Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Output-Based Controller Synthesis for Networked Control Systems with Periodic Protocols and Time-Varying Transmission Intervals and Delays ⋆

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Abstract: In this paper, we will present LMI-based synthesis conditions for designing dynamic output-based controllers for networked control systems (NCSs). In particular, the controller is designed to render a linear time-invariant plant stable given a network that causes the transmission intervals and delays to be time varying and that precludes that all sensors and actuators data can be sent simultaneously. The latter fact necessitates the use of a scheduling protocol and, in this paper, we will focus on a periodic protocol. We will formulate the networked plant model as a discrete-time switched linear parameter-varying system and we will use convex overapproximation techniques to arrive at a model that is amenable for controller synthesis. The controller synthesis result is inspired from controller synthesis results for switched systems and yields a switched controller that is robust for the aforementioned network phenomena. We will illustrate the synthesis results on a benchmark example of a batch reactor, which shows that our novel results can outperform existing results on controller synthesis.

1. INTRODUCTION

Networked control systems (NCSs) are feedback control systems, in which the control loops are closed over a shared communication network. Compared to traditional control systems, in which the sensors, controllers and actuators are connected through dedicated point-to-point connections, NCSs offer advantages, such as, e.g., increased flexibility and maintainability of the system architecture and reduced wiring. These advantages explain why NCSs have received a significant amount of attention in the literature in the recent years. However, NCSs introduce new challenges that need to be overcome before the advantages they offer can be fully exploited. In particular, the communication network introduces time-varying delays, time-varying sampling/transmission intervals and packet dropouts in the control loop. Moreover, as all sensors and actuators are connected to the controller(s) through the same network, it is no longer possible to transmit all information simultaneously. Therefore, controller design methods are needed that explicitly deal with these artifacts.

To study the impact of these networked-induced phenomena, several models have been developed. Roughly speaking, three different approaches towards modelling and stability analysis have been considered in the literature, i.e., an approach based on discrete-time parameter-varying systems, see, e.g., [Cloosterman et al., 2009, Donkers et al., 2011, Hetel et al., 2008], a continuous-time approach based on impulsive or jump-flow systems, see, e.g., [Heemels et al., 2010a, Nešić and Teel, 2004] and an approach based on delayed (impulsive) differential equations, see, e.g., [Gao et al., 2008, Naghshtabrizi et al., 2010, Suplin et al., 2007]. Unfortunately, most of these results focus on stability and performance analysis and do not offer direct network-aware controller synthesis methods. As a consequence, in the context of controller design, this means that these methods can only be used in an emulation-based setting, in which the controller is designed by ignoring the network artifacts and an a posteriori stability and performance analysis has to be done to conclude if the controller still works satisfactorily in the presence of the communication network.

Although some results exist that allow for direct network-aware controller synthesis for NCSs, they can only be applied under certain specific conditions. Namely, it is either assumed that the full state is available for feedback [Al-Areqi et al., 2011, Antunes et al., 2012, Cloosterman et al., 2010, Zhang and Yu, 2008] or that a static-output feedback controller can be used [Hao and Zhao, 2010], that all sensors and actuators can transmit data simultaneously [Hao and Zhao, 2010, Jungers et al., 2013, Zhang and Yu, 2008], that delays and transmission interval are con-

⋆ This work is supported by the Innovational Research Incentives Scheme under the VICI grant “Wireless control systems: A new frontier in automation” (no. 11382) awarded by NWO (Dutch Organisation for Scientific Research) and STW (Dutch Science Foundation), and the European Union Seventh Framework Programme [FP7/2007-2013] under grant agreement no. 257462 HYCON2 “Network of excellence”

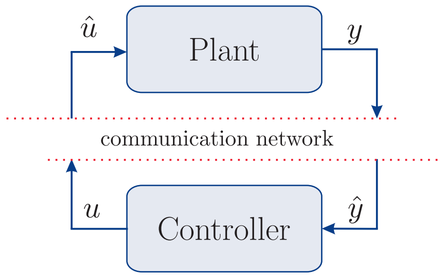

Fig. 1. Networked control system schematic.

stant [Antunes et al., 2012, Dačić and Nešić, 2007, Zhang and Hristu-Varsakelis, 2006], or that an observer/state-feedback structure can be imposed a priori [Bauer et al., 2013, Zhang and Hristu-Varsakelis, 2006]. Hence, to the best of the authors' knowledge, synthesis of stabilising dynamic output-based controllers for NCSs for given scheduling protocols, not imposing an observer-based controller structure a priori and allowing for time-varying delays and transmission intervals, is an open problem.

In this paper, we will propose a solution to this open problem. In particular, we assume that the plant is linear-time invariant, the scheduling is done by a periodic scheduling protocol (see, e.g., [Donkers et al., 2011, Nešić and Teel, 2004]), and the sampling intervals and delays are bounded, but can be time varying. This will allows us to formulate the plant and communication network as a discrete-time switched linear parameter-varying system (see, e.g., [Donkers et al., 2011]). Using this modelling setup, we propose conditions to synthesise a stabilising output-based controller, which are based on extensions of the results in [Bernussou et al., 2005, Deaecto et al., 2011] towards switched and parameter-varying systems. We will illustrate the results on a benchmark example of a batch reactor [Bauer et al., 2013, Dačić and Nešić, 2007, Donkers et al., 2011, Heemels et al., 2010a, Nešić and Teel, 2004], showing that our novel results can outperform existing results on controller synthesis [Bauer et al., 2013, Dačić and Nešić, 2007].

The remainder of this paper is organised as follows. After introducing the necessary notational conventions, we introduce the model of the NCS in Section 2 and propose a method to write it as a discrete-time switched linear uncertain system. Subsequently, in Section 3, we present the controller synthesis conditions in terms of LMIs. Finally, we illustrate the results using a numerical benchmark example in Section 4 and we draw conclusions in Section 5.

Nomenclature The following notational conventions will be used. $\text{diag}(A_1, \dots, A_n)$ denotes a block-diagonal matrix with the entries $A_1, \dots, A_n$ on the diagonal and $A^\top \in \mathbb{R}^{m\times n}$ denotes the transpose of matrix $A \in \mathbb{R}^{n\times m}$. For a vector $x \in \mathbb{R}^n$, we denote by x^i the i-th component and, for brevity, we sometimes write symmetric matrices of the form $\begin{bmatrix} A & B^\top \\ B & C \end{bmatrix}$, as $\begin{bmatrix} A & \bullet \\ B & C \end{bmatrix}$. Finally, by $\lim_{s\downarrow t}$, we denote the limit as s approaches t from above, and the convex hull a set $\mathcal{A}$ is denoted by $\text{co}\mathcal{A}$.

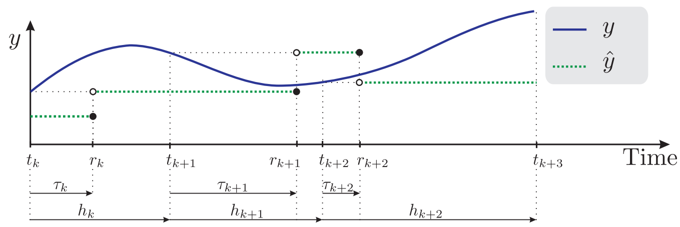

Fig. 2. Illustration of a typical evolution of y and $\hat{y}$.

2. THE NETWORKED CONTROL SYSTEM MODEL

In this section, we introduce the networked control system (NCS) under study, which is schematically shown in Fig. 1. We will first formulate the NCS and make a discrete-time model of the plant and the communication network, see also Fig. 1. Subsequently, we will overapproximate this model to make it amenable for controller synthesis.

2.1 Description of the Networked Control System

Let us consider the linear time-invariant (LTI) continuous-time plant given by

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}t}x^p(t) = A^p x^p(t) + B^p \hat{u}(t) \\ \quad y(t) = C^p x^p(t), \end{cases} \tag{1}$$

where $x^p \in \mathbb{R}^{n_p}$ denotes the state of the plant, $\hat{u} \in \mathbb{R}^{n_u}$ the most recently received control variable, $y \in \mathbb{R}^{n_y}$ the (measured) output of the plant and $t \in \mathbb{R}^+$ the time. Since the plant and controller are communicating via a network, the actual input of the plant $\hat{u} \in \mathbb{R}^{n_u}$ is not equal to the output of the controller u and the actual input of the controller $\hat{y} \in \mathbb{R}^{n_y}$ is not equal to the output of the plant y. The situation described above is illustrated in Fig. 1 and Fig. 2 and we call $\hat{u}$ and $\hat{y}$ the 'networked versions' of u and y, respectively. The control input is implemented using a zero-order hold

$$\hat{u}(t) = \hat{u}(t_k), \quad \forall t \in (r_k, r_{k+1}] \tag{2}$$

In this expression, $t_k, k \in \mathbb{N}$, denote the transmission instants at which (parts of) the (sampled) outputs of the plant $y(t_k)$ and controller $u(t_k)$ have been transmitted over the network and r_k, $k \in \mathbb{N}$, denotes the arrival instant, see also Fig. 2.

To introduce these networked versions $\hat{u}$ and $\hat{y}$ properly, we have to explain the functioning of the network, see also [Bauer et al., 2013, Dačić and Nešić, 2007, Donkers et al., 2011, Heemels et al., 2010a, Nešić and Teel, 2004]. The plant is equipped with sensors and actuators that are grouped into N nodes. At each transmission instant t_k, $k \in \mathbb{N}$, one node, denoted by $\sigma_k \in \{1, 2, \dots, N\}$, obtains access to the network and transmits its corresponding values. These transmitted values are received and implemented on the controller or the plant at arrival instant r_k. As in [Donkers et al., 2011], a new transmission only occurs after the previous transmission has arrived, i.e., $t_{k+1} > r_k \geqslant t_k$, for all $k \in \mathbb{N}$. In other words, we consider the sampling interval to be lower bounded and the delays to be smaller than the transmission interval. After each transmission and reception, the values in $\hat{y}$ and $\hat{u}$ are updated with the newly received values, while the other values in $\hat{y}$ and $\hat{u}$ remain the same, as no additional information is received.

By defining $y_k := y(t_k)$ and $\hat{u}_k := \lim_{t\downarrow r_k} \hat{u}(t)$, we can express this constrained data exchange as

$$\begin{cases} \hat{y}_k = \Gamma^y_{\sigma_k} y_k + (I - \Gamma^y_{\sigma_k})\hat{y}_{k-1} \\ \hat{u}_k = \Gamma^u_{\sigma_k} u_k + (I - \Gamma^u_{\sigma_k})\hat{u}_{k-1}, \end{cases} \tag{3}$$

for all $k \in \mathbb{N}$, where $\Gamma_{\sigma_k} := \operatorname{diag}(\Gamma^y_{\sigma_k}, \Gamma^u_{\sigma_k})$ is a diagonal matrix, given by

$$\Gamma_i = \operatorname{diag}(\gamma_{i,1}, \ldots, \gamma_{i,n_y+n_u}). \tag{4}$$

when $\sigma_k = i$. In (4), the elements $\gamma_{i,j}$, with $i \in \{1, 2, \ldots, N\}$ and $j \in \{1, 2, \ldots, n_y\}$, are equal to one, if plant output y^j is in node i, elements $\gamma_{i,j+n_y}$, with $i \in \{1, 2, \ldots, N\}$ and $j \in \{1, 2, \ldots, n_u\}$, are equal to one, if controller output u^j is in node i, and are zero elsewhere.

The value of $\sigma_k \in \{1, 2, \ldots, N\}$ in (3) indicates which node is given access to the network at transmission instant t_k, $k \in \mathbb{N}$. Indeed, (3) reflects that the values in $\hat{u}$ and $\hat{y}$ corresponding to node σ_k are updated with the corresponding transmitted values, while the others remain the same. A scheduling protocol determines the sequence $(\sigma_0, \sigma_1, \ldots)$ and in this paper, we will assume a periodic protocol is used (see, e.g., Donkers et al. [2011]), which for ease of exposition, is restricted to be the well-known Round-Robin protocol (see, e.g., [Nešić and Teel, 2004]) given by

$$\sigma_{k+1} = \begin{cases} \sigma_k + 1 & \text{when} \quad \sigma_k < N \\ 1 & \text{when} \quad \sigma_k = N \end{cases} \tag{5}$$

for all $k \in \mathbb{N}$, for some $\sigma_0 \in \{1, 2, \ldots, N\}$, and for a given number of nodes $N \in \mathbb{N}$.

The transmission instants t_k, as well as the arrival instants r_k, $k \in \mathbb{N}$, are not necessarily distributed equidistantly in time. In fact, both the transmission intervals $h_k := t_{k+1} - t_k$ and the transmission delays $\tau_k := r_k - t_k$ are varying in time, as is also illustrated in Fig. 2. We assume that the variations in the transmission interval and delays are bounded and contained in the sets $[\underline{h}, \overline{h}]$ and $[\underline{\tau}, \overline{\tau}]$, respectively, with $\overline{h} \geqslant \underline{h} > 0$ and $\overline{\tau} \geqslant \underline{\tau} \geqslant 0$. Since we assumed that each transmission delay τ_k is smaller than the corresponding transmission interval h_k, we have that $(h_k, \tau_k) \in \Theta$, for all $k \in \mathbb{N}$, where

$$\Theta := \left\{(h, \tau) \in \mathbb{R}^2 \mid h \in [\underline{h}, \overline{h}], \tau \in [\underline{\tau}, \min\{h, \overline{\tau}\})\right\}. \tag{6}$$

2.2 The Discrete-Time Networked Plant Model

To design a discrete-time controller for this plant and communication network, we discretise (1) with (2) at the transmission times t_k, i.e., $x^p_k := x^p(t_k)$, resulting in

$$\begin{aligned} x^p_{k+1} &= e^{A^p h_k} x^p_k + \textstyle\int_0^{h_k} e^{A^p(h_k-s)} B^p \hat{u}(t_k + s)\mathrm{d}s \\ &= e^{A^p h_k} x^p_k + \textstyle\int_0^{\tau_k} e^{A^p(h_k-s)}\mathrm{d}s B^p \hat{u}_{k-1} \\ &\qquad + \textstyle\int_{\tau_k}^{h_k} e^{A^p(h_k-s)}\mathrm{d}s B^p \hat{u}_k, \end{aligned} \tag{7}$$

for all $k \in \mathbb{N}$. Substituting (3) into (7) yields

$$\begin{aligned} x^p_{k+1} &= e^{A^p h_k} x^p_k + \textstyle\int_0^{h_k-\tau_k} e^{A^p s}\mathrm{d}s B^p \Gamma^u_{\sigma_k} u_k \\ &\quad + (\textstyle\int_0^{h_k} e^{A^p s}\mathrm{d}s B^p - \int_0^{h_k-\tau_k} e^{A^p s}\mathrm{d}s B^p \Gamma^u_{\sigma_k})\hat{u}_{k-1}. \end{aligned} \tag{8}$$

The combined model of the plant and the network effects can be obtained by combining (8) and (3) and by defining $\bar{x}_k = [(x^p_k)^\top \ \hat{y}^\top_{k-1} \ \hat{u}^\top_{k-1}]^\top$. This results in a discrete-time model given by

$$\begin{cases} \bar{x}_{k+1} = A_{\sigma_k,h_k,\tau_k}\bar{x}_k + B_{\sigma_k,h_k,\tau_k} u_k \\ \begin{bmatrix}\hat{y}_k \\ \hat{u}_k\end{bmatrix} = C_{\sigma_k}\bar{x}_k + D_{\sigma_k} u_k, \end{cases} \tag{9}$$

in which

$$A_{\sigma,h,\tau} = \begin{bmatrix} e^{A^p h} & 0 & E_h - E_{h-\tau}\Gamma^u_\sigma \\ \Gamma^y_\sigma C^p & (I - \Gamma^y_\sigma) & 0 \\ 0 & 0 & (I - \Gamma^u_\sigma) \end{bmatrix}, \quad B_{\sigma,h,\tau} = \begin{bmatrix} E_{h-\tau}\Gamma^u_\sigma \\ 0 \\ \Gamma^u_\sigma \end{bmatrix}$$

$$C_\sigma = \begin{bmatrix} \Gamma^y_\sigma C^p & (I - \Gamma^y_\sigma) & 0 \\ 0 & 0 & (I - \Gamma^u_\sigma) \end{bmatrix}, \qquad D_\sigma = \begin{bmatrix} 0 \\ \Gamma^u_\sigma \end{bmatrix} \tag{10}$$

with $E_\rho = \int_0^\rho e^{A^p s}\mathrm{d}s B^p$, $\rho \in \mathbb{R}$.

2.3 Convex Overapproximations

The networked plant model in the form of a switched uncertain discrete-time system (9) with (5) and $(h_k, \tau_k) \in \Theta$, as in (6) and $k \in \mathbb{N}$, cannot be directly used to develop controller synthesis techniques, due to the exponential appearance of h_k and τ_k in the matrices of (9). To make the system amenable controller synthesis, a procedure presented in [Donkers et al., 2011] is used to overapproximate system (9) by a polytopic system with norm-bounded additive uncertainty of the form

$$\bar{x}_{k+1} = \sum_{l=1}^{L} \alpha^l_k \big((\bar{A}_{\sigma_k,l} + \bar{F}\Delta_k \bar{G}_{\sigma_k})\bar{x}_k + (\bar{B}_{\sigma_k,l} + \bar{F}\Delta_k \bar{H}_{\sigma_k})u_k\big), \tag{11}$$

where $\bar{A}_{\sigma,l}$, $\bar{B}_{\sigma,l}$, $\bar{F}$, $\bar{G}_\sigma$, $\bar{H}_\sigma$ are matrices of appropriate dimensions, $\sigma \in \{1, 2, \ldots, N\}$ and $l \in \{1, 2, \ldots, L\}$, with L the number of vertices of the polytope. The vector $\alpha_k = [\alpha^1_k \ \ldots \ \alpha^L_k]^\top \in \mathcal{A}$, $k \in \mathbb{N}$, is time-varying with

$$\mathcal{A} = \Big\{\alpha \in \mathbb{R}^L \,\Big|\, \sum_{l=1}^{L} \alpha^l = 1, \alpha^l \geqslant 0, \text{ for } \ l \in \{1, 2, \ldots, L\}\Big\} \tag{12}$$

and $\Delta_k \in \boldsymbol{\Delta}$, $k \in \mathbb{N}$, with the norm-bounded additive uncertainty set $\boldsymbol{\Delta} = \{\Delta \in \mathbb{R}^{2n_p \times 2n_p} \,|\, \|\Delta\| < 1\}$. The system (11) is an overapproximation of (9) in the sense that for all $\sigma \in \{1, 2, \ldots, N\}$, it holds that

$$\begin{aligned} &\{[A_{\sigma,h,\tau} \ \ B_{\sigma,h,\tau}] \mid (h, \tau) \in \Theta\} \\ &\subseteq \Big\{ \sum_{l=1}^{L} \alpha^l([\bar{A}_{\sigma,l} \ \ \bar{B}_{\sigma,l}] + \bar{F}\Delta[\bar{G}_\sigma \ \ \bar{H}_\sigma]) \,|\, \alpha \in \mathcal{A}, \Delta \in \boldsymbol{\Delta}\Big\}. \end{aligned} \tag{13}$$

Due to this inclusion, stabilisation of (11) for all $\alpha_k \in \mathcal{A}$ and $\Delta_k \in \boldsymbol{\Delta}, k \in \mathbb{N}$, implies stabilisation of (9) for all $(h_k, \tau_k) \in \Theta$, $k \in \mathbb{N}$. Although many overapproximation techniques are available, see, e.g., the survey [Heemels et al., 2010b], we employ here the gridding-based procedure of [Donkers et al., 2011] to overapproximate system (9), such that (13) holds. This choice is motivated by the fact that this gridding-based procedure makes an overapproximation that is arbitrarily tight in an appropriate sense and does not introduce conservatism if the number of grid points is sufficiently large and they are well distributed, see [Donkers et al., 2011]. Below, we briefly summarise the main ideas and introduce the relevant notation required later to formulate the main synthesis results.

To construct an overapproximation of (9) in the form (11) using a gridding-based approach, a set of grid points $\{(\tilde{h}_1, \tilde{\tau}_1), ..., (\tilde{h}_L, \tilde{\tau}_L)\} \in \Theta$ is chosen such that

$\text{co}(\cup_{l=1}^{L}\{\tilde{h}_1, \tilde{\tau}_1)\}) = \Theta$. The choice of the grid points directly influences the tightness of the overapproximation. In [Donkers et al., 2011], a procedure is given to determine the set of grid points $\{(\tilde{h}_1, \tilde{\tau}_1), ..., (\tilde{h}_L, \tilde{\tau}_L)\} \in \Theta$ by iteratively placing each grid point at the location of the worst-case approximation error, thus, iteratively tightening the overapproximation. For the sake of brevity we do not provide the procedure here but instead refer the reader to [Donkers et al., 2011] for details.

A procedure similar to that of [Donkers et al., 2011] leads to a new overapproximation (11) of (9) satisfying (13), with $\bar{A}_{\sigma,l} := A_{\sigma,\tilde{h}_l,\tilde{\tau}_l}$, $\bar{B}_{\sigma,l} := B_{\sigma,\tilde{h}_l,\tilde{\tau}_l}$, for $\sigma \in \{1, 2, \ldots, N\}$, $l \in \{1, 2, \ldots, L\}$. Moreover, we define

$$\bar{G}_\sigma := \begin{bmatrix} T^{-1}A^p & 0 & T^{-1}B^p \\ 0 & 0 & -T^{-1}B^p\Gamma_\sigma^u \end{bmatrix}, \quad \bar{H}_\sigma := \begin{bmatrix} 0 \\ T^{-1}B^p\Gamma_\sigma^u \end{bmatrix}, \tag{14}$$

for $\sigma \in \{1, 2, \ldots, N\}$, and

$$\bar{F} := \begin{bmatrix} T & T \\ 0 & 0 \\ 0 & 0 \end{bmatrix} U. \tag{15}$$

The matrix T stems from the real Jordan form decomposition of the matrix A^p, see, e.g., [Horn and Johnson, 1985], i.e., $A^p := T\Lambda T^{-1}$, where T is an invertible matrix and $\Lambda = \text{diag}(\Lambda_1, \ldots, \Lambda_Q)$ with $\Lambda_q \in \mathbb{R}^{n_{\lambda_q} \times n_{\lambda_q}}$, $q \in \{1, 2, \ldots, Q\}$, the q-th real Jordan block of A^p. Finally, the matrix U is given by

$$U := \text{diag}(\delta_1^{E_h} I_1, \ldots, \delta_Q^{E_h} I_Q, \delta_1^{E_{h-\tau}} I_1, \ldots, \delta_Q^{E_{h-\tau}} I_Q), \tag{16}$$

where I_q is the $n_{\lambda_q} \times n_{\lambda_q}$ identity matrix and $\delta_q^{E_h}$ and $\delta_q^{E_{h-\tau}}$ are the worst-case approximation errors for each real Jordan block $q \in \{1, 2, ..., Q\}$. They can be computed using [Donkers et al., 2011, Eqn. (35b) and (35c)]. The reason that we do not need (35a) of [Donkers et al., 2011] is that we can write $e^{A^p h} = I + \int_0^h e^{A^p s} A^p \text{d}s$, making (35a) and (35b) of [Donkers et al., 2011] actually a scaled version of each other.

3. OUTPUT-BASED CONTROLLER DESIGN

In this section, we will provide controller synthesis conditions for a stabilising dynamic output-feedback controller for the overapproximated system (11) with

$$\begin{bmatrix} \hat{y}_k \\ \hat{u}_k \end{bmatrix} = C_{\sigma_k}\bar{x}_k + D_{\sigma_k}u_k \tag{17}$$

and a given protocol (5). The controller is a switched dynamic output-feedback controller of the form

$$\begin{cases} x_{k+1}^c = A_{\sigma_k}^c x_k^c + B_{\sigma_k}^c \begin{bmatrix} \hat{y}_k \\ \hat{u}_k \end{bmatrix} \\ u_k = C_{\sigma_k}^c x_k^c \end{cases} \tag{18}$$

which results in the closed-loop system

$$\begin{bmatrix} \bar{x}_{k+1} \\ x_{k+1}^c \end{bmatrix} = \Big(\sum_{l=1}^{L} \alpha_k^l \mathcal{A}_{\sigma_k,l} + \mathcal{F}\Delta_k\mathcal{G}_{\sigma_k} \Big) \begin{bmatrix} \bar{x}_k \\ x_k^c \end{bmatrix} \tag{19}$$

where

$$\mathcal{A}_{\sigma,l} = \begin{bmatrix} \bar{A}_{\sigma,l} & \bar{B}_{\sigma,l}C_\sigma^c \\ B_\sigma^c C_\sigma & A_\sigma^c + B_\sigma^c D_\sigma C_\sigma^c \end{bmatrix}, \quad \mathcal{F} = \begin{bmatrix} \bar{F} \\ 0 \end{bmatrix}, \quad \mathcal{G}_\sigma = [\bar{G}_\sigma \;\; \bar{H}_\sigma C_\sigma^c]. \tag{20}$$

The objective is to synthesise (18), such that (19) becomes asymptotically stable for a given switching sequence (5) and for all $\alpha_k \in \mathcal{A}$ and $\Delta_k \in \mathbf{\Delta}$, $k \in \mathbb{N}$.

The results are based on the existence of a Lyapunov function for the closed-loop system (19). Finding a Lyapunov function for a given controller (18) can be done using the following lemma, which is adopted from Theorem IV.5 in [Donkers et al., 2011].

Lemma 1. Assume there exist symmetric matrices $\mathcal{P}_i$, $i \in \{1, 2, \ldots, N\}$, such that the following inequalities

$$\begin{bmatrix} \mathcal{P}_i & \bullet & \bullet & \bullet \\ 0 & I & \bullet & \bullet \\ \mathcal{P}_{i+1}\mathcal{A}_{i,l} & \mathcal{P}_{i+1}\mathcal{F} & \mathcal{P}_{i+1} & \bullet \\ \mathcal{G}_i & \mathbf{0} & 0 & I \end{bmatrix} \succ 0 \tag{21}$$

are satisfied for all $i \in \{1, 2, \ldots, N\}$ and $l \in \{1, 2, \ldots, L\}$, with $\mathcal{P}_{N+1} = \mathcal{P}_1$. Then, the closed-loop system (19) with switching sequence (5) is asymptotically stable for all $\alpha_k \in \mathcal{A}$ and $\Delta_k \in \mathbf{\Delta}$, $k \in \mathbb{N}$.

To solve the controller synthesis problem, we face the difficulty that conditions of Lemma 1 are nonlinear with respect to the dynamic output-feedback controller entries and the matrices of the quadratic Lyapunov function. Ideas from [Bernussou et al., 2005] are used to overcome this difficulty. In particular, we will use the following lemma from [Deaecto et al., 2011] as an intermediate result.

Lemma 2. Given a nonsingular matrix V_i and symmetric matrices Y_i and X_i, $i \in \{1, 2, \ldots, N\}$, satisfying

$$\begin{bmatrix} Y_i & \bullet \\ I & X_i \end{bmatrix} \succ 0. \tag{22}$$

Then, there exist nonsingular matrices U_i and symmetric matrices $\hat{Y}_i$ and $\hat{X}_i$, such that

$$\mathcal{S}_i^{-1} = \begin{bmatrix} Y_i & \bullet \\ V_i^\top & \hat{Y}_i \end{bmatrix} \succ 0, \quad \text{and} \quad \mathcal{S}_i = \begin{bmatrix} X_i & \bullet \\ U_i^\top & \hat{X}_i \end{bmatrix} \succ 0 \tag{23}$$

with $U_i = (I - X_iY_i)V_i^{-\top}$, $\hat{Y}_i = V_i^\top(Y_i - X_i^{-1})^{-1}V_i$ and $\hat{X}_i = V_i^{-1}(Y_iX_iY_i - Y_i)V_i^{-\top}$, $i \in \{1, 2, \ldots, N\}$.

The main result of this section is given in the theorem below.

Theorem 3. If there exist symmetric positive definite matrices Y_i, X_i, and matrices K_i, M_i, W_i and Ψ_i, $i \in \{1, 2, \ldots, N\}$, of appropriate dimensions such that

$$\begin{bmatrix} Y_i & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ I & X_i & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & I & \bullet & \bullet & \bullet & \bullet & \bullet \\ Y_{i+1}\bar{A}_{i,l} + K_iC_i & M_i & Y_{i+1}\bar{F} & Y_{i+1} & \bullet & \bullet & \bullet & \bullet \\ \bar{A}_{i,l} & \bar{A}_{i,l}X_i + \bar{B}_{i,l}W_i & \bar{F} & I & X_{i+1} & \bullet & \bullet & \bullet \\ \bar{G}_i & \bar{G}_iX_i + \bar{H}_iW_i & 0 & 0 & 0 & I & \bullet & \bullet \\ 0 & \Xi_{i,l} & 0 & 0 & 0 & 0 & I & \bullet \\ 0 & 0 & 0 & Y_{i+1} & 0 & 0 & 0 & I \end{bmatrix} \succ 0 \tag{24}$$

for all $i \in \{1, 2, \ldots, N\}$, and $l \in \{1, 2, \ldots, L\}$, with $X_{N+1} = X_1$, $Y_{N+1} = Y_1$, and $\Xi_{i,l} = \bar{A}_{i,l}X_i + \bar{B}_{i,l}W_i + \Psi_i$. Then, the dynamic output-feedback controller given by (18) with

$$A_i^c = V_{i+1}^{-1}(M_i - K_iC_iX_i - K_iD_iW_i + Y_{i+1}\Psi_i) \times (I - Y_iX_i)^{-1}V_i \tag{25a}$$

$$B_i^c = V_{i+1}^{-1}K_i \tag{25b}$$

$$C_i^c = W_i(I - Y_iX_i)^{-1}V_i \tag{25c}$$

with any nonsingular V_i, $i \in \{1, 2, \ldots, N\}$, renders the closed-loop system (19) with switching sequence (5) asymptotically stable for all $\alpha_k \in \mathcal{A}$ and $\Delta_k \in \mathbf{\Delta}$, $k \in \mathbb{N}$.

Remark 4. In this paper, we focus on controller synthesis for a given periodic protocol, while the results of [Deaecto et al., 2011] could in principle also be used for simultaneous synthesis of a controller and a *dynamic protocol* (such as the maximum-error first (MEF), or the try-once-discard (TOD) protocol, see, e.g., [Dačić and Nešić, 2007, Donkers et al., 2011, Heemels et al., 2010a, Nešić and Teel, 2004]). However, the extension of the current results towards dynamic protocols will be useless for any practical NCS.

To explain this, observe that positive definiteness of (24) can only occur if

$$\begin{bmatrix} Y_i & \bullet \\ Y_{i+1}A_{i,l}+K_iC_i & Y_{i+1} \end{bmatrix} \succ 0 \quad \text{and} \quad Y_{N+1} = Y_1. \tag{26}$$

Note that this condition is the same as the synthesis condition of a switched observer gain $Y_{i+1}^{-1}K_i$ for a Luenburger observer and requires certain observability conditions to be satisfied. Now when the results of [Deaecto et al., 2011] are extended towards NCSs, or when the results of this paper are extended towards synthesis of dynamic protocols (such as the MEF/TOD protocols), the resulting stability conditions could only be verified if

$$\begin{bmatrix} Y & \bullet \\ YA_{i,l}+K_iC_i & Y \end{bmatrix} \succ 0 \tag{27}$$

can hold for all $i \in \{1, 2, \ldots, N\}$. Since the matrix Y cannot depend on the node i, the condition (27) can only be satisfied if every pair (A_i, C_i) is detectable. Hence, straightforward extensions to the results of this paper towards joint synthesis of a controller and a quadratic protocol using ideas from [Deaecto et al., 2011] can only result in a stabilising controller if the complete NCS is detectable from every node. For NCSs that are detectable from every node, scheduling transmissions of networked plant outputs is not needed in case only stability has to be guaranteed. In this case, a trivial solution to the scheduling problem is to choose always the same sensor node. □

4. NUMERICAL EXAMPLE

In this section, we illustrate the presented theory using a well-known benchmark example in the NCS literature, see, e.g., [Bauer et al., 2013, Dačić and Nešić, 2007, Donkers et al., 2011, Heemels et al., 2010a, Nešić and Teel, 2004], consisting of a model of a batch reactor. The details of the linearised model of the batch reactor model used in this example and the continuous-time controller can be found in the aforementioned references.

The batch reactor example has been used primarily to compare conservatism in stability analysis techniques, where the dynamic output-based stabilising controller is assumed to be given. Notable exceptions are [Dačić and Nešić, 2007], in which dynamic output-feedback controllers were synthesised together with the scheduling protocol for this example using bilinear matrix inequalities (BMIs), but with a constant transmission interval, and [Bauer et al., 2013], in which output-based controllers were synthesised by imposing an observer-based control structure in the controller, for a given periodic protocol, but for time-varying transmission intervals. We will compare our synthesis results with these two results.

For this numerical example, we will synthesise output-based controllers using Theorem 3. To do so, we will

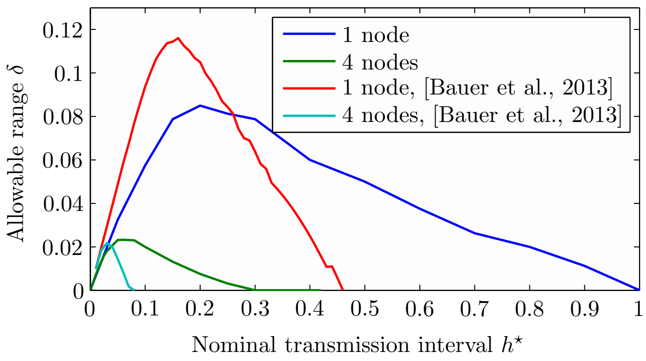

Fig. 3. Range for with a controller can be synthesised.

use the networked plant model (9), and we consider two cases. In the first case, we assume that all sensors and actuators transmit simultaneously, meaning that $\Gamma_k = I$ for all k. In the second case, it is assumed that every sensor and actuator sends data after each other, which corresponds to $\Gamma_1 = \text{diag}(1, 0, 0, 0)$, $\Gamma_2 = \text{diag}(0, 1, 0, 0)$, $\Gamma_3 = \text{diag}(0, 0, 1, 0)$, $\Gamma_4 = \text{diag}(0, 0, 0, 1)$ and $N = 4$. For both cases, we assume that delays are absent, i.e., $\tau_k = 0$ for all $k \in \mathbb{N}$, and take $h \in [h_\star - \delta, h_\star + \delta]$, for some $h^\star$ and δ satisfying $h^\star > \delta \geqslant 0$ and take 20 equidistantly distributed grid points to compute the convex overapproximation. The parameter $h_\star$ can be considered a nominal transmission interval and δ a relative perturbation of this nominal transmission interval. We can now maximise δ for a given $h^\star$ such that a stabilising controller can still be found. The results are shown in Fig. 3.

To compare our results with [Bauer et al., 2013], we have also computed the maximal δ for a given $h^\star$ such that a stabilising controller can be found using the results from [Bauer et al., 2013], see also Fig. 3. We can observe that for both cases, the novel results allow for larger nominal transmission intervals $h^\star$, but that for smaller nominal transmission intervals $h^\star$, the results from [Bauer et al., 2013] allow for a larger δ. It is difficult to analyse what exactly causes this difference, but as the overapproximation presented in Section 2.3 does not introduce conservatism, Theorem 3 sometimes introduces more conservatism than its counterpart in [Bauer et al., 2013]. In particular, we believe that conservatism in the results of this paper is introduced through $\Xi_{i,l}$ in (24), which is needed to have that (25) is independent of l. Based on the observations on conservatism, it is advisable to use both the method from this paper and the method from [Bauer et al., 2013] to synthesise controllers for a specific networked plant and to choose the most appropriate result.

Let us now also compare our results with [Dačić and Nešić, 2007], in which dynamic output-based controllers were designed together with a dynamic scheduling protocol, as well as for a given dynamic scheduling protocol. As our novel results do not allow for a joint dynamic protocol/controller synthesis, we select a periodic protocol and we compute the maximal constant transmission interval h for which the conditions of Theorem 3 are feasible. Since [Dačić and Nešić, 2007] does not consider time-varying transmission intervals, we do not have to make a convex overapproximation. For this comparison, we consider, as was done in [Dačić and Nešić, 2007], the case of three nodes, in which $\Gamma_1 = \text{diag}(1, 0, 0, 0)$, $\Gamma_2 = \text{diag}(0, 1, 0, 0)$,

$\Gamma_3 = \text{diag}(0,0,1,1)$ and $N = 3$, and the case of two nodes, in which $\Gamma_1 = \text{diag}(1,0,1,1)$, $\Gamma_2 = \text{diag}(0,1,1,1)$ and $N = 2$. Using the results of [Dačić and Nešić, 2007], the maximal allowable transmission interval of $h = 0.067$, is obtained for the case of three nodes and $h = 0.81$ is obtained for the case of two nodes [1]. For these cases, our novel synthesis results yield $h = 0.63$ and $h = 1$, respectively, for the given periodic protocol. As the dynamic protocols that are considered in [Dačić and Nešić, 2007] typically result in larger maximal allowable transmission intervals guaranteeing closed-loop stability than periodic protocols, see, e.g., [Donkers et al., 2011, Nešić and Teel, 2004], we believe that the improvements in maximal allowable transmission intervals are due to the fact that [Dačić and Nešić, 2007] requires solving linearised bilinear matrix inequalities, even if the dynamic protocol is given, which not always gives satisfactory results. Note that, for our design approach, the maximal allowable transmission interval for two nodes is equal to the case where all sensors and actuators send simultaneously (see Fig. 3), which might be due to the fact that the batch reactor example is fully detectable from either output.

5. CONCLUSIONS

In this paper, we have presented LMI-based synthesis conditions for designing stabilising dynamic output-based controllers for networked control systems (NCSs). In particular, the controller is designed for a network that causes the transmission intervals and delays to be time-varying and precludes all sensors and actuators to send their information simultaneously. The latter facts necessitates the use of a scheduling protocol and, in this paper, we have focussed on a periodic protocol. We formulated the networked plant model as a discrete-time switched linear parameter-varying system and used convex overapproximation techniques to arrive at a model that is amenable for controller synthesis. The controller synthesis is done using ideas from controller synthesis for switched systems, resulting in a switched controller that is robust for the aforementioned network phenomena. We have illustrated the synthesis results on a benchmark example of a batch reactor, which showed that our novel results can outperform existing results on controller synthesis.

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[1] Note that in [Dačić and Nešić, 2007] the maximal allowable transmission interval for the three-node case is obtained by jointly synthesising the controller and the dynamic scheduling protocol, while for the two-node case a larger maximal allowable transmission interval was obtained by a priori selecting the dynamic protocol and synthesising only the controller.

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