efficiency or a larger lifetime of actuators that are subject to wear and tear. See Fig. 1 for an illustration of sporadically changing and sparse input profiles. For instance, in [16] sparse thrust actuation signals are considered to use fuel in the control of a spacecraft in an efficient manner. In [10] sporadically changing actuation is used in the control of an autonomous underwater vehicle in order to decrease fuel consumption and increase the deployment time. Also in overactuated systems it can be desirable to have a smart control allocation policy that does not require each actuator to be updated continuously in time in order to realize a desired level of performance. One specific example includes the usage of sparse actuation signals to control the roll of an overactuated ship [12].

Another domain in which sparse control solutions are relevant is the domain of networked control systems (NCSs). In NCSs sensors, controllers and actuators are spatially distributed and communication takes place via packet-based (often wireless) networks. In a networked control environment controllers are no longer implemented using dedicated communication channels, but use (shared) communication networks. Since the control tasks have to share the communication resources with other tasks, the availability of these resources is limited and might even change over time [20]. In these NCSs it is of interest to reduce the number of times the control law is updated to lower the network resource utilization. This is also highly relevant in the context of control in case of battery powered wireless sensors in order to increase the life time of the batteries.

In both sparse control applications and NCSs it is of importance to take the resource constraints into account in the controller design. These resource constraints are imposed by limitations on the available computational power, communication bandwidth, battery power (e.g., for battery powered sensors) and actuation resources. The class of resource constraints for these applications, and considered in this paper, are the ones requiring the actuation input to be sparse or sporadically changing. However, despite the fact that resources can

* Corresponding author.
E-mail address: t.gommans@gmail.com (T.M.P. Gommans).

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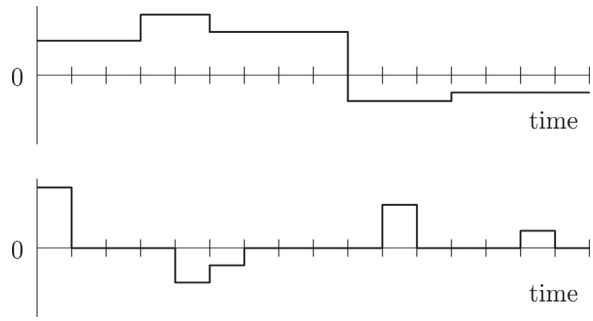

Fig. 1. Example of sporadically changing (top) and sparse (bottom) input profiles.

be scarce, controllers are typically still implemented in a time-triggered fashion, in which control tasks are executed periodically. This design choice often leads to over-utilization of the available resources, as it might not be necessary to execute the control task every period to guarantee certain closed-loop performance. In fact, when designing control strategies for systems with limited actuation and/or communication resources, it is expected that the resulting resource-aware control strategies abandon the periodic time-triggered control paradigm and use aperiodic execution of the control tasks.

Two approaches that abandon this periodic communication pattern in NCSs are event-triggered control (ETC) and self-triggered control (STC), see [17] for a recent survey. In ETC and STC, the control law consists of two elements, namely, a feedback controller that computes the control input, and a triggering mechanism that determines when the control input has to be updated. The difference between ETC and STC is that the former is reactive, while the latter is proactive. ETC is reactive in the sense that the triggering consists of verifying a specific condition continuously and when it becomes true, the control task is triggered. In STC the next update time is pre-computed at the current update time based on predictions using previously received data and knowledge on the plant dynamics.

In this paper we are particularly interested in the design of resource-aware (aperiodic) control strategies for systems subject to constraints on states and inputs. One of the most widely used control strategies for systems with such constraints is model predictive control (MPC). MPC is a form of optimal control based on solving, at each sampling instant, a constrained finite horizon open-loop optimal control problem, using the current state of the plant as the initial state. The optimization yields an optimal control sequence and the first control value in the sequence is applied to the plant [24].

Normally, the receding horizon implementation of MPC is time-triggered. However, there are several works in the literature that address control of state and input constrained systems with scarce resources. In [6], an ETC scheme based on using the sequence of optimal controls obtained from the MPC optimization problem is presented, for discrete-time linear systems. In addition to sending the computed MPC input sequence to the actuators, in [6] also the predicated optimal state (or output) sequence is transmitted to the sensors. An event-triggering mechanism, positioned at the sensors, is chosen such that it detects if the true states deviate from the predicted states in the MPC problem, using absolute thresholds. In [29], another ETC scheme is proposed for the control of an irrigation canal. Here, the canal operators are considered as moving sensors and actuators and the resource limitations arise due to their travel times. Self-triggered MPC schemes that maximize the time until the next control execution while satisfying state and input constraints in the presence of additive disturbances are proposed in [22,9]. In [11] a self-triggered decentralized MPC framework is presented that aims at reducing the communication between agents, as well as the number of times the agents update their control values. In [21] a self-triggered estimation and a predictive control scheme are proposed for the control of linear constrained systems subject to additive noise and the stability and robustness properties of such a scheme are analyzed. In [18,19], self-triggered MPC approaches for unconstrained discrete-time linear systems are proposed. The approaches [18,19] provide sporadically changing input profiles by solving an MPC problem, involving standard quadratic control costs and a penalty related to sampling the system and updating the control law. Another well known approach is based on modifying the MPC problem by appending the original MPC control cost with an ℓ_1 penalty on the input in order to obtain sparse input signals. Regularizing by the ℓ_1-norm is known to induce sparsity, in the sense that individual components of the input signal will be equal to zero, see, e.g., [1,12,13,25,27] and [26] in which the focus is on discrete-time and continuous-time linear systems, respectively. Also different types of sum-of-norms regularization can be used to obtain so-called group sparsity, meaning that at many time instants the entire input vector becomes zero, see, e.g., [28].

Although the methods mentioned above, including ℓ_1 regularization have proven to be effective in obtaining sparse input profiles, no performance guarantees are given in terms of the original MPC cost function due to the additional penalties included in the cost function. We have previously proposed a self-triggered MPC framework for constrained linear [5] and nonlinear [14] systems. The approaches in [5,14] are such that a reduction in resource utilization can be realized without modifying the cost function by input regularization or explicitly penalizing resource usage. The control laws and triggering mechanisms are synthesized so that a priori chosen performance levels (in terms of the original control cost function) are guaranteed by design next to asymptotic stability and constraint satisfaction.

In this paper, we propose two stabilizing MPC strategies for systems with state and input constraints that are based on so-called rollout approaches. In rollout algorithms optimal decisions are made along a (finite) lookahead horizon, assuming that, from then on a "base policy" is used for which the cost-to-go is typically simple to compute, see [7]. Exploiting the idea of rollout algorithms in resource-aware control systems was used before in [2–4] and related ideas were proposed in [18,19] but only for linear systems *without* any constraints. In this paper, the objective is to present resource-aware MPC strategies that can handle hard constraints on states and inputs. The two novel strategies we propose solve the co-design problem of determining both the time instants on which the updating/communication of the control action take place and selecting the new (continuous) control inputs. The first approach provides performance guarantees by design (in terms of the original control cost function), in the sense that it allows the user to select a desired suboptimal level of performance. The degree of suboptimality provides a trade-off between the guaranteed closed-loop control performance on the one hand and utilization of

resources on the other hand. The second approach provides a guaranteed (average) resource utilization, while cleverly allocating these resources in order to maximize the control performance. We illustrate the effectiveness of both approaches using a numerical example.

The remainder of this paper is organized as follows. After indicating the notational conventions used in this paper, in Section 2 we provide the problem formulation and modeling preliminaries. For sparse input profiles, we present in Sections 3 and 4 our rollout approaches with a priori guarantees on control performance and guaranteed resource utilization rate, respectively. In Section 5 we discuss how the framework can be used to create sporadically changing input profiles. The effectiveness of the proposed approaches is demonstrated by means of a numerical example in Section 6. Section 7 contains a discussion on the complexity of the presented algorithms. Finally, in Section 8 we present the conclusions.

1.1. Nomenclature

By $\mathbb{R}$ and $\mathbb{N}$ we denote the set of real numbers and the set of non-negative integers (including zero), respectively. For $s, t \in \mathbb{N}$, the notation $\mathbb{N}_{[s,t]}$ is used to denote the set $\{r \in \mathbb{N} \mid s \le r \le t\}$. The empty set is denoted by $\emptyset$. The inequalities $\prec$, $\preceq$, $\succ$ and $\succeq$ are used for matrices, i.e., for a square matrix $X \in \mathbb{R}^{n\times n}$ we write $X \prec 0$, $X \preceq 0$, $X \succ 0$ and $X \succeq 0$ if X is symmetric and, in addition, X is negative definite, negative semi-definite, positive definite and positive semi-definite, respectively. Sequences of vectors are indicated by bold letters, e.g., $\mathbf{u} = (u_0, u_1, \ldots, u_M)$ with $u_i \in \mathbb{R}^{n_u}$, $i \in \{0, 1, \ldots, M\}$, where $M \in \mathbb{N} \cup \{\infty\}$ will be clear from the context. The projection operators $\Pi_i : (\mathbb{R}^{n_u})^M \to \mathbb{R}^{n_u}$ and $\Pi_{i:j} : (\mathbb{R}^{n_u})^M \to (\mathbb{R}^{n_u})^{j-i+1}$ for $i, j, M \in \mathbb{N}$ with $0 \le i \le j \le M-1$, are defined by $\Pi_i \mathbf{u} := u_i$ and $\Pi_{i:j}\mathbf{u} := (u_i, u_{i+1}, \ldots, u_j)$, respectively, for $\mathbf{u} = (u_0, u_1, \ldots, u_{M-1}) \in (\mathbb{R}^{n_u})^M$.

2. Problem formulation and preliminaries

In this section, we provide the class of systems considered in this work. Moreover, we present two problem formulations that are of interest in designing resource-aware controllers. Finally, in this section we present the modeling preliminaries that are used throughout the remainder of the paper.

2.1. System description

In this paper we consider a discrete-time linear system

$$x_{t+1} = Ax_t + Bu_t, \tag{1}$$

where $x_t \in \mathbb{R}^{n_x}$ and $u_t \in \mathbb{R}^{n_u}$ are the state and the input, respectively, at time $t \in \mathbb{N}$. The system (1) is subject to input and state constraints given by

$$u_t \in \mathbb{U} \quad \text{and} \quad x_t \in \mathbb{X}, \quad t \in \mathbb{N}, \tag{2}$$

where $\mathbb{X} \subseteq \mathbb{R}^{n_x}$ and $\mathbb{U} \subseteq \mathbb{R}^{n_u}$ are convex and compact sets containing the origin in their interiors. For $N \in \mathbb{N}$, $x_k(x, \mathbf{u})$ denotes the solution to (1) at time $k \in \mathbb{N}_{[0,N]}$ initialized at $x_0 = x$ and with control input sequence given by $\mathbf{u} = (u_0, u_1, \ldots, u_{N-1})$.

2.2. Problem formulation for resource-aware control

Standard stabilizing MPC techniques require the use of communication and/or actuation resources at *each* time $t \in \mathbb{N}$ to update the controller, see, e.g., [23,24]. This may be undesirable in applications where these resources are limited. In this work, we propose resource-aware controllers that not only compute the control inputs, but also decide at which times $t \in \mathbb{N}$ resources need to be used to update the inputs to the plant u_t in order to guarantee stability, constraint satisfaction and a desired level of performance. It is convenient to introduce $\sigma_t \in \{0, 1\}$, $t \in \mathbb{N}$, as a decision variable, indicating if at time t resources are used to update the controller ($\sigma_t = 1$) or not ($\sigma_t = 0$). We are interested in designing a policy $\pi = (\mu_0, \mu_1, \ldots)$ which is defined as a sequence of functions $\mu_t = (\mu_t^u, \mu_t^\sigma)$, $t \in \mathbb{N}$, that map the state at each time t into control actions and resource decisions, i.e.,

$$(u_t, \sigma_t) = \mu_t(x_t), \quad t \in \mathbb{N}.$$

Performance of the resource-aware controllers will be measured in terms of a multi-objective cost criterion, as it should reflect both the infinite horizon control cost

$$J_{\text{control}}(x_0, \pi) = \sum_{t=0}^{\infty} x_t^\top Q x_t + u_t^\top R u_t, \tag{3}$$

for $Q \succeq 0$ and $R \succ 0$, where $(A, Q^{1/2})$ is detectable, and the (average) resource utilization

$$J_{\text{resource}}(x_0, \pi) = \lim_{\gamma\to\infty} \frac{1}{\gamma} \sum_{t=0}^{\gamma-1} \sigma_t. \tag{4}$$

In designing resource-aware controllers for system (1) subject to the constraints (2) we consider two different multi-objective problem formulations, namely

(A) $\min_\pi J_{\text{resource}}(x_0, \pi)$ such that $J_{\text{control}}(x_0, \pi) \le c_{\text{control}}(x_0)$,
(B) $\min_\pi J_{\text{control}}(x_0, \pi)$ such that $J_{\text{resource}}(x_0, \pi) = c_{\text{resource}}$,

for some control specification given by $c_{\text{control}} : \mathbb{R}^{n_x} \to \mathbb{R}_{\geq 0}$ and resource specification given by $c_{\text{resource}} \in \mathbb{R}_{(0,1]}$. For each problem, we assume that a *periodic* solution exists, which yields a finite cost, i.e., it stabilizes the system (1), and satisfies the state and input constraints, i.e., (2). These periodic solutions are used as a feasible base policy in our rollout approaches, where we aim to improve over these policies in terms of Problem (A) or (B). In Section 3, we present a strategy that addresses (A) by reducing the required resource utilization with respect to the periodic base policy, while guaranteeing a certain level of performance (in terms of the cost of the periodic base policy). In Section 4, we present a strategy that addresses (B) by reducing the cost with respect to a given periodic base policy, while guaranteeing the same resource utilization as the periodic base policy. Note that for Problem (B), the existence of a feasible periodic solution boils down to the existence of a periodic controller that, on average, updates the input once every $1/c_{\text{resource}}$ time instants. Hence, $c_{\text{resource}} \in \mathbb{R}_{(0,1]}$ should be selected large enough to guarantee feasibility (using periodic feedback) for the set of states of interest. Similarly, for Problem (A), the upper bound on the control cost $c_{\text{control}}(x_0)$ should be such that the set of states that are feasible for a periodic solution is sufficiently large.

2.3. Preliminaries

The problems (A) and (B) are intractable due to the (infinite) number of discrete choices $(\sigma_0, \sigma_1, \sigma_2, \ldots)$ in the policies over which the optimization takes place. Consequently, the true optimal policy is hard to determine. This work provides approximate solutions by solving a (tractable) finite-horizon problem in a receding horizon fashion. We do this by considering optimization problems over a fixed prediction horizon N and restrict the number of times that resources are used to update the control input within the prediction horizon N. In fact, $N_q \in \mathbb{N}_{[1,N]}$ updates are allowed. To formulate our optimization problems, we require a parameterization of the input profiles of length N with N_q control input updates. More specifically, we write $\mathbf{s} = (s_0, \quad s_1, \quad \ldots, \quad s_{N_q-1}) \in \mathbb{S}_{N,N_q}$, with $\mathbb{S}_{N,N_q} := \{(s_0, s_1, \ldots, s_{N_q-1}) \in (\mathbb{N}_{[0,N-1]})^{N_q} \mid s_0 < s_1 < \cdots < s_{N_q-1}\}$ to denote the collection of (ordered) time schedules at which the control values are updated within the prediction horizon N and $\mathbf{v} = (v_0, \quad v_1, \quad \ldots, \quad v_{N_q-1}) \in (\mathbb{R}^{n_u})^{N_q}$ to denote the corresponding control values.

There are many possible strategies for selecting the new control value at the times when no input update is specified, i.e., when $k \in \mathbb{N}_{[0,N-1]} \backslash \{s_0, s_1, \ldots, s_{N_q-1}\}$. A strategy resulting in sparse input profiles is based on setting the control value to zero if $k \in \mathbb{N}_{[0,N-1]} \backslash \{s_0, s_1, \ldots, s_{N_q-1}\}$; this will be referred to as the "*zero*" strategy. Sporadically changing input profiles can be obtained by holding the previously applied control value if no input update is satisfied, i.e., if $k \in \mathbb{N}_{[0,N-1]} \backslash \{s_0, s_1, \ldots, s_{N_q-1}\}$. This will be referred to as the "*hold*" strategy. For ease of exposition, we focus on obtaining sparse input signals using a zero strategy, although sporadically changing input signals can also be pursued, see Section 5 for further details.

We introduce the mappings $M_{N,i} : \mathbb{S}_{N,i} \times (\mathbb{R}^{n_u})^i \to (\mathbb{R}^{n_u})^N$, for $N \in \mathbb{N}$ and $i \in \mathbb{N}_{[1,N]}$, converting the information in $\mathbf{s} \in \mathbb{S}_{N,i}$ and $\mathbf{v} \in (\mathbb{R}^{n_u})^i$ to obtain the corresponding input sequence $\mathbf{u} = (u_0, u_1, \ldots, u_{N-1}) \in (\mathbb{R}^{n_u})^N$ for (1). Given $\mathbf{s} = (s_0, s_1, \ldots, s_{i-1}) \in \mathbb{S}_{N,i}$ and $\mathbf{v} = (v_0, v_1, \ldots, v_{i-1}) \in (\mathbb{R}^{n_u})^i$ we define $\mathbf{u} := M_{N,i}(\mathbf{s}, \mathbf{v}) = (u_0, u_1, \ldots, u_{N-1}) \in (\mathbb{R}^{n_u})^N$ with

$$u_k = \begin{cases} v_j, & \text{when} \quad k = s_j \quad \text{for} \quad \text{some} \quad j \in \{0, 1, \ldots, i-1\}, \\ 0, & \text{when} \quad k \in \mathbb{N}_{[0,N-1]} \backslash \{s_0, s_1, \ldots, s_{i-1}\}, \end{cases}$$

for $k \in \mathbb{N}_{[0,N-1]}$.

Standard receding horizon techniques in MPC are based on sampling the state and computing an optimal sequence of control inputs for each $t \in \mathbb{N}$, and applying only the first element of the computed optimal sequence to the plant. the procedure is then repeated at time $t+1$. We consider a more general receding horizon setup where the state is sampled and an optimization problem is solved at periodic *scheduling times* $t_l = lH$, $l \in \mathbb{N}$, for some $H \in \mathbb{N}_{[1,N]}$. In Section 3 we present an MPC approach for Problem (A) with $H = 1$ and in Section 4 we present an MPC approach for Problem (B) for a fixed $H \in \mathbb{N}_{[1,N]}$.

The approaches in Sections 3 and 4 are of a self-triggered nature, meaning that at the scheduling times $t = t_l$, $l \in \mathbb{N}$, the update times until t_{l+1} are determined.

3. Problem (A): Guarantees on control performance

In this section, we propose a rollout MPC scheme that employs a one-step lookahead to improve over a standard MPC setup, in terms of the optimization Problem (A). Our solution is based on sampling the state deciding at each time $t \in \mathbb{N}$ if the next control value can be zero or not (i.e., $H = 1$) and, consequently, each time $t \in \mathbb{N}$ is a scheduling time.

At each (scheduling) time $t \in \mathbb{N}$, given state x_t, our approach aims at reducing the number of times the control input is nonzero by solving an optimization problem based on two different schedules. The two schedules only differ in whether the input in the first step of the prediction horizon is zero or not, assuming resources are used in the remainder of the steps (i.e., the inputs are nonzero). Hence, we consider the set of schedules $\mathbb{S}_N^{\text{os}} := \{\mathbf{s}_{\text{os}}^0, \mathbf{s}_{\text{os}}^1\}$, where $\mathbf{s}_{\text{os}}^0 := (0, 1, \ldots, N-1) \in \mathbb{S}_{N,N}$ and $\mathbf{s}_{\text{os}}^1 := (1, 2, \ldots, N-1) \in \mathbb{S}_{N,N-1}$, note that $\mathbb{S}_{N,N} = \{\mathbf{s}_{\text{os}}^0\}$.

As mentioned at the end of Section 2.2, our approach relies on the existence of a stabilizing periodic solution. In this case, this periodic solution is a standard MPC scheme, which we introduce in the next section before stating our rollout approach and its theoretical properties in Sections 3.2 and 3.3, respectively.

3.1. A standard stabilizing MPC setup

For the system (1), we now consider the MPC setup given by the following optimization problem. For a fixed prediction horizon $N \in \mathbb{N}_{\geq 1}$, given state $x_t = x \in \mathbb{X}$ at time $t \in \mathbb{N}$,

$$\min J_N(x, \mathbf{u}) \tag{5a}$$

$$\text{w.r.t.} \quad \mathbf{u} \in \mathcal{U}_N(x) := \left\{ \mathbf{u} \in \mathbb{U}^N \middle| \forall k \in \{0, 1, 2, \ldots, N-1\}, x_k(x, \mathbf{u}) \in \mathbb{X} \quad \text{and} \quad x_N(x, \mathbf{u}) \in \mathbb{T} \right\}. \tag{5b}$$

Here

$$J_N(x, \mathbf{u}) := F(x_N(x, \mathbf{u})) + \sum_{k=0}^{N-1} L(x_k(x, \mathbf{u}), \Pi_k \mathbf{u}), \tag{6}$$

and $\mathbb{T} \subseteq \mathbb{X}$ is the terminal set, which is assumed to be compact, convex and to contain the origin in its interior. In this work we focus on the case where the running cost $L : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \to \mathbb{R}_{\geq 0}$ is chosen as a quadratic function of the states and inputs and for $x \in \mathbb{R}^{n_x}$ and $u \in \mathbb{R}^{n_u}$ is given by

$$L(x, u) = x^\top Q x + u^\top R u \tag{7}$$

with $Q \succeq 0$ and $R \succ 0$, where $(A, Q^{1/2})$ is detectable. Furthermore, the terminal cost $F : \mathbb{R}^{n_x} \to \mathbb{R}_{\geq 0}$ is taken for $x \in \mathbb{R}^{n_x}$ as

$$F(x) = x^\top P x, \tag{8}$$

where $P \succ 0$ will be defined later in this section. A state $x \in \mathbb{X}$ is said to be feasible for the optimization problem (5) if $\mathcal{U}_N(x) \neq \emptyset$, i.e., there is at least one admissible input sequence. The set of feasible states is denoted by $\mathbb{X}_f$, i.e.,

$$\mathbb{X}_f = \left\{ x \in \mathbb{X} \mid \mathcal{U}_N(x) \neq \emptyset \right\}. \tag{9}$$

As N is finite, for $x \in \mathbb{X}_f$, the given conditions ensure the existence of a unique minimizer $\mathbf{u}^*(x) = \left(u_0^*(x), \ldots, u_{N-1}^*(x)\right)$ for the optimization problem (5) for given $x \in \mathbb{X}_f$. For $x \in \mathbb{X}_f$, $V(x)$ denotes the corresponding minimum value for the optimization problem (5). Hence, $V : \mathbb{X}_f \to \mathbb{R}_{\geq 0}$ is the MPC value function given by

$$V(x) := \min_{\mathbf{u} \in \mathcal{U}_N(x)} J_N(x, \mathbf{u}) = J_N(x, \mathbf{u}^*(x)). \tag{10}$$

The resulting MPC law $u^{\mathrm{mpc}} : \mathbb{X}_f \to \mathbb{U}$ is defined for $x \in \mathbb{X}_f$ as

$$u^{mpc}(x) := u_0^*(x), \tag{11}$$

which is implemented in a receding horizon fashion, leading to the policy

$$u_t = u^{\mathrm{mpc}}(x_t), \tag{12}$$

$$\sigma_t = 1, \tag{13}$$

for $t \in \mathbb{N}$.

To guarantee recursive feasibility and closed-loop stability we use the terminal cost and set method [24,23], which typically assumes for the linear case considered here, that there is a $K \in \mathbb{R}^{n_u \times n_x}$ such that

$$(A + BK)^\top P (A + BK) - P \preceq -K^\top R K - Q, \tag{14a}$$

$$(A + BK)\mathbb{T} \subseteq \mathbb{T}, \tag{14b}$$

$$K\mathbb{T} \subseteq \mathbb{U} \text{ and } \mathbb{T} \subseteq \mathbb{X}. \tag{14c}$$

Here, we take P as the solution to the discrete algebraic Ricatti equation (DARE)

$$P = A^\top P A - (A^\top P B)(R + B^\top P B)^{-1}(B^\top P A) + Q, \tag{15}$$

and select

$$K = -(R + B^\top P B)^{-1} B^\top P A. \tag{16}$$

Note that P satisfies (14a) with equality.

Under these standing assumptions, the optimal MPC law as in (12) results in a closed-loop system given by (1) and (12), which

(i) is recursively feasible in the sense that for each $x_0 \in \mathbb{X}_f$ the corresponding solution to (1) and (12), denoted by $\{x_t\}_{t \in \mathbb{N}}$, exists for all $t \in \mathbb{N}$;
(ii) is asymptotically stable in $\mathbb{X}_f$, in the sense that
- [Lyapunov stability]: for any $\varepsilon > 0$ there is a $\delta > 0$ such that if $x_0 \in \mathbb{X}_f$ and $\|x_0\| \leq \delta$ then the corresponding trajectory $\{x_t\}_{t \in \mathbb{N}}$ to (1) and (12) satisfies $\|x_t\| \leq \varepsilon$;
- [attractivity]: for any $x_0 \in \mathbb{X}_f$ the corresponding trajectory $\{x_t\}_{t \in \mathbb{N}}$ to (1) and (12) satisfies $\lim_{t \to \infty} x_t = 0$;

(iii) satisfies input and state constraints, i.e., (2);
(iv) satisfies the performance guarantee

$$\sum_{t=0}^{\infty} \left(x_t^\top Q x_t + u_t^\top R u_t \right) \leq V(x_0). \tag{17}$$

See [24,23] and the references therein, for the proofs and further details on the terminal set and cost method.

3.2. Approach

The rollout strategy proposed in this section aims at reducing the utilization of resources compared to the standard MPC setup in Section 3.1, while still meeting certain performance guarantees, next to asymptotic stability and constraint satisfaction. More specifically, we require that

$$\sum_{t=0}^{\infty}(x_t^\top Q x_t + u_t^\top R u_t) \leq (1+\beta)V(x_0), \tag{18}$$

for some $\beta \geq 0$, with $V(x_0)$ in (10). Hence, in the formulation of Problem (A) in Section 2.2 it holds that $c_{\text{control}}(x_0) = (1+\beta)V(x_0)$ for $x_0 \in \mathbb{X}_f$. Here, $\beta \geq 0$ is a design parameter that allows to relax the obtained performance guarantee with respect to the performance (17) of the stabilizing MPC setup in Section 3.1, in order to reduce the utilization of resources.

To realize the constraint (18) and to achieve a reduction in resource utilization, we multiply the cost function in (6) by $(1+\beta)$ and include a discount for not updating the *first* control value in the prediction horizon. Hence, for $x \in \mathbb{R}^{n_x}$ and for $\mathbf{s} \in \mathbb{S}_N^{\text{os}}$ and $\mathbf{v} \in (\mathbb{R}^{n_u})^{N_q}$ we consider the cost function

$$J_N^{\text{os}}(x, M_{N,N_q}(\mathbf{s},\mathbf{v})) = (1+\beta)J_N(x, M_{N,N_q}(\mathbf{s},\mathbf{v})) - \Pi_0\mathbf{s}\beta L(x, \Pi_0 M_{N,N_q}(\mathbf{s},\mathbf{v})). \tag{19}$$

Note that $\Pi_0\mathbf{s}_{\text{os}}^0 = 0$ and $\Pi_0\mathbf{s}_{\text{os}}^1 = 1$. Hence, the schedule $\mathbf{s}_{\text{os}}^1$ in which the first step uses the zero-input gets a discount of $-\beta L(x, \Pi_0 M_{N,N_q}(\mathbf{s}_{\text{os}}^1, \mathbf{v}))$ compared to the schedule $\mathbf{s}_{\text{os}}^0$ in which a non-zero input is used at the first step.

For fixed $N \in \mathbb{N}_{\geq 1}$ and given state $x_t = x \in \mathbb{X}$ at time $t \in \mathbb{N}$, we solve the MPC optimization problem

$$\min J_N^{\text{os}}(x, M_{N,N_q}(\mathbf{s},\mathbf{v})) \tag{20a}$$

$$\text{w.r.t.} \quad (\mathbf{s},\mathbf{v}) \in \mathcal{SV}_N^{\text{os}}(x), \tag{20b}$$

where the set of admissible control and schedule values are given by the union of the admissible $N_q = N$ control values for the schedule $\mathbf{s}_{\text{os}}^0$ and the admissible $N_q = N-1$ control values for the schedule $\mathbf{s}_{\text{os}}^1$, i.e.,

$$\mathcal{SV}_N^{\text{os}}(x) = \left(\{\mathbf{s}_{\text{os}}^0\} \times \mathcal{U}_N(x)\right) \cup \mathcal{SV}_{N,N-1}^0(x),$$

where

$$\mathcal{SV}_{N,N-1}^0(x) = \{(\mathbf{s},\mathbf{v}) \in \mathcal{SV}_{N,N-1}(x) \mid \Pi_0\mathbf{s} = 1\},$$

and where for fixed $N \in \mathbb{N}_{\geq 1}$ and $N_q \in \mathbb{N}_{[1,N]}$, and given state $x \in \mathbb{X}$ we define $\mathcal{SV}_{N,N_q}(x)$ as the set of schedules of length N with N_q control value updates and their respective control values when in state x, i.e.,

$$\mathcal{SV}_{N,N_q}(x) := \{(\mathbf{s},\mathbf{v}) \in \mathbb{S}_{N,N_q} \times \mathbb{U}^{N_q} \mid \forall k \in \{0,1,2,\ldots,N-1\}, x_k(x, M_{N,N_q}(\mathbf{s},\mathbf{v})) \in \mathbb{X}, \quad x_N(x, M_{N,N_q}(\mathbf{s},\mathbf{v})) \in \mathbb{T}\}. \tag{21}$$

Observe that the set of admissible control values where the first control value is not updated (i.e., $\mathbf{s}_{\text{os}}^1$) is contained in the set of admissible control values for updating the input at every step (i.e., $\mathbf{s}_{\text{os}}^0$) as $0 \in \mathbb{U}$ is an admissible control value in the first step. Hence, for $x \in \mathbb{R}^{n_x}$,

$$\{\mathbf{u} \in \mathbb{U}^N \mid \exists(\mathbf{s},\mathbf{v}) \in \mathcal{SV}_{N,N-1}^0(x), \mathbf{u} = M_{N,N-1}(\mathbf{s},\mathbf{v})\} \subseteq \{\mathbf{u} \in \mathbb{U}^N \mid \exists(\mathbf{s},\mathbf{v}) \in \{\mathbf{s}_{\text{os}}^0\} \times \mathcal{U}_N(x), \mathbf{u} = M_{N,N}(\mathbf{s},\mathbf{v})\} = \mathcal{U}_N(x) \tag{22}$$

and, consequently, the feasible states for (20) are

$$\mathbb{X}_f^{\text{os}} := \{x \in \mathbb{X} \mid \mathcal{SV}_N^{\text{os}}(x) \neq \emptyset\} = \mathbb{X}_f.$$

Hence, the set of feasible states $\mathbb{X}_f^{\text{os}}$ for the MPC problem (20) is the same as for the standard MPC problem discussed in Section 3.1. For $x \in \mathbb{X}_f$, a minimizer to (20) is known to exist, and we write $(\mathbf{s}_{\text{os}}^*(x), \mathbf{v}_{\text{os}}^*(x))$ to denote a particular one. For $x \in \mathbb{X}_f$, $V^{\text{os}}(x)$ denotes the corresponding value for the optimization problem (20). Hence, $V^{\text{os}} : \mathbb{X}_f \to \mathbb{R}_{\geq 0}$ is the rollout MPC value function given by

$$V^{\text{os}}(x) = \min_{(\mathbf{s},\mathbf{v}) \in \mathcal{SV}_N^{\text{os}}(x)} J_N^{\text{os}}(x, M_{N,N_q}(\mathbf{s},\mathbf{v})), = J_N^{\text{os}}(x, M_{N,N_q}(\mathbf{s}_{\text{os}}^*(x), \mathbf{v}_{\text{os}}^*(x))).$$

An optimal sequence of inputs corresponding to $(\mathbf{s}_{\text{os}}^*(x), \mathbf{v}_{\text{os}}^*(x))$ is given for $x \in \mathbb{X}_f$ by

$$\mathbf{u}_{\text{os}}^*(x) := M_{N,N_q(\mathbf{s}_{\text{os}}^*(x))}(\mathbf{s}_{\text{os}}^*(x), \mathbf{v}_{\text{os}}^*(x)). \tag{23}$$

For $t \in \mathbb{N}$, the rollout policy for system (1) is now given by

$$u_t = \Pi_0\mathbf{u}_{\text{os}}^*(x_t), \tag{24a}$$

$$\sigma_t = 1 - \Pi_0\mathbf{s}_{\text{os}}^*(x_t). \tag{24b}$$

3.3. Theoretical properties

The rollout MPC policy given by (24) has the following important properties.

Theorem 1 (Recursive feasibility and constraint satisfaction). *For $\beta \geq 0$ and under the conditions (14) with K in (16), the control law given by (24) is recursively feasible for all $x_0 \in \mathbb{X}_f$ in the sense that the closed-loop system (1) and (24) leads to a trajectory $\{x_t\}_{t\in\mathbb{N}}$ defined for all $x_0 \in \mathbb{X}_f$. Moreover, the closed-loop system (1) and (24) satisfies the constraints (2).*

Proof. The proof is based on showing that for each $x \in \mathbb{X}_f$, it holds that $x_1(x, \mathbf{u}^*_{\text{os}}(x)) \in \mathbb{X}_f$.

Let $\mathbf{u}^*_{\text{os}}(x) := (u^*_{\text{os},0}(x), u^*_{\text{os},1}(x), \ldots, u^*_{\text{os},N-1}(x))$ as in (23). Due to (22) it holds that $\mathbf{u}^*_{\text{os}}(x) \in \mathcal{U}_N(x)$. We define now $\bar{\mathbf{u}}(x) = (u^*_{\text{os},1}(x), \ldots, u^*_{\text{os},N-1}(x), Kx_N(x, \mathbf{u}^*_{\text{os}}(x)))$. Since $\mathbf{u}^*_{\text{os}}(x) \in \mathcal{U}_N(x)$, we have that $x_N(x, \mathbf{u}^*_{\text{os}}(x)) \in \mathbb{T}$. We invoke the conditions (14b) and (14c) related to the terminal cost and set method to see that $Kx_N(x, \mathbf{u}^*_{\text{os}}(x)) \in \mathbb{U}$ and $x_1(x_N(x, \mathbf{u}^*_{\text{os}}(x)), Kx_N(x, \mathbf{u}^*_{\text{os}}(x))) \in \mathbb{T}$. Note that both $\mathbf{u}^*_{\text{os}}(x)$ and $\bar{\mathbf{u}}(x)$ contain the same sequence $(u^*_{\text{os},1}(x), \ldots, u^*_{\text{os},N-1}(x))$. Consequently, $\bar{\mathbf{u}}(x) \in \mathcal{U}_N(x_1(x, u^*_{\text{os},0}(x)))$ and thus $(\mathbf{s}^0_{\text{os}}, \bar{\mathbf{u}}(x)) \in \mathcal{SV}^{\text{os}}_N(x_1(x, u^*_{\text{os},0}(x)))$. Hence, we have that $x_1(x, \mathbf{u}^*_{\text{os}}(x)) \in \mathbb{X}_f$, which proves recursive feasibility. As $u_t = \Pi_0 \mathbf{u}^*_{\text{os}}(x_t)$ for $t \in \mathbb{N}$, we can conclude that the input and state constraints as in (2) are satisfied. □

Theorem 2 (Performance guarantee). *For $\beta \geq 0$ and given the conditions in (14), the closed-loop system given by (1) and (24) satisfies the performance guarantee in (18) for all $x_0 \in \mathbb{X}_f$.*

Proof. Let $\mathbf{u}^*_{\text{os}}(x) = (u^*_{\text{os},0}(x), u^*_{\text{os},1}(x), \ldots, u^*_{\text{os},N-1}(x)) \in \mathcal{U}_N(x)$ as in (23) and $\bar{\mathbf{u}}(x) = (u^*_{\text{os},1}(x), \ldots, u^*_{\text{os},N-1}(x), Kx_N(x, \mathbf{u}^*_{\text{os}}(x))) \in \mathcal{U}_N(x_1(x, \mathbf{u}^*_{\text{os}}(x)))$, where the latter inclusion is established in the proof of Theorem 1. Moreover, let $\bar{x}_N := x_N(x, \mathbf{u}^*_{\text{os}}(x)) \in \mathbb{T}$.

To show (18), we use that for $x \in \mathbb{X}_f$

$$(1+\beta)V(x) = J^{\text{os}}_N(x, \mathbf{u}^*(x)) \geq J^{\text{os}}_N(x, \mathbf{u}^*_{\text{os}}(x)) = V^{\text{os}}(x), \tag{25}$$

where $\mathbf{u}^*(x) = \left(u^*_0(x), \ldots, u^*_{N-1}(x)\right)$ is the optimal input sequence of the standard MPC approach, see (10). Moreover, for all $x \in \mathbb{X}_f$

$$V^{\text{os}}(x) - V^{\text{os}}(x_1(x, \mathbf{u}^*_{\text{os}}(x))) \geq V^{\text{os}}(x) - J^{\text{os}}_N(x_1(x, \mathbf{u}^*_{\text{os}}(x)), M_{N,N}(\mathbf{s}^0_{\text{os}}, \bar{\mathbf{u}}(x))). \tag{26}$$

Rewriting the right-hand side of the inequality (26), using the definition of $V^{\text{os}}(x)$ with $\mathbf{u}^*_{\text{os}}(x)$ as defined in (23) leads to

$$\begin{aligned}
&V^{\text{os}}(x) - V^{\text{os}}(x_1(x, \mathbf{u}^*_{\text{os}}(x))) \geq J^{\text{os}}_N(x, \mathbf{u}^*_{\text{os}}(x)) - J^{\text{os}}_N(x_1(x, \mathbf{u}^*_{\text{os}}(x)), M_{N,N}(\mathbf{s}^0_{\text{os}}, \bar{\mathbf{u}}(x))) = (1+\beta(1-\Pi_0\mathbf{s}^*_{\text{os}}(x)))L(x, u^*_{\text{os},0}(x)))\\
&\quad + (1+\beta)\left(\sum_{k=1}^{N-1} L(x_k(x, \mathbf{u}^*_{\text{os}}(x)), u^*_{\text{os},k}(x)) + F(x_N(x, \mathbf{u}^*_{\text{os}}(x))) - F(x_N(x_1(x, \mathbf{u}^*_{\text{os}}(x)), \bar{\mathbf{u}}(x)))\right.\\
&\quad \left. - \sum_{k=0}^{N-1} L(x_k(x_1(x, \mathbf{u}^*_{\text{os}}(x)), \bar{\mathbf{u}}(x)), \Pi_k M_{N,N}(\mathbf{s}^0_{\text{os}}, \bar{\mathbf{u}}(x))\right) = (1+\beta(1-\Pi_0\mathbf{s}^*_{\text{os}}(x)))L(x, u^*_{\text{os},0}(x)) + (1+\beta)(F(\bar{x}_N) - L(\bar{x}_N, K\bar{x}_N)\\
&\quad - F(x_1(\bar{x}_N, K\bar{x}_N))) = (1+\beta(1-\Pi_0\mathbf{s}^*_{\text{os}}(x)))\left(x^\top Qx + (u^*_{\text{os},0}(x))^\top Ru^*_{\text{os},0}(x)\right) - (1+\beta)\bar{x}_N^\top\left(Q + K^\top RK - P + (A+BK)^\top P(A+BK)\right)\bar{x}_N.
\end{aligned}$$

Here, for the first equality we used the definition of $J^{\text{os}}_N(x)$ in (19). To obtain the second equality, note that

$$\sum_{k=1}^{N-1} L(x_k(x, \mathbf{u}^*_{\text{os}}(x)), u^*_{\text{os},k}(x)) = \sum_{k=0}^{N-2} L(x_k(x_1(x, \mathbf{u}^*_{\text{os}}(x)), \bar{\mathbf{u}}(x)), \Pi_k M_{N,N}(\mathbf{s}^0_{\text{os}}, \bar{\mathbf{u}}(x)),$$

moreover, we use the definition of $\bar{x}_N$ and note that $x_N(x_1(x, \mathbf{u}^*_{\text{os}}(x)), \bar{\mathbf{u}}(x)) = \bar{x}_N$ and that $x_N(x_1(x, \mathbf{u}^*_{\text{os}}(x)), \bar{\mathbf{u}}(x)) = x_1(\bar{x}_N, K\bar{x}_N)$. Finally, the third equality follows from substitution of the functions $L(x, u)$ and $F(x)$ from (7) and (8), respectively, and grouping terms for $\bar{x}_N$. Because $\bar{x}_N = x_N(x, \mathbf{u}^*_{\text{os}}(x)) \in \mathbb{T}$, using (14a) related to the terminal cost and set method gives

$$V^{\text{os}}(x) - V^{\text{os}}(x_1(x, \mathbf{u}^*_{\text{os}}(x))) \geq (1+\beta(1-\Pi_0\mathbf{s}^*_{\text{os}}(x)))\left(x^\top Qx + (u^*_{\text{os},0}(x))^\top Ru^*_{\text{os},0}(x)\right).$$

Since $\Pi_0\mathbf{s}^0_{\text{os}} = 0$, $\Pi_0\mathbf{s}^1_{\text{os}} = 1$ and $\beta \geq 0$, using (24), for $t \in \mathbb{N}$ it holds that

$$(x^{\text{os}}_t)^\top Qx^{\text{os}}_t + (u^{\text{os}}_t)^\top Ru^{\text{os}}_t \leq V^{\text{os}}(x^{\text{os}}_t) - V^{\text{os}}(x^{\text{os}}_{t+1}), \tag{27}$$

where $\{x^{\text{os}}_t\}_{t\in\mathbb{N}_{[0,T]}}$ and $\{u^{\text{os}}_t\}_{t\in\mathbb{N}_{[0,T]}}$ are the state and input trajectories produced by the closed-loop system given by (1) and (24) for some $T \in \mathbb{N}$, with $x^{\text{os}}_0 = x_0 \in \mathbb{X}_f$. Summing (27) from $t=0$ to $t=T$ leads to

$$\begin{array}{rcl}
x_0^\top Qx_0 + (u^{\text{os}}_0)^\top Ru^{\text{os}}_0 & \leqslant & V^{\text{os}}(x_0) - V^{\text{os}}(x^{\text{os}}_1)\\
(x^{\text{os}}_1)^\top Qx^{\text{os}}_1 + (u^{\text{os}}_1)^\top Ru^{\text{os}}_1 & \leqslant & V^{\text{os}}(x^{\text{os}}_1) - V^{\text{os}}(x^{\text{os}}_2)\\
\vdots & & \vdots\\
(x^{\text{os}}_T)^\top Qx^{\text{os}}_T + (u^{\text{os}}_T)^\top Ru^{\text{os}}_T & \leqslant & V^{\text{os}}(x^{\text{os}}_T) - V^{\text{os}}(x^{\text{os}}_{T+1})\\
\hline
\sum_{t=0}^{T}\left((x^{\text{os}}_t)^\top Qx^{\text{os}}_t + (u^{\text{os}}_t)^\top Ru^{\text{os}}_t\right) & \leqslant & V^{\text{os}}(x_0) - V^{\text{os}}(x^{\text{os}}_{T+1}) \leqslant V^{\text{os}}(x_0),
\end{array}$$

where in the latter inequality we used that V^{os} takes only nonnegative values. Using $V^{\text{os}}(x_0) \leq (1+\beta)V(x_0)$ as established in (25), and letting $T \to \infty$, we obtain

$$\sum_{t=0}^{\infty}\left((x^{\text{os}}_t)^\top Qx^{\text{os}}_t + (u^{\text{os}}_t)^\top Ru^{\text{os}}_t\right) \leq (1+\beta)V(x_0), \tag{28}$$

which is the performance guarantee in (18). □

Theorem 3 (Closed-loop stability). *For $\beta \geq 0$, and given the conditions in (14), the closed-loop system given by (1) and (24) is asymptotically stable for initial conditions in $\mathbb{X}_f$.*

Proof. From (28) and $Q \succeq 0$ with $(A, Q^{1/2})$ detectable, we obtain the existence of a $c_1 > 0$ such that for all $x_0 \in \mathbb{X}_f$

$$\sum_{t=0}^{\infty} \|x_t^{os}\|^2 \leq c_1 W(x_0) + c_1 \sum_{t=0}^{\infty} \left((x_t^{os})^\top Q x_t^{os} + \|u_t^{os}\|^2 \right),$$

where $W(x) = x^\top V x$, with $V \succ 0$ such that for some L (which exists due to detectability of $(A, Q^{1/2})$)

$$(A - LQ^{1/2})^\top V (A - LQ^{1/2}) - V \prec 0.$$

Here $\{x_t^{os}\}_{t \in \mathbb{N}}$ and $\{u_t^{os}\}_{t \in \mathbb{N}}$ are the state and input trajectories produced by the closed-loop system (1) and (24) with $x_0^{os} = x_0 \in \mathbb{X}_f$. Using now that $R \succ 0$, we obtain that there exists $c_2 > 0$ such that

$$\sum_{t=0}^{\infty} \|x_t^{os}\|^2 \leq c_2 \|x_0\|^2 + c_2 \sum_{t=0}^{\infty} \left((x_t^{os})^\top Q x_t^{os} + (u_t^{os})^\top R u_t^{os} \right).$$

Using (28) this yields that

$$\sum_{t=0}^{\infty} \|x_t^{os}\|^2 \leq c_2 \|x_0\|^2 + c_2 (1 + \beta) V(x_0). \tag{29}$$

From (29) we immediately obtain that $\lim_{t \to \infty} x_t^{os} = 0$. In addition, since we have $V(x) \leq c_3 \|x\|^2$ for all $x \in \mathbb{T}$ and since 0 is in the interior of $\mathbb{T}$ we also obtain Lyapunov stability from (29). Therefore, the closed-loop system given by (1) and (24) is asymptotically stable for all $x_0 \in \mathbb{X}_f$. □

Remark 1 *(Required feedback).* Note that the approach presented in this section produces sparse input profiles, however it requires access to the state x_t for all $t \in \mathbb{N}$. In the context of NCS, this means lower communication resource utilization is only obtained between the controllers and actuators, whereas the communication resources between the sensors and the controller are still used at each time $t \in \mathbb{N}$. Our solution to Problem (B) also achieves reduced resource utilization between sensors and controller, see Section 4.

4. Problem (B): Guarantees on resource utilization

In this section, we propose a rollout MPC scheme that addresses Problem (B) with (average) *actuation* resource utilization $c_{\text{resource,actuation}} = 1/q$, $q \in \mathbb{N}$. The communication resources are used at the scheduling times $t_l = lH$, $l \in \mathbb{N}$ with $H \in \mathbb{N}_{[1,N]}$. Hence, the *communication* rate $c_{\text{resource,communication}} = 1/H$, where we require the following assumption on the variables N, q and H.

Assumption 1. $N \in \mathbb{N}_{\geq 1}$, $q \in \mathbb{N}_{\geq 1}$ and $H \in \mathbb{N}_{\geq 1}$ are such that $N_q := (N/q) \in \mathbb{N}_{[1,N]}$, $H_q := (H/q) \in \mathbb{N}_{[1,N]}$ and $N_H := (N/H) \in \mathbb{N}_{[1,N]}$.

Before stating the proposed rollout MPC scheme in Section 4.2, we introduce a periodic (multi-rate) MPC scheme that is base policy in Section 4.1. Important theoretical properties of our scheme are discussed in Section 4.3.

4.1. Periodic solution to Problem (B): Multi-rate MPC

We consider a multi-rate MPC approach that allows the periodic utilization of actuation resources at a rate $\frac{1}{q}$. The schedule for the multi-rate approach is given by $\bar{\mathbf{s}} = (0, q, 2q, \ldots, (N_q - 1)q) \in \mathbb{S}_{N,N_q}$ and we define $\mathbb{S}_{N,N_q}^{\text{mr}} = \{\bar{\mathbf{s}}\} \subseteq \mathbb{S}_{N,N_q}$.

For fixed N, q and H satisfying Assumption 1, and given state $x_{t_l} = x \in \mathbb{X}$ at time $t_l = lH$, $l \in \mathbb{N}$, we consider

$$\min_{\mathbf{v}} J_N(x, M_{N,N_q}(\bar{\mathbf{s}}, \mathbf{v})) \tag{30a}$$

$$\text{s.t.} \quad (\bar{\mathbf{s}}, \mathbf{v}) \in \mathcal{SV}_{N,N_q}^{\text{mr}}(x), \tag{30b}$$

where

$$\mathcal{SV}_{N,N_q}^{\text{mr}}(x) := \{(\bar{\mathbf{s}}, \mathbf{v}) \in \mathbb{S}_{N,N_q}^{\text{mr}} \times \mathbb{U}^{N_q} \mid \forall k \in \{0, 1, 2, \ldots, N-1\}, x_k(x, M_{N,N_q}(\bar{\mathbf{s}}, \mathbf{v})) \in \mathbb{X}, \quad x_N(x, M_{N,N_q}(\bar{\mathbf{s}}, \mathbf{v})) \in \mathbb{T}_{\text{mr}}\}$$

and where J_N is given by (6), for L in (7) with $Q \succeq 0$ and $R \succ 0$, with $(A, Q^{1/2})$ is detectable and where $F(x) = x^\top P_{\text{mr}} x$, for $x \in \mathbb{R}^{n_x}$, with $P_{\text{mr}} = P_{\text{mr}}^\top \succ 0$ taken as the solution to the following discrete algebraic Riccati equation

$$P_{\text{mr}} = Q_q + (A^q)^\top P_{\text{mr}} A^q - ((A^q)^\top P_{\text{mr}} B_q + S_q)(R_q + B_q^\top P_{\text{mr}} B_q)^{-1} (B_q^\top P_{\text{mr}} A^q + S_q^\top). \tag{31}$$

Here, we define $B_q := A^{q-1} B$, $Q_q := \sum_{k=0}^{q-1} (A^k)^\top Q A^k$, $S_q := \sum_{k=1}^{q-1} (A^k)^\top Q A^{k-1} B$ and $R_q := (\sum_{k=1}^{q-1} B^\top (A^{k-1})^\top Q A^{k-1} B) + R$, see, e.g., [15].

By $\mathbb{T}_{\text{mr}} \subseteq \mathbb{X}$ we denote the terminal set, which is assumed to be compact, convex and to contain the origin in its interior. The set of feasible states is denoted by $\mathbb{X}_f^{\text{mr}}$, i.e.,

$$\mathbb{X}_f^{\text{mr}} = \{x \in \mathbb{X} \mid \mathcal{SV}_{N,N_q}^{\text{mr}}(x) \neq \emptyset\}. \tag{32}$$

For $x \in \mathbb{X}_f^{\text{mr}}$, the given conditions ensure that a unique minimizer $\mathbf{v}_{\text{mr}}^*(x) = \left(v_{\text{mr},0}^*(x), v_{\text{mr},1}^*(x), \ldots, v_{\text{mr},N_q-1}^*(x) \right)$ to the optimization problem (30) exists. By $V^{\text{mr}}(x)$ we denote the corresponding minimum in (30), i.e., $V^{\text{mr}} : \mathbb{X}_f^{\text{mr}} \to \mathbb{R}_{\geq 0}$ is the multi-rate MPC value function given by

$$V^{\text{mr}}(x) = \min\{J_N(x, M_{N,N_q}(\bar{\mathbf{s}}, \mathbf{v})) \mid (\bar{\mathbf{s}}, \mathbf{v}) \in \mathcal{SV}_{N,N_q}^{\text{mr}}(x)\} \tag{33}$$

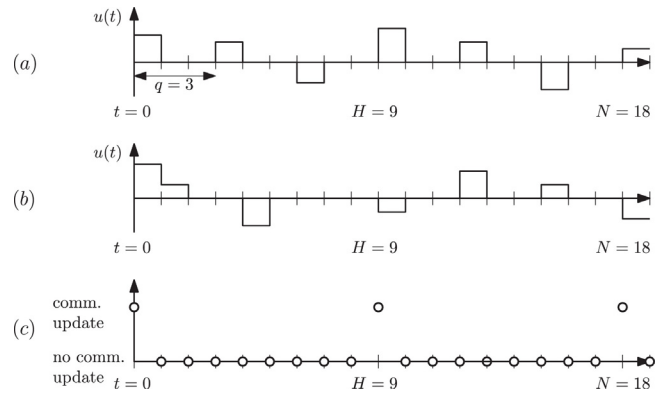

Fig. 2. Input profiles for a multi-rate MPC scheme (a) and a rollout scheme (b). The communication updates of both schemes are depicted in (c).

$$= J_N(x, M_{N,N_q}(\bar{\mathbf{s}}, \mathbf{v}^*(x))). \tag{34}$$

The corresponding optimal sequence of control inputs for the multi-rate approach is given by

$$\mathbf{u}^*_{\mathrm{mr}}(x) = M_{N,N_q}(\bar{\mathbf{s}}, \mathbf{v}^*_{\mathrm{mr}}(x)). \tag{35}$$

For $t \in \mathbb{N}$, the resulting MPC policy for (1) is now given by

$$u_t = \Pi_{t-t_l}\mathbf{u}^*_{\mathrm{mr}}(x_{t_l}), \quad t \in \mathbb{N}_{[t_l,t_{l+1})}, \quad l \in \mathbb{N}, \tag{36a}$$

$$\sigma_t = \begin{cases} 1, & \text{if } \ t = kq, \quad k \in \mathbb{N}, \\ 0, & \text{otherwise.} \end{cases} \tag{36b}$$

To guarantee recursive feasibility and closed-loop stability of this multi-rate MPC scheme we use a terminal set and cost method. We assume that there is a $K_{\mathrm{mr}} \in \mathbb{R}^{n_x \times n_u}$ such that

$$\overline{A}_q^\top P_{\mathrm{mr}}\overline{A}_q - P_{\mathrm{mr}} \preceq -Q_q - K_{\mathrm{mr}}^\top (S_q)^\top - S_q K_{\mathrm{mr}} - K_{\mathrm{mr}}^\top R_q K_{\mathrm{mr}}, \tag{37a}$$

$$\overline{A}_q \mathbb{T}_{\mathrm{mr}} \subseteq \mathbb{T}_{\mathrm{mr}}, \tag{37b}$$

$$\overline{A}_k \mathbb{T}_{\mathrm{mr}} \subseteq \mathbb{X}, \quad k = 1, 2, \ldots, q-1, \tag{37c}$$

$$K_{\mathrm{mr}} \mathbb{T}_{\mathrm{mr}} \subseteq \mathbb{U}, \tag{37d}$$

where $\overline{A}_k := A^k + \overline{B}_k K_{\mathrm{mr}}$ and $\overline{B}_k := A^{k-1}B$, $k = 1, 2, \ldots, q$. Note that $\overline{B}_q = B_q$. For our approach, we use a particular choice of K_{mr}, namely

$$K_{\mathrm{mr}} = -\left(R_q + B_q^\top P_{\mathrm{mr}} B_q\right)^{-1}\left(B_q^\top P_{\mathrm{mr}} A_q + S_q^\top\right), \tag{38}$$

which satisfies (37a) with equality.

It is not hard to show that the multi-rate MPC policy proposed in this section provides input profiles using the actuation resources at a rate of $1/q$ and the communication resources at a rate $1/H$, while still satisfying constraints on states and inputs for all $t \in \mathbb{N}$, i.e., (2). The schedule of update times $\bar{\mathbf{s}}$ is fixed and as a result, the obtained resource-aware input profiles for the multi-rate MPC scheme are *periodic* with times $t_l = lH$, $l \in \mathbb{N}$, and actuation updates at kq, $k \in \mathbb{N}$.

Remark 2. Note that here we considered periodic schedules that have actuation updates at the equidistant times kq, $k \in \mathbb{N}$. It is also possible to use other periodic update patterns in which one still obtains average actuation rates of $1/q$, but the actuation update times are not necessarily equidistantly distributed along the time axis. For ease of exposition, we did not consider the latter case although we believe that these can be handled as well using similar techniques. Moreover, the resulting aperiodic MPC scheme proposed in the next section can be obtained also for this setup in a rather straightforward manner.

4.2. Approach

In this section, we propose an MPC scheme that addresses variant (B), with $c_{\mathrm{resource}} = 1/q$, $q \in \mathbb{N}_{[1,N]}$, which is the same (average) actuation rate as the multi-rate MPC scheme in Section 4.1. However, the MPC scheme we propose allows for control update patterns that are *aperiodic*, and in doing so we aim to obtain better performance than the multi-rate scheme at the same update rate $1/q$. For the proposed rollout MPC strategy, we consider the class of schedules

$$\mathbb{S}^{\mathrm{ro}}_{N,N_q,H_q} = \{(s_0, s_1, \ldots, s_{N_q-1}) \in \mathbb{S}_{N,N_q} \mid s_0 = 0 < s_1 < s_2 < \ldots < s_{H_q-1} < H, \text{and} \ \ \forall i \in \{H_q, H_q+1, \ldots, N_q-1\}, \quad s_i = iq\}. \tag{39}$$

Note that the set of schedules $\mathbb{S}^{\mathrm{ro}}_{N,N_q,H_q}$ contains all schedules of length N with N_q control updates, where the last $N_q - H_q$ input updates are periodic with period q. Fig. 2 provides an illustration of the input profiles we consider here. Fig. 2(a) shows a (periodic) the input profile of a periodic multi-rate MPC scheme that updates the input at a rate $1/q = 1/3$ and utilizes communication resources at a rate of $1/H = 1/6$. Fig. 2(b) shows one of the aperiodic input profiles with a schedule contained in $\mathbb{S}^{\mathrm{ro}}_{N,N_q,H_q}$ that also utilizes communication resources at a rate of $1/H$ and updates the input at a rate of $1/q$ on average.

Remark 3. Note that the class of schedules (39) is periodic after time H and therefore contains $H!/((H-H_q)!\,H_q!)$ schedules of length $N \in \mathbb{N}$ with $N_q \in \mathbb{N}_{[1,N]}$ input updates, where H, N, q, H_q, and N_q satisfy Assumption 1. Here, N_q should be chosen large enough, such that $\mathbb{X}_f^{\mathrm{mr}} \neq \emptyset$. At scheduling times $t_l = lH$, $l \in \mathbb{N}$, communication resources are used to send a sequence of H_q control values to the actuators. Besides the communication rate, H and H_q also determine the number of schedules considered by the rollout strategy, and hence the computational complexity. See Remark 4 for a variant based on a reduced class of schedules.

For fixed N, q and $H \in \mathbb{N}_{\geq 1}$ satisfying Assumption 1 and given state $x_{t_l} = x \in \mathbb{X}$ at time t_l, $l \in \mathbb{N}$, we consider the following optimization problem

$$\min J_N(x, M_{N,N_q}(\mathbf{s}, \mathbf{v})) \tag{40a}$$

$$\text{w.r.t.} \quad (\mathbf{s}, \mathbf{v}) \in \mathcal{SV}^{\mathrm{ro}}_{N,N_q,H_q}(x), \tag{40b}$$

where

$$\mathcal{SV}^{\mathrm{ro}}_{N,N_q,H_q}(x) := \{(\mathbf{s}, \mathbf{v}) \in \mathbb{S}^{\mathrm{ro}}_{N,N_q,H_q} \times \mathbb{U}^{N_q} \mid \forall k \in \{0, 1, 2, \ldots, N-1\}, x_k(x, M_{N,N_q}(\mathbf{s}, \mathbf{v})) \in \mathbb{X} \text{ and } x_N(x, M_{N,N_q}(\mathbf{s}, \mathbf{v})) \in \mathbb{T}_{\mathrm{mr}}\}.$$

Note that $\mathbb{S}^{\mathrm{mr}}_{N,N_q} \subseteq \mathbb{S}^{\mathrm{ro}}_{N,N_q,H_q}$, and that, as a consequence, for $x \in \mathbb{X}$, it holds that $\mathcal{SV}^{\mathrm{mr}}_{N,N_q}(x) \subseteq \mathcal{SV}^{\mathrm{ro}}_{N,N_q,H_q}(x)$. Moreover,

$$\mathbb{X}_f^{\mathrm{mr}} \subseteq \mathbb{X}_f^{\mathrm{ro}} := \{x \in \mathbb{X} \mid \mathcal{SV}^{\mathrm{ro}}_{N,N_q,H_q}(x) \neq \emptyset\},$$

hence, the set of feasible states $\mathbb{X}_f^{\mathrm{ro}}$ for the rollout problem (40) will be larger than or equal to the set of feasible states $\mathbb{X}_f^{\mathrm{mr}}$ of the multi-rate MPC problem (30). For $x \in \mathbb{X}_f^{\mathrm{ro}}$, a combination of an optimal sequence of schedules and control values is known to exist, and we denote $\mathbf{s}^*_{\mathrm{ro}}(x) = \left(s^*_{\mathrm{ro},0}(x), s^*_{\mathrm{ro},1}(x), \ldots, s^*_{\mathrm{ro},N_q-1}(x)\right)$ and $\mathbf{v}^*_{\mathrm{ro}}(x) = \left(v^*_{\mathrm{ro},0}(x), v^*_{\mathrm{ro},1}(x), \ldots, v^*_{\mathrm{ro},N_q-1}(x)\right)$ as a particular one, i.e.,

$$(\mathbf{s}^*_{\mathrm{ro}}(x), \mathbf{v}^*_{\mathrm{ro}}(x)) \in \underset{(\mathbf{s},\mathbf{v}) \in \mathcal{SV}^{\mathrm{ro}}_{N,N_q,H_q}(x)}{arg\min} J_N(x, M_{N,N_q}(\mathbf{s}, \mathbf{v})). \tag{41}$$

For $x \in \mathbb{X}_f^{\mathrm{ro}}$, we use $V^{\mathrm{ro}}(x)$ to denote the corresponding minimum of (40). Hence, $V^{\mathrm{ro}} : \mathbb{X}_f^{\mathrm{ro}} \to \mathbb{R}_{\geq 0}$ is the MPC value function given by

$$V^{\mathrm{ro}}(x) = \min_{(\mathbf{s},\mathbf{v}) \in \mathcal{SV}^{\mathrm{ro}}_{N,N_q,H_q}(x)} J_N(x, M_{N,N_q}(\mathbf{s}, \mathbf{v})), = J_N(x, M_{N,N_q}(\mathbf{s}^*_{\mathrm{ro}}(x), \mathbf{v}^*_{\mathrm{ro}}(x))). \tag{42}$$

The optimal sequence of control inputs corresponding to the optimal schedule $\mathbf{s}^*_{\mathrm{ro}}(x) = (s^*_{\mathrm{ro},0}(x), s^*_{\mathrm{ro},1}(x), \ldots, s^*_{\mathrm{ro},N_q-1}(x))$ is given by

$$\mathbf{u}^*_{\mathrm{ro}}(x) := M_{N,N_q}(\mathbf{s}^*_{\mathrm{ro}}(x), \mathbf{v}^*_{\mathrm{ro}}(x)). \tag{43}$$

As $x \in \mathbb{X}_f^{\mathrm{mr}} \subseteq \mathbb{X}_f^{\mathrm{ro}}$ and $\mathcal{SV}^{\mathrm{mr}}_{N,N_q}(x) \subseteq \mathcal{SV}^{\mathrm{ro}}_{N,N_q,H_q}(x)$, it holds that

$$V^{\mathrm{ro}}(x) \leq V^{\mathrm{mr}}(x). \tag{44}$$

For $t \in \mathbb{N}$, the resulting MPC policy for (1) is now given for $t \in \mathbb{N}_{[t_l,t_{l+1})}$ with $l \in \mathbb{N}$ by

$$u_t = \Pi_{t-t_l} \mathbf{u}^*_{\mathrm{ro}}(x_{t_l}), \tag{45a}$$

$$\sigma_t = \begin{cases} 1, \text{if} \quad t - t_l = s^*_{\mathrm{ro},k}(x_{t_l}) \quad \text{for} \quad \text{some} \quad k = 0, 1, \ldots, N_q - 1, \\ 0, \text{otherwise.} \end{cases} \tag{45b}$$

4.3. Theoretical properties

The rollout MPC policy given by (45) has the following important properties.

Theorem 4 (Recursive feasibility and constraint satisfaction). *Under Assumption 1 and the conditions (37) with K_{mr} as in (38), it holds that the control law given by (45) is recursively feasible for all $x \in \mathbb{X}_f^{\mathrm{ro}}$ in the sense that the closed-loop system (1) and (45) leads to a trajectory $\{x_t\}_{t \in \mathbb{N}}$ defined for all $t \in \mathbb{N}$. Moreover, the closed-loop system (1) and (45) satisfies the constraints (2).*

Proof. Let $\hat{\mathbf{s}} = (0, q, \ldots, (H_q - 1)q) \in \mathbb{S}^{\mathrm{mr}}_{H,H_q}$ and $\hat{\mathbf{v}}(x) = (K_{\mathrm{mr}}\hat{x}_0, K_{\mathrm{mr}}\hat{x}_1, \ldots, K_{\mathrm{mr}}\hat{x}_{H_q-1}) \in (\mathbb{R}^{n_u})^{H_q}$, where $\hat{x}_0 := x_N(x, \mathbf{u}^*_{\mathrm{ro}}(x))$ and $\hat{x}_{i+1} = \overline{A}_q \hat{x}_i$, $i = 0, 1, \ldots, H_q - 2$. We denote $\tilde{\mathbf{s}}(x) = (\Pi_{H:N-1}\mathbf{s}^*_{\mathrm{ro}}(x), \hat{\mathbf{s}})$ and $\tilde{\mathbf{v}}(x) = (\Pi_{H:N-1}\mathbf{v}^*_{\mathrm{ro}}(x), \hat{\mathbf{v}}(x))$.

The proof is based on showing that for each $x \in \mathbb{X}_f^{\mathrm{mr}}$, it holds that $x_H(x, \mathbf{u}^*_{\mathrm{ro}}(x)) \in \mathbb{X}_f^{\mathrm{ro}}$, meaning our strategy is recursively feasible. In fact, we show that $M_{N,N_q}(\tilde{\mathbf{s}}(x), \tilde{\mathbf{v}}(x))$ is an admissible input sequence at $x_H(x, \mathbf{u}^*_{\mathrm{ro}}(x)) \in \mathbb{X}_f^{\mathrm{ro}}$.

We first show that $(\tilde{\mathbf{s}}(x), \tilde{\mathbf{v}}(x)) \in \mathcal{SV}^{\mathrm{ro}}_{N,N_q,H_q}(x_H(x, \mathbf{u}^*_{\mathrm{ro}}(x)))$. As $(\mathbf{s}^*_{\mathrm{ro}}(x), \mathbf{v}^*_{\mathrm{ro}}(x)) \in \mathcal{SV}^{\mathrm{ro}}_{N,N_q,H_q}(x)$, we have that $\hat{x}_0 = x_N(x, \mathbf{u}^*_{\mathrm{ro}}(x)) \in \mathbb{T}_{\mathrm{mr}}$. Due to (37b), this implies $\hat{x}_i \in \mathbb{T}_{\mathrm{mr}}$, $i = 1, 2, \ldots, H_q - 1$, and, consequently, $\hat{\mathbf{v}}(x) \in \mathbb{U}^H$, using (37d) and the fact that the origin is contained in $\mathbb{U}$. As $\hat{x}_i \in \mathbb{T}_{\mathrm{mr}}$, $i = 0, 1, \ldots, H_q - 1$, we can invoke (37c) to see that $x_j(\hat{x}_i, M_{q,1}(0, K_{\mathrm{mr}}\hat{x}_i)) = \overline{A}_j \hat{x}_i \in \mathbb{X}$, $j = 1, 2, \ldots, q-1$, $i = 0, 1, \ldots, H_q - 1$. Lastly, note that $x_N(x_H(x, \mathbf{u}^*_{\mathrm{ro}}(x)), M_{N,N_q}(\tilde{\mathbf{s}}(x), \tilde{\mathbf{v}}(x)) = x_H(\hat{x}_0, M_{H,H_q}(\hat{\mathbf{s}}, \hat{\mathbf{v}}(x)) = (\overline{A}_q)^{H_q}\hat{x}_0 \in \mathbb{T}_{\mathrm{mr}}$ due to (37b). Hence, $(\tilde{\mathbf{s}}(x), \tilde{\mathbf{v}}(x)) \in \mathcal{SV}^{\mathrm{ro}}_{N,N_q,H_q}(x_H(x, \mathbf{u}^*_{\mathrm{ro}}(x)))$ and thus $x_H(x, \mathbf{u}^*_{\mathrm{ro}}(x)) \in \mathbb{X}_f^{\mathrm{ro}}$. Because $x_0 \in \mathbb{X}_f^{\mathrm{ro}}$ and $u_t = \Pi_{t-t_l}\mathbf{u}^*_{\mathrm{ro}}(x_{t_l})$, for $t \in \mathbb{N}_{[t_l,t_{l+1})}$, $l \in \mathbb{N}$, we can conclude that the input and state constraints as in (2) are satisfied. □

Theorem 5 (Performance). *Under Assumption 1 and the conditions (37) with P_{mr} taken as the solution to (31) and K_{mr} as in (38), for all $x_0 \in \mathbb{X}_f^{\mathrm{mr}}$ we have that*

$$\sum_{t=0}^{\infty} L(x_t^{\mathrm{ro}}, u_t^{\mathrm{ro}}) \leq V^{\mathrm{ro}}(x_0) \leq V^{\mathrm{mr}}(x_0), \tag{46}$$

and for all $x_0 \in \mathbb{T}_{\mathrm{mr}}$ we have that

$$\sum_{t=0}^{\infty} L(x_t^{\mathrm{ro}}, u_t^{\mathrm{ro}}) \leq \sum_{t=0}^{\infty} L(x_t^{\mathrm{mr}}, u_t^{\mathrm{mr}}) = x_0^\top P_{\mathrm{mr}} x_0, \tag{47}$$

where $\{x_t^{\mathrm{mr}}\}_{t\in\mathbb{N}}$, $\{u_t^{\mathrm{mr}}\}_{t\in\mathbb{N}}$ and $\{x_t^{\mathrm{ro}}\}_{t\in\mathbb{N}}$, $\{u_t^{\mathrm{ro}}\}_{t\in\mathbb{N}}$ are the state and input trajectories produced by the closed-loop system (1) and (36), and the closed-loop system (1) and (45), respectively.

Proof. We start by showing (46) for $x_0 \in \mathbb{X}_f^{\mathrm{ro}}$. Note that for $x_0 \in \mathbb{X}_f^{\mathrm{ro}}$ the second inequality follows immediately from (44). To show the first inequality, let $\hat{\mathbf{u}}_i(x) = (K_{\mathrm{mr}}\hat{x}_i, 0, 0, \ldots, 0) \in (\mathbb{R}^{n_u})^q$, $i = 0, 1, \ldots, H_q - 1$, where $\hat{x}_0 := x_N(x, \mathbf{u}_{\mathrm{ro}}^*(x))$ and $\hat{x}_{i+1} = x_q(\hat{x}_i, \hat{\mathbf{u}}_i(x)) = \overline{A}_q\hat{x}_i$, $i = 0, 1, \ldots, H_q - 2$. We denote $\hat{\mathbf{u}}(x) = (\hat{\mathbf{u}}_0(x), \hat{\mathbf{u}}_1(x), \ldots, \hat{\mathbf{u}}_{H_q-1}(x))$ and define $\tilde{\mathbf{u}}(x) := (\Pi_{H:N-1}\mathbf{u}_{\mathrm{ro}}^*(x), \hat{\mathbf{u}}(x))$. For all $x \in \mathbb{X}_f^{\mathrm{ro}}$ we have

$$\begin{aligned}
V^{\mathrm{ro}}(x_H(x, \mathbf{u}_{\mathrm{ro}}^*(x))) - V^{\mathrm{ro}}(x) &\leq J_N(x_H(x, \mathbf{u}_{\mathrm{ro}}^*(x)), \tilde{\mathbf{u}}(x)) - J_N(x, \mathbf{u}_{\mathrm{ro}}^*(x)) = F(x_N(x_H(x, \mathbf{u}_{\mathrm{ro}}^*(x)), \tilde{\mathbf{u}}(x))) - \sum_{k=0}^{N-1} L(x_k(x, \mathbf{u}_{\mathrm{ro}}^*(x)), \Pi_k\mathbf{u}_{\mathrm{ro}}^*(x)) \\
&+ \sum_{k=0}^{N-1} L(x_k(x_H(x, \mathbf{u}_{\mathrm{ro}}^*(x)), \tilde{\mathbf{u}}(x)), \Pi_k\tilde{\mathbf{u}}(x)) - F(\hat{x}_0) = F(x_H(\hat{x}_0, \hat{\mathbf{u}}(x))) + \sum_{k=0}^{H-1} L(x_k(\hat{x}_0, \hat{\mathbf{u}}(x)), \Pi_k\hat{\mathbf{u}}(x)) - F(\hat{x}_0) \\
&- \sum_{k=0}^{H-1} L(x_k(x, \mathbf{u}_{\mathrm{ro}}^*(x)), \Pi_k\mathbf{u}_{\mathrm{ro}}^*(x)) = -\sum_{k=0}^{H-1} L(x_k(x, \mathbf{u}_{\mathrm{ro}}^*(x)), \Pi_k\mathbf{u}_{\mathrm{ro}}^*(x)) + \hat{x}_0^\top((\overline{A}_q^\top)^{H_q} P \overline{A}_q^{H_q} - P_{\mathrm{mr}} \\
&+ \sum_{i=0}^{H_q-1} ((\overline{A}_q^\top)^i \left(Q_q + K_{\mathrm{mr}}^\top S_q^\top + S_q K_{\mathrm{mr}} + K_{\mathrm{mr}}^\top R_q K_{\mathrm{mr}}\right) \overline{A}_q^i))\hat{x}_0 \leq -\sum_{k=0}^{H-1} L(x_k(x, \mathbf{u}_{\mathrm{ro}}^*(x)), \Pi_k\mathbf{u}_{\mathrm{ro}}^*(x)),
\end{aligned} \tag{48}$$

where in the last step we used that

$$(\overline{A}_q^\top)^{H_q} P_{\mathrm{mr}} \overline{A}_q^{H_q} - P_{\mathrm{mr}} \preceq -\sum_{i=0}^{H_q-1} ((\overline{A}_q^\top)^i \left(Q_q + K_{\mathrm{mr}}^\top S_q^\top + S_q K_{\mathrm{mr}} + K_{\mathrm{mr}}^\top R_q K_{\mathrm{mr}}\right) \overline{A}_q^i), \tag{49}$$

which can be derived from (37a) by pre-multiplication with $(\overline{A}_q^\top)^i$ and post-multiplication with $\overline{A}_q^i$ and observing that

$$(\overline{A}_q^\top)^{i+1} P_{\mathrm{mr}} \overline{A}_q^{i+1} - (\overline{A}_q^\top)^i P_{\mathrm{mr}} \overline{A}_q^i \preceq -(\overline{A}_q^\top)^i \left(Q_q + K_{\mathrm{mr}}^\top (S_q)^\top + S_q K_{\mathrm{mr}} + K_{\mathrm{mr}}^\top R_q K_{\mathrm{mr}}\right) \overline{A}_q^i, \tag{50}$$

for $i \in \{0, 1, \ldots, H_q - 1\}$. By summing (50) from 0 to $H_q - 1$ we obtain indeed (49).

At scheduling time $t_l = lH$, $l \in \mathbb{N}$, given $x_{t_l} \in \mathbb{X}_f^{\mathrm{ro}}$, note that $x_{t_{l+1}} = x_H(x_{t_l}, \mathbf{u}_{\mathrm{ro}}^*(x_{t_l}))$ and thus from (48) we obtain

$$V^{\mathrm{ro}}(x_{t_l}) - V^{\mathrm{ro}}(x_{t_{l+1}}) \geq \sum_{k=0}^{H-1} L(x_k(x_{t_l}, \mathbf{u}_{\mathrm{ro}}^*(x_{t_l})), \Pi_k\mathbf{u}_{\mathrm{ro}}^*(x_{t_l})). \tag{51}$$

Summing (51) from $l = 0$ to L and using $t_0 = 0$ we obtain

$$V^{\mathrm{ro}}(x_0) - V^{\mathrm{ro}}(x_{t_{L+1}}) \geq \sum_{l=0}^{L}\sum_{k=0}^{H-1} L(x_k(x_{t_l}, \mathbf{u}_{\mathrm{ro}}^*(x_{t_l})), \Pi_k\mathbf{u}_{\mathrm{ro}}^*(x_{t_l})), = \sum_{t=0}^{LH-1} ((x_t^{\mathrm{ro}})^\top Q x_t^{\mathrm{ro}} + (u_t^{\mathrm{ro}})^\top R u_t^{\mathrm{ro}}), \quad t \in \mathbb{N},$$

where $\{x_t^{\mathrm{ro}}\}_{t\in\mathbb{N}}$ and $\{u_t^{\mathrm{ro}}\}_{t\in\mathbb{N}}$ are the state and input trajectories produced by the closed-loop system (1) and (45) for $x_0^{\mathrm{ro}} = x_0$. Taking the limit $L \to \infty$ and using the fact that $V^{\mathrm{ro}}(x)$ only takes nonnegative values, we obtain

$$V^{\mathrm{ro}}(x_0) \geq \sum_{t=0}^{\infty} ((x_t^{\mathrm{ro}})^\top Q x_t^{\mathrm{ro}} + (u_t^{\mathrm{ro}})^\top R u_t^{\mathrm{ro}}). \tag{52}$$

To show (47) for $x_0 \in \mathbb{T}_{mr}$, note that the first inequality follows from (46), as $x_0 \in \mathbb{T}_{mr} \subseteq \mathbb{X}_f^{mr}$. With P_{mr} taken as the solution to (31) and K_{mr} as in (38), we have that (37a) holds with equality and, consequently, for $x_0 \in \mathbb{T}_{mr}$, it holds that $V^{\mathrm{mr}}(x_0) = \sum_{t=0}^{\infty} L(x_t^{\mathrm{mr}}, u_t^{\mathrm{mr}}) = x_0^\top P_{\mathrm{mr}} x_0$. □

In terms of Problem (B), both the proposed rollout MPC approach and the multi-rate MPC approach provide a solution with (average) actuation resource utilization $c_{\mathrm{resource}} = 1/q$ where the rate at which communication resources are used is $1/H$, i.e., only at the scheduling times $t_l = lH$, $l \in \mathbb{N}$. However, the proposed rollout MPC approach provides a guaranteed upper bound on the control cost $J_{\mathrm{control}}(x_0)$ that is not worse than the upper bound provided by the multi-rate MPC approach. Moreover, for $x_0 \in \mathbb{T}_{\mathrm{mr}}$ the control cost $J_{\mathrm{control}}(x_0)$ of the proposed rollout MPC approach cost is less than, or equal to the control cost of the multi-rate MPC scheme.

Theorem 6 (Closed-loop stability). *Under Assumption 1 and the conditions (37) with K_{mr} as in (38), the closed-loop system given by (1) and (45) is asymptotically stable for initial conditions in $\mathbb{X}_f^{\mathrm{ro}}$.*

Proof. Using $Q \succeq 0$ and $R \succ 0$, with $(A, Q^{1/2})$ detectable, and (52), the proof follows similar arguments as provided in the proof of Theorem 3 to show that for all $x_0 \in \mathbb{X}_f^{\mathrm{ro}}$ we have $\lim_{t\to\infty} x_t^{\mathrm{ro}} = 0$. In addition, since we have $V^{\mathrm{ro}}(x) \le c\,\|x\|^2$ for $x \in \mathbb{T}_{\mathrm{mr}}$ and since 0 is in the interior of $\mathbb{T}_{\mathrm{mr}}$ we also obtain Lyapunov stability from (52). Therefore, the closed-loop system given by (1) and (24) is asymptotically stable for all $x_0 \in \mathbb{X}_f$. □

Remark 4. One can impose restrictions on the set of schedules $\mathbb{S}^{\mathrm{ro}}_{N,N_q,H_q}$, for instance for computational reasons. In fact, all theorems in this section are also valid for a reduced set of schedules $\mathbb{S}^{\mathrm{ro,red}}_{N,N_q,H_q}$ that satisfies $\mathbb{S}^{\mathrm{mr}}_{N,N_q} \subseteq \mathbb{S}^{\mathrm{ro,red}}_{N,N_q,H_q} \subseteq \mathbb{S}^{\mathrm{ro}}_{N,N_q,H_q}$, see Section 7 for a discussion on the computational complexity of the presented algorithms.

5. Sporadically changing input profiles

The focus so far has been on providing resource-aware MPC schemes generating sparse input profiles. However, the rollout approaches in both Sections 3 and 4 can also be used to obtain sporadically changing input profiles. As mentioned before, these sporadically changing input profiles can be obtained by employing a "hold" strategy between input updates. For the hold strategy we introduce now the mappings $M^{\mathrm{hold}}_{N,i} : \mathbb{U} \times \mathbb{S}_{N,i} \times \mathbb{U}^i \to \mathbb{U}^N$, for $N \in \mathbb{N}$ and $i \in \mathbb{N}_{[1,N]}$, converting the information in $v_{\mathrm{old}} \in \mathbb{U}$, $\mathbf{s} \in \mathbb{S}_{N,i}$ and $\mathbf{v} \in \mathbb{U}^i$ to obtain the corresponding input sequence $\mathbf{u} = (u_0, u_1, \ldots, u_{N-1}) \in \mathbb{U}^N$ for (1) based on the hold strategy. Given $v_{\mathrm{old}} \in \mathbb{U}$, $\mathbf{s} = (s_0, s_1, \ldots, s_{i-1}) \in \mathbb{S}_{N,i}$ and $\mathbf{v} = (v_0, v_1, \ldots, v_{i-1}) \in \mathbb{U}^i$ we define $\mathbf{u} := M^{\mathrm{hold}}_{N,i}(v_{\mathrm{old}}, \mathbf{s}, \mathbf{v}) = (u_0, u_1, \ldots, u_{N-1}) \in \mathbb{U}^N$ with

$$u_k = \begin{cases} v_{\mathrm{old}}, & \text{when } \; k < s_0, \\ v_j, & \text{when } \; s_j \le k < s_{j+1}, \quad \text{for } \; j \in \{0, 1, \ldots, i-1\}, \end{cases}$$

for $k \in \mathbb{N}_{[0,N-1]}$, and some $v_{\mathrm{old}} \in \mathbb{R}^{n_u}$. Note that for the hold strategy it is necessary to keep track of the last input value, as this value is to be applied until the first update s_0 in the schedule $\mathbf{s}$.

The base policy used in Section 4 is a periodic policy with $c_{\mathrm{resource}} = 1/q$. In obtaining sparse control profiles using the zero strategy, this implies that the input profile contains one nonzero control value every q time steps. However, when aiming for sporadically changing input profiles using a hold strategy, this base policy is a periodic policy updating the control value once every q steps, but keeping the value constant between updates. For the hold strategy, the parameters $B_q := \sum_{k=0}^{i-1} A^k B$, $R_q := (\sum_{k=1}^{q-1} B^\top \sum_{i=0}^{k-1} (A^i)^\top Q \sum_{i=0}^{k-1} (A^i) B + R) + R$ and $S_q := \sum_{k=1}^{q-1} (A^k)^\top Q \sum_{i=0}^{k-1} A^i B$ are to be used for obtaining P_{mr} from the Riccati equation (31) and K_{mr} in (38). Moreover, for the conditions related to the terminal set and terminal cost, i.e., (37), $\overline{A}_k := A^k + \overline{B}_k K_{\mathrm{mr}}$ where $\overline{B}_k := \sum_{i=0}^{k-1} A^k B$, $k = 1, 2, \ldots, q$. Note that no changes are needed for the base policy used in Section 3, as it is based on a standard MPC setup that updates at every time.

The theoretical properties derived in Sections 3 and 4 also apply (mutatis mutandis) for our strategies when sporadically changing inputs are used. In the next section, we demonstrate the effectiveness of our rollout approaches in obtaining both sparse and sporadically changing input profiles.

6. Numerical examples

To illustrate the effectiveness of the rollout approaches proposed in this paper, we consider an open-loop unstable discrete-time system as in (1) with

$$A = \begin{bmatrix} 1.0221 & 0.25 \\ -0.01 & 0.9988 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Additionally, state constraints $-10 \le x_t^{(i)} \le 10$, $i = 1, 2$, where $x_t = [x_t^{(1)} \quad x_t^{(2)}]^\top$, and input constraints $-2 \le u_t \le 2$ are imposed. The weighting matrices of the running cost are chosen as

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad R = 0.1.$$

Section 6.1 shows the result for the rollout strategy in Section 3, where we consider both sparse and sporadically changing input profiles. The results of the rollout strategy in Section 4, considering sporadically changing input profiles, are presented in Section 6.2.

6.1. Rollout approach variant (A)

In this subsection we illustrate the approach of Section 3. The prediction horizon is fixed to $N = 60$. The terminal cost and terminal set satisfying (14), are given by $P = \begin{bmatrix} 6.2683 & 1.3785 \\ 1.3785 & 1.4530 \end{bmatrix}$, $K = [-0.8979 \; -1.1564]$ and $\mathbb{T}$ is chosen as $\{x \in \mathbb{X} \mid Dx \le E\}$, where

$$D = \begin{bmatrix} -0.9525 & 0.3044 \\ 0.9525 & -0.3044 \\ 0.6133 & 0.7898 \\ -0.6133 & -0.7898 \end{bmatrix} \quad \text{and} \quad E = \begin{bmatrix} 14.4157 \\ 14.4157 \\ 1.3660 \\ 1.3660 \end{bmatrix}.$$

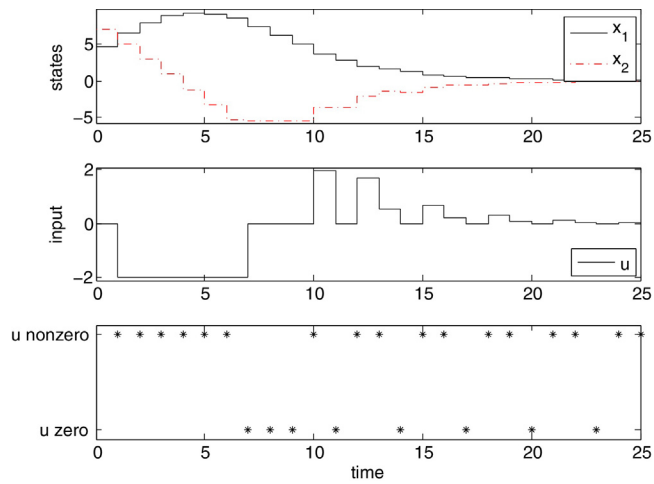

Fig. 3. Sparse input signal and resulting states versus time for $x_0 = [4.7239\ 7.0892]^\top$ and $\beta = 0.05$.

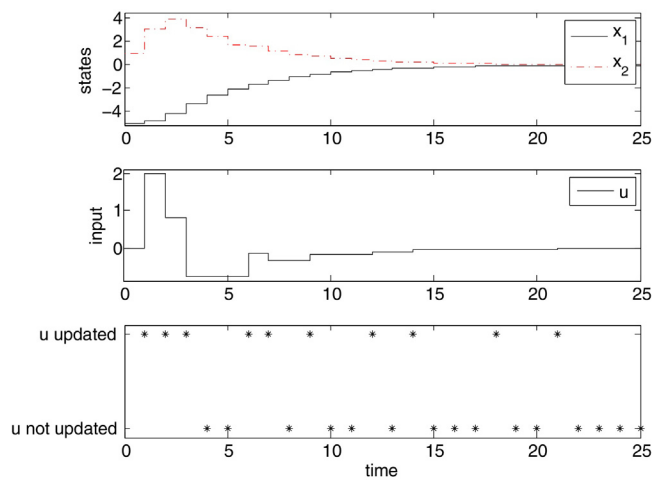

Fig. 4. Sporadically changing input signal and resulting states versus time for $x_0 = [-4.9630\ 0.9892]^\top$ and $\beta = 0.01$.

We consider 25 initial conditions randomly and uniformly distributed in $\mathbb{X}_f$. For the hold strategy, we set $u_{-1} := 0$. For $\beta = 0.05$, Figs. 3 and 4 show the state trajectories and corresponding input signals versus time, for sparse and sporadically changing input signals, respectively. We observe that even for a slight decrease in performance (i.e., at most 1% and 5% of the guaranteed upper bound on the infinite horizon control cost (see (18)), for sparse and sporadically changing input profiles), both approaches obtain input profiles with a significant reduction in resource utilization. Note that the obtained sparse input profile in Fig. 3 tends to a periodic profile when the state converges the origin (and constraints no longer play a role).

Figs. 5 and 6 show both the guaranteed upper bound on the control cost $(1+\beta)V(x_0)$ as well as the control cost (3) evaluated over a finite but sufficiently long simulation horizon[1] versus the average resource cost (4), averaged over 25 initial conditions, for sparse input profiles with $\beta \in \{0, 0.025, 0.050, \ldots, 0.800\}$ and sporadically changing input profiles with $\beta \in \{0, 0.0025, 0.0050, \ldots, 0.0500, 0.0750, 0.1000, \ldots, 0.1000\}$, respectively. From Fig. 5 we observe that for $\beta = 0.075$ we obtain an average resource utilization J_{resource} of 0.53. For higher values of β the resource utilization is even lower, e.g., for $\beta = 0.8$ we obtain $J_{\text{resource}} = 0.39$. However for $\beta = 0.8$ this also implies a significantly increased control cost. From Fig. 6 we observe that for $\beta = 0.00325$ we obtain an average resource utilization of 0.62, indicating a tremendous decrease in resource utilization at only a very slight increase in performance. However, similarly to the case of sparse input profiles, increasing β further only yields minor reductions in resource utilization. These results show that the proposed rollout approach can realize significant reductions in the utilization of resources, at only a slight degradation in performance.

6.2. Rollout approach variant (B)

In this subsection we illustrate the approach of Section 4. The prediction horizon is fixed to $N = 48$, and we choose $H = 12$ and $q = 4$ (satisfying Assumption 1). This means that $c_{\text{resource}} = 1/4$ and that $\mathbb{S}^{\text{ro}}_{N,N_q,H_q}$ contains 220 schedules that are used for our rollout approach.

[1] For each simulation, the simulation horizon t_{sim} is given by $t_{\text{sim}} = \min\{t \in \mathbb{N} \mid \|x_t\|_2 \leq 10^{-5}\}$, that is, the simulation is terminated when the 2-norm of the state is smaller than 10^{-5}.

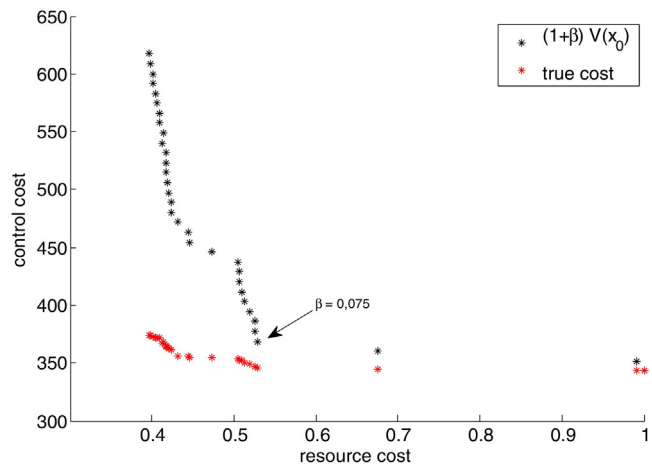

Fig. 5. Control cost $J_{\text{control}}(x_0)$ versus the (average) actuation rate J_{resource} for sparse input profiles for $\beta \in \{0, 0.025, 0.050, \ldots, 0.800\}$.

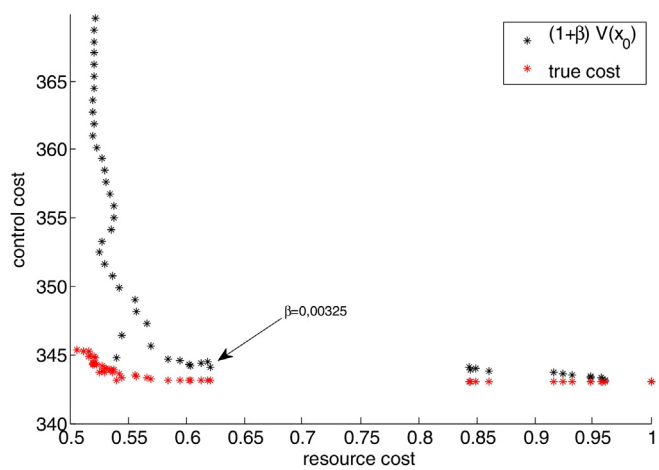

Fig. 6. Control cost $J_{\text{control}}(x_0)$ versus the (average) actuation rate J_{resource} for sporadically changing input profiles for $\beta \in \{0, 0.0025, 0.0050, \ldots, 0.0500, 0.0750, 0.1000, \ldots, 0.1000\}$.

To obtain sporadically changing input profiles using the zero strategy, the terminal cost and terminal set satisfying (37) are given by $P_{\text{mr}} = \begin{bmatrix} 6.4514 & 1.4028 \\ 1.4028 & 1.4594 \end{bmatrix}$, $K_{\text{mr}} = [-0.6928 - 1.1614]$ and $\mathbb{T}_{\text{mr}}$ is chosen as $\{x \in \mathbb{X} \mid D_z x \leq E_z\}$, where

$$D_z = \begin{bmatrix} -0.9785 & -0.2064 \\ 0.9785 & 0.2064 \\ 0.5123 & 0.8588 \\ -0.5123 & -0.8588 \end{bmatrix} \quad \text{and} \quad E_z = \begin{bmatrix} 4.2056 \\ 4.2056 \\ 1.4789 \\ 1.4789 \end{bmatrix}.$$

We consider 25 initial conditions randomly and uniformly distributed in $\mathbb{X}_f^{\text{mr}}$. Fig. 7 shows the sparse input signal and corresponding state trajectories versus time, for $x_0 = [5.1174\ 3.2030]^\top$. The average control cost over 25 initial conditions is 423.98 for the multi-rate strategy and 241.11 for the rollout strategy. This clearly shows the advantage of using aperiodic control updates, as with the same resource utilization the rollout approach obtains a significant reduction in the control cost.

7. Discussion

This section provides a discussion on the computational complexity and the implementation of both rollout approaches, where we assume the sets $\mathbb{X}$, $\mathbb{U}$ and $\mathbb{T}$ are described by polytopes.

At each time $t \in \mathbb{N}$, the rollout approach presented in Section 3 samples the state x_t and solves 2 quadratic programming (QP) problems (one for each of the schedules $\mathbf{s}_{\text{os}}^0$ and $\mathbf{s}_{\text{os}}^1$) with N and $N-1$ free control variables in order to obtain sparse input profiles and reduced communication from the controller to the actuators. Although sampling the state at each time $t \in \mathbb{N}$ requires that the communication resources between the sensors and the controller are used at each time step, see Remark 1, having feedback can be beneficial in case disturbances act on the system or if there is a mismatch between the system model (1) and the physical plant.

The rollout approach presented in Section 4 samples the state at times $t_l = lH$, $l \in \mathbb{N}$, with $H \in \mathbb{N}_{[q,N]}$. At the sampling times it solves $H!/((H-H_q)!\,H_q!)$ QPs (one for each schedule contained in $\mathbb{S}_{N,N_q,H_q}^{\text{ro}}$) with N_q free control variables, where ! denotes the factorial operator.

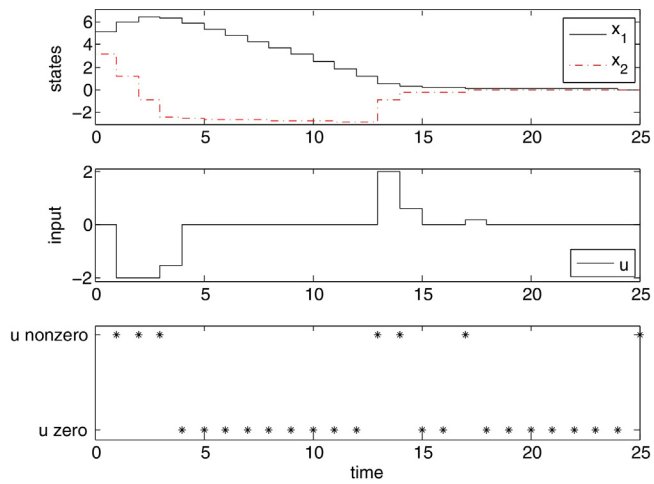

Fig. 7. Sparse input signal and resulting states versus time for $x_0 = [5.1174\ 3.2030]^\top$.

Table 1
Average actuation, communication and scheduling rates for variants of the rollout approach in Section 4.

	OL in H	CL in H	CL in H WR
Actuation rate	$\frac{1}{q}$	$\frac{1}{q}$	$\frac{1}{q}$
Communication rate	$\frac{1}{H}$	$\frac{1}{q}$	$\frac{1}{q}$
Scheduling rate	$\frac{1}{H}$	$\frac{1}{H}$	$\frac{1}{q}$

However, note that the next H time steps no computations are required. Moreover, (suboptimal) variants based on solving less QPs can also be used, by using less schedules, see Remark 4. The obtained optimal control sequence is then applied to the system in open-loop, for the next H time steps, see (45). We refer to this approach as open-loop in H (OL in H). From a NCS perspective, OL in H implies that communication resources are used at a rate $\frac{1}{H}$. When disturbances act on the system, we envision that variants based on more frequent feedback may provide better robustness with respect to these disturbances. Two of these variants are discussed briefly below, and their average actuation, communication and scheduling rates are summarized in Table 1.

Closed-loop in H (CL in H): Instead of applying the optimal rollout control sequence obtained at time t_l, $l \in \mathbb{N}$, in open loop for H steps, one can *sample* the state at the H_q times when the input is to be updated (according to the optimal schedule at time t_l) until time t_{l+1}. Based on the sampled state, the remainder of the control *values* until time t_{l+1} can be re-computed using the remainder of the optimal schedule decided at time t_l, see, e.g., [8].

Closed-loop in H with rescheduling (CL in H WR): This variant is also based on *sampling* the state at the H_q times when the input is to be updated. However, next to re-computing the control *values* also the remaining *schedule* until time t_{l+1} is re-computed based on the obtained samples of the state.

The principles of implementing these variants are similar to the ones discussed in Section 4.

8. Conclusions and future work

In this paper, we proposed two resource-aware MPC policies for discrete-time linear systems subject to state and input constraints. The strategies solve the co-design problem of both determining the time instants on which the updating/communication related to the control tasks take place, and selecting the new (continuous) control inputs. The first approach provides performance guarantees by design, in the sense that it allows the user to select a desired level of suboptimality for the performance (in terms of the original control cost function), where the degree of suboptimality provides a trade-off between the guaranteed closed-loop control performance on the one hand and the utilization of (communication/actuation) resources on the other hand. The second approach provides a guaranteed (average) resource utilization, while cleverly allocating these resources in order to maximize the control performance (i.e., minimize the control cost). Interestingly, the presented framework is flexible in the sense that it can be configured to generate both sparse and sporadically changing input profiles thereby having the ability to serve various application domains in which different resources are scarce or expensive, e.g., communication bandwidth in networked control systems, battery power in wireless control, or fuel in space or underwater vehicles. The effectiveness of both approaches was illustrated by means of numerical examples. In particular, the first rollout strategy led to significant reductions in the usage of the system's resources, without trading much of the guaranteed achievable performance. Moreover, the rollout strategy with guaranteed resource utilization shows that allocating these resources cleverly can lead to significant reductions of the control cost. As such, the proposed approaches provides viable control strategies for systems where both control performance and limited resource utilization are important and hard constraints on states and inputs have to be taken into account.

For ease of exposition, and not to blur the main ideas by further technicalities, the full state was assumed available for feedback. In the case where only a subset (output) of the state is available for feedback, a state estimator can be used. An important topic for future work is addressing the possibility of extending our formal results in this paper to this case as well. Although the focus of this paper is on the design of resource-aware control strategies for constrained linear systems and their stability and performance analysis, the inclusion of disturbances in the proposed schemes and guaranteeing formal (robust) stability and performance properties is another important topic for future work.

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