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Event-Triggered Consensus for Multi-Agent Systems with Guaranteed Robust Positive Minimum Inter-Event Times

V.S. Dolk R. Postoyan W.P.M.H. Heemels

Abstract— **We study consensus seeking single-integrator multi-agent systems equipped with packet-based communication channels. As the communication bandwidth of such channels is typically limited, it is essential to consider control schemes that lead to the desired performance while not overusing the communication resources. For this purpose, we propose a distributed dynamic event-triggered control scheme that results in aperiodic information exchange between agents, asymptotic consensus, strictly positive lower bounds on the inter-event times (strong Zeno-freeness) and robustness to unknown, non-uniform, and time-varying transmission delays. The proposed framework is such that the local control laws and event-triggering mechanisms can be directly obtained from the number of connected agents and local tuning parameters. The proposed design framework is therefore applicable to large-scale multi-agent systems.**

I. Introduction

Consensus problems for multi-agent systems (MASs) have attracted a lot of attention in recent years, see *e.g.*, [22] for an overview of some early approaches. One particular topic of interest has been consensus of MASs in which the exchange of information is realized by digital packet-based communication networks. Event-triggered control, see also [16] and the references therein, is relevant in this context as it uses smart state- or output-based event generators, which only trigger transmission "when this is needed", thereby having the potential to reduce the number of transmissions compared to periodic time-triggered control significantly. This implementation paradigm is therefore likely to reduce the utilization of the communication network in order to adhere to bandwidth limitations and avoid packet losses.

A considerable amount of works has considered the event-triggered consensus problem, see, for instance, [6], [10], [15], [17]–[19], [24], [26] and [21] for a recent overview. However, both interestingly and surprisingly, to date there are only a few works on distributed event-triggered communication schemes available that cover all the following desirable (or, in fact, indispensable) properties:

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(i) asymptotic consensus, *i.e.*, all the agents converge asymptotically to the same values,
(ii) each agent determines independently and asynchronously (without any form of clock synchronization) when to transmit their information based on locally available information only,
(iii) communication resources are saved in the sense that an agent communicates only when needed,
(iv) robust non-Zenoness is guaranteed for the MASs in the sense that each agent has a positive lower bound on the inter-communication times, also in the presence of (arbitrarily) small disturbances and communication artefacts such as varying communication delays.

Clearly, all of these properties are of high relevance to realize real-life implementations. In particular, property (iv) is a necessity in the context of event-triggered control due to the following two reasons. Firstly, Zeno behavior prohibits the actual implementation of the scheme as an infinite number of transmissions would be needed in a finite time span whereas hardware limitations always impose a positive lower bound between consecutive transmissions. Secondly, the principal idea of event-triggered schemes is saving valuable communication resource, which would not be satisfied at all if arbitrarily small inter-transmission times are possible.

The two most common approaches to avoid Zeno behavior in event-triggered control systems while ensuring asymptotic stability properties, are via time-regularization in which the triggering condition is only verified after a specific time duration, see also [1], [9] for time-regularized ETC strategies for MASs, or via periodic event-triggered control (PETC) in which the triggering condition is only verified at fixed equidistant time instants, see for instance [11], [19], [20] for PETC strategies for MASs.

In this paper, inspired by the ideas in [7], we present an approach based on time-regularized ETC that leads to systematic designs of distributed dynamic event-triggered cooperation schemes for MASs with single-integrator dynamics connected over an indirected fixed graph satisfying properties (i)-(iv). Compared to the work in [9], in which the approach is based on LMI conditions (for general linear agent dynamics), the proposed design framework leads to local control laws and event-triggering mechanisms (ETMs) that can be designed independently per agent. To be more specific, the design of the control laws and ETMs only rely on the number of connected neighbors and local tuning parameters, without the need for numerical optimization methods, which, in general, become intractable in case of large-scale MASs.

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Existing works that are closest in nature to our results, are presented in [1], [4], [26]. In [4], [26] distributed event-triggered consensus schemes (for linear agent dynamics) are presented that have properties (i)-(iii). and partially have property (iv) in the sense that indeed non-Zenoness is shown. However, a positive lower bound on the inter-communication times is not established, which is crucial for implementability as already discussed. Recently, in [1], an approach based on clock-like event generators, as proposed in [23], is presented that has properties (i)-(iv). Compared to [1], we present a new solution using different event generators in line with dynamic event-generators as in [13] and [7]. Secondly, we show the existence of a positive lower bound on the inter-communication times, even in the presence of the inevitable presence of (non)-uniform[1] communication delays, which are omnipresent in practice and which may have a major impact on the systems performance, if not handled carefully.

For the sake of presentation, we focus in this paper on the case of integrator agent dynamics and lay down the main ideas on how to apply the framework of distributed dynamic event-triggered control in [7] in an analytical manner. However, the underlying ideas are applicable to more general problems in resource-aware communication for MAS, in terms of agents dynamics, communication protocols, graph topology, dropouts and even the presence of denial-of-service attacks, see also Chapter 7 of [8], which will be the topic of future work.

II. Definitions and Preliminaries

A. Definitions

The set $\mathbb{N}$ denotes the set of non-negative integers, $\mathbb{N}_{>0}$ is the set of all positive integers, $\mathbb{R}$ is the field of all real numbers and $\mathbb{R}_{\geqslant 0}$ is the set of all non-negative reals. For N vectors $x_i \in \mathbb{R}^{n_i}, i \in \{1, 2, \ldots, N\}$, we denote the vector obtained by stacking all vectors in one (column) vector $x \in \mathbb{R}^n$ with $n = \sum_{i=1}^{N} n_i$ by $(x_1, x_2, \ldots, x_N)$, *i.e.*, $(x_1, x_2, \ldots, x_N) = \left[\begin{array}{cccc} x_1^\top & x_2^\top & \cdots & x_N^\top \end{array}\right]^\top$. The vector in $\mathbb{R}^N$ whose elements are all ones is denoted by 1_N. By $\|\cdot\|$ and $\langle \cdot, \cdot \rangle$ we denote the Euclidean norm and the usual inner product of real vectors, respectively. Moreover, for $x \in \mathbb{R}^n$ and a given non-empty set $\mathcal{A} \subset \mathbb{R}^n$, the distance of x to $\mathcal{A}$ is defined as $\|x\|_{\mathcal{A}} = \inf_{y \in \mathcal{A}} \|x - y\|$. The notation $F : X \rightrightarrows Y$, indicates that F is a set-valued mapping from X to Y with $F(x) \subset Y$ for all $x \in X$.

B. Graph Theory Notions

Here we recall some basic definitions and properties from graph theory [2], [5]. A *graph* is a pair $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ composed of a vertex set $\mathcal{V}$ and a set of edges $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$. The cardinality of $\mathcal{V}$, denoted by $N \in \mathbb{N}_{>0}$, is the number of vertices in $\mathcal{V}$. An ordered pair $(i, j) \in \mathcal{E}$ with $i, j \in \mathcal{V}$ is said to be an edge *directed* from i to j. A graph is called *undirected* if it holds that $(i, j) \in \mathcal{E}$ if and only if $(j, i) \in \mathcal{E}$. A vertex j is said to be a neighbor of i if $(j, i) \in \mathcal{E}$. The set of neighbors of a vertex i is denoted by $\mathcal{V}_i^{\text{in}}$ and defined as $\mathcal{V}_i^{\text{in}} := \{j \in \mathcal{V} \mid (j, i) \in \mathcal{E}\}$ and the set of vertices for which vertex i is a neighbor is denoted by $\mathcal{V}_i^{\text{out}}$ and defined as $\mathcal{V}_i^{\text{out}} := \{j \in \mathcal{V} \mid (i, j) \in \mathcal{E}\}$. The cardinality of $\mathcal{V}_i^{\text{in}}$, $i \in \mathcal{V}$ is denoted by N_i. An edge $(i, i) \in \mathcal{E}$ is called a *self-loop*. A *path* from i to j is a (finite) sequence of edges starting in i and ending at j. A graph $\mathcal{G}$ is connected if there exists a path, regardless of its direction, between all vertices $i, j \in \mathcal{V}$. The Laplacian matrix L is defined as $L := D - A$ with A the *adjacency* matrix of graph $\mathcal{G}$, which is defined as $A := (a_{i,j})$ with

$$a_{i,j} = \begin{cases} 1, & \text{if } j \in \mathcal{V}_i^{\text{in}}, \\ 0, & \text{otherwise}, \end{cases} \tag{1}$$

and $D := (d_{i,j})$ the *degree* matrix of graph $\mathcal{G}$, which is defined as a diagonal matrix with diagonal elements $d_{i,i} = N_i$, $i \in \mathcal{V}$.

III. Problem formulation

Consider a collection of agents with single integrator dynamics, which are interconnected according to an undirected and connected graph (without self-loops) $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with $\mathcal{V} := \{1, 2, \ldots, N\}$. Hence, the graph $\mathcal{G}$ consists of N nodes, where the i-th node is related to agent i with integrator dynamics

$$\dot{x}_i = u_i, \text{ for all } i \in \mathcal{V}, \tag{2}$$

where $x_i \in \mathbb{R}$ is the state of agent $\mathcal{A}_i$, and $u_i \in \mathbb{R}$ its control input. The consensus problem for the multi-agent system (MAS) described in (2) is to find appropriate control laws determining the local inputs u_i, $i \in \mathcal{V}$, such that all agents asymptotically reach a common consensus state, *i.e.*, $\lim_{t \to \infty} |x_i(t) - x_j(t)| = 0$ for all $i, j \in \mathcal{V}$ and any initial conditions $x_i(0) \in \mathbb{R}$, for all $i \in \mathcal{V}$. A well-known control law that results in consensus is given by

$$u_i = -\sum_{j \in \mathcal{V}_i^{\text{in}}} (x_i - x_j), \tag{3}$$

see also [22]. In vector notation, we can write (3) as

$$u = -Lx, \tag{4}$$

where $x := (x_1, x_2, \ldots, x_N)$.

The controller in (3) requires that agents have continuous access to the state information of neighboring agents. In case of a packet-based digital communication network, this cannot be achieved as information can only be communicated at discrete time instants. To be precise, we denote the transmission times of agent $\mathcal{A}_i$ by t_k^i, $k \in \mathbb{N}$. For now, we ignore the transmission delays (and get back to this in Section VI below) and we indicate the latest broadcast state information of agent $\mathcal{A}_i$, which is received by all connected agents $\mathcal{A}_j$, $j \in \mathcal{V}_i^{\text{out}}$, by $\hat{x}_i$. The information $\hat{x}_i$ evolves in a zero-order hold (ZOH) fashion, $\hat{x}_i(t) = x_i(t_k^i)$, when $t \in [t_k^i, t_{k+1}^i)$, $k \in \mathbb{N}$. Let us emphasize that agents $j \in \mathcal{V}_i^{\text{out}}$ only have access to $\hat{x}_i(t)$ at time $t \in \mathbb{R}_{\geq 0}$, and do not know

[1]By non-uniform delays, we mean that when an agent broadcasts its state to its neighbors, the message is in general received by the neighbors at different time instants and thus not synchronously.

$x_i(t)$. For this reason, instead of (4), we consider the control law as in, *e.g.*, [1], [6], [24],

$$u = -L\hat{x}, \tag{5}$$

where $\hat{x} := (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_N)$.

In this paper, we are interested in the design of distributed event-triggered communication schemes, which determine when transmissions of the individual agents to their neighbors take place. We adapt for this purpose the approach originally developed in [7] for point stabilization of nonlinear single systems. We thus propose to let the transmission times t_k^i, $k \in \mathbb{N}$ of agent $\mathcal{A}_i$ be determined by

$$t_{k+1}^i = \inf\{t > t_k^i + \tau_{miet}^i \mid \eta_i(t) \leqslant 0\}, \tag{6}$$

where η_i is a local variable available at agent $\mathcal{A}_i$ satisfying

$$\dot{\eta}_i = \Psi_i(o_i), \text{ when } t \neq t_k^i, \tag{7a}$$
$$\eta_i^+ = \eta_i^0(o_i), \text{ when } t = t_k^i \text{ for some } k \in \mathbb{N}. \tag{7b}$$

Here, Ψ_i and η_i^0 are mappings that only have local information o_i as their arguments, which can consist of $\hat{x}_i, x_i, \hat{x}_j, j \in \mathcal{V}_i^{\text{in}}, \eta_i$ or other variables that are locally available at agent $\mathcal{A}_i$. Moreover, $\tau_{miet}^i > 0$, $i \in \mathcal{V}$, is a lower bound on the inter-transmission times for agent $\mathcal{A}_i$ and it is a design parameter. In that way, two successive broadcasts by agent $\mathcal{A}_i$, $i \in \mathcal{V}$ are separated by at least τ_{miet}^i units of time, and thereby the Zeno phenomenon is excluded.

The objective is to synthesize Ψ_i, η_i^0 and τ_{miet}^i for the event-triggering mechanism (ETM) in (6), such that the mentioned properties (i)-(iv) in Section I are satisfied. We first ignore the possible occurrence of transmission delays in Sections IV and V. We then extend the results to be robust to such delays in Section VI.

IV. Hybrid model

To facilitate the analysis, we describe the event-triggered multi-agent scheme presented above in terms of a hybrid dynamical system $\mathcal{H}(\mathcal{C}, F, \mathcal{D}, G)$ in the formalism of [14], of the form

$$\dot{\xi} \in F(\xi) \quad \xi \in \mathcal{C}, \quad \xi^+ \in G(\xi) \quad \xi \in \mathcal{D}, \tag{8}$$

where $\mathcal{C}$ denotes the flow set, F the flow map, $\mathcal{D}$ the jump set and G the jump map, see [14] for more details on this formalism. To do so, we define the state vector $\xi := (x, e, \eta, \tau)$ where $e := (e_1, e_2, \ldots, e_N) = \hat{x} - x$ denotes the network-induced error. Observe that $e_i((t_k^i)^+) = 0$, for all $i \in \mathcal{V}$ and $k \in \mathbb{N}$. Moreover, we introduce $\tau := (\tau_1, \tau_2, \ldots, \tau_N)$ with τ_i, $i \in \mathcal{V}$, a timer-variable that keeps track on the time elapsed since the most recent transmission of agent $\mathcal{A}_i$. Hence, in between the transmissions of agent $\mathcal{A}_i$, $\dot{\tau}_i = 1$ and at a transmission τ_i is set to zero, *i.e.*, $\tau_i((t_k^i)^+) = 0$, for all $i \in \mathcal{V}$ and $k \in \mathbb{N}$.

By means of (2), (5), (7a) and the aforementioned definitions, we find that the flow dynamics are given by

$$\dot{x} = -Lx - Le, \qquad \dot{e} = Lx + Le \tag{9a}$$
$$\dot{\eta} \in \Psi(o), \qquad \dot{\tau} = 1_N, \tag{9b}$$

where $\Psi(o) := (\Psi_1, \Psi_2, \ldots, \Psi_N)$ with $o = (o_1, o_2, \ldots, o_N)$ and o_i, $i \in \mathcal{V}$, to be specified. Observe that $\dot{e} = -\dot{x}$ since $\dot{\hat{x}}_i = 0$ for all $i \in \mathcal{V}$ when $t \neq t_k^i, k \in \mathbb{N}$. Given (9), we define the flow map, for all $\xi \in \mathbb{X} := \mathbb{R}^N \times \mathbb{R}^N \times \mathbb{R}_{\geq 0}^N \times \mathbb{R}_{\geq 0}^N$,

$$F(\xi) := (-Lx - Le, Lx + Le, \Psi(o), 1_N), \tag{10}$$

The corresponding flow set is given by

$$\mathcal{C} := \mathbb{X}. \tag{11}$$

The jump events in the system correspond to transmission instants by one of the agents. In case agent $\mathcal{A}_i$, $i \in \mathcal{V}$, transmits its information due to a trigger governed by (6), the network-induced error e_i and the timer τ_i jump to zero. Given the latter and by recalling (7b), we find that the jump dynamics, corresponding to agent $\mathcal{A}_i$ communicating, is given by

$$x^+ = x, \qquad e^+ = \bar{\Gamma}_i e \tag{12a}$$
$$\eta^+ = \bar{\Gamma}_i \eta + \Gamma_i \eta^0(o), \qquad \tau^+ = \bar{\Gamma}_i 1_N \tag{12b}$$

where Γ_i is the $N \times N$ matrix consisting of only zeros except the i-th diagonal element is 1, $\bar{\Gamma}_i = I_N - \Gamma_i$ and $\eta^0(o) = (\eta_1^0(o_1), \eta_2^0(o_2), \ldots, \eta_N^0(o_N))$. Observe that, indeed, (12) is such that $e_i^+ = 0$, $\tau_i^+ = 0$ and $\eta_i^+ = \eta_i^0(o_i)$. By means of (12), we can define the jump map of the entire MAS as

$$G(\xi) := \bigcup_{i \in \bar{N}} G_i(\xi), \tag{13}$$

where

$$G_i(\xi) = \begin{cases} \{(x, \bar{\Gamma}_i e, \bar{\eta} + \Gamma_i \eta^0(o), \bar{\Gamma}_i 1_N)\}, & \text{when } \xi \in \mathcal{D}_i \\ \emptyset, & \text{when } \xi \notin \mathcal{D}_i \end{cases} \tag{14}$$

with $\mathcal{D}_i := \{\xi \in \mathbb{X} \mid \tau_i \geq \tau_{miet}^i\}$. The set $\mathcal{D}_i$ captures the ETM presented in (6) (in fact it also allows jumps for $\eta_i > 0$). Map G in (13) means that, when $\xi \in \mathcal{D}$, at least one of the agents can broadcast its state to its neighbors. In case multiple agents are in this situation, the hybrid model generates several instantaneous jumps. This construction is important to ensure that the jump map in (13) is outer-semicontinuous, which is one of the hybrid basic conditions of the formalism, see also [14]. The corresponding jump set is given by

$$\mathcal{D} = \bigcup_{i \in \bar{N}} \mathcal{D}_i. \tag{15}$$

By means of (10), (11), (13) and (15), we can compose the overall hybrid model $\mathcal{H} := \mathcal{H}(\mathcal{C}, F, \mathcal{D}, G)$. We can now proceed with the design of Ψ_i, η_i^0 and τ_{miet}^i, for all $i \in \mathcal{V}$, and the analysis of the induced hybrid system.

V. Design of the dynamic event generators

In this section, we present the design of the functions Ψ_i, η_i^0 and the time-constant τ_{miet}^i for all $i \in \mathcal{V}$. Before establishing the main results, we recall some properties available in literature, which are exploited in the design and analysis.

A. Preliminaries

Let us recall the following lemma.

Lemma 1. *[6], [12] Consider* $V(x) := \frac{1}{2}x^\top Lx$ *for any* $x \in \mathbb{R}^N$. *There exist positive constants* $\underline{\beta}, \overline{\beta} \in \mathbb{R}_{>0}$ *such that for all* $x \in \mathbb{R}^N$,

- $\underline{\beta}\|x\|^2_{\mathcal{A}_x} \leq x^\top Lx \leq \overline{\beta}\|x\|^2_{\mathcal{A}_x}$,

where $\mathcal{A}_x$ *denotes the consensus set,* i.e.,

$$\mathcal{A}_x := \{x \in \mathbb{R}^n \mid x_1 = x_2 = \ldots = x_N\}.$$

Moreover, given the dynamics in (10)*, it holds, for all* $x \in \mathbb{R}^N$*, that*

- $\langle \nabla V(x), -Lx - Le\rangle = -\|Lx\|^2 - x^\top LLe$,
- $\langle \nabla V(x), -Lx - Le\rangle = -\|L\hat{x}\|^2 + \hat{x}^\top LLe$.

By defining $z := (z_1, z_2, \ldots, z_N) := Lx$, and by using Lemma 1 and Young's inequality, we obtain that, for any $x, e \in \mathbb{R}^N$,

(a) $\langle \nabla V(x), -Lx - Le\rangle \leq \sum_i -(1-aN_i)z_i^2 + \frac{1}{a}N_ie_i^2$,
(b) $\langle \nabla V(x), -Lx - Le\rangle \leq \sum_i -(1-aN_i)u_i^2 + \frac{1}{a}N_ie_i^2$,

where, as before, N_i denotes the cardinality of $\mathcal{V}_i^{\text{in}}$, $i \in \mathcal{V}$ and where for both cases, a has to satisfy $0 < a < \frac{1}{N_i}$ for all $i \in \mathcal{V}$. Let us remark that, under the assumption that the maximum number of neighbors per agent is bounded by a fixed constant N_{max}, the parameter a can be chosen such that $0 < a < \frac{1}{N_{max}}$ and thus can be specified without prior knowledge of the graph $\mathcal{G}$. The aforementioned assumption is reasonable to make as a larger number of neighbors in general leads to smaller inter-event times. Given the fact that communication resources are limited, it will always be necessary to employ such a bound in practice.

The developments above can directly be used to establish an ETM. In fact, in [6], item (a) is exploited to obtain the triggering condition

$$t_{k+1}^i = \inf\{t > t_k^i \mid \sigma e_i^2(t) \geqslant \frac{a(1-aN_i)}{N_i}z_i^2(t)\}. \quad (16)$$

Due to the presence of z_i, this triggering condition requires continuously access to the state of neighboring agents. This is an issue when each agents has access to the state of its neighbors via packet-based communication, and not via its own sensors. In [12], this requirement was avoided by using item (b) to obtain the triggering condition

$$t_{k+1}^i = \inf\{t > t_k^i \mid \sigma_i e_i^2(t) \geqslant \frac{a(1-aN_i)}{N_i}u_i^2(t)\}. \quad (17)$$

The event-triggering mechanisms in (16) and (17) can be classified as relative triggering conditions, as originally proposed in [25]. As shown in [3], these type of triggering conditions lead to Zeno-behavior in presence of arbitrarily small disturbances.

B. Design and analysis

In this subsection, we present design conditions for the distributed event-generators as given in (7), that do not exhibit Zeno behavior.

In view of (b) in Section V.A, we design the triggering mechanism in (6) and (7) with

$$\begin{aligned}\Psi_i(u_i, e_i, \eta_i, \tau_i) := (1-\alpha_i)c_iu_i^2 \\ - \omega_i(\tau_i)\gamma_i^2[1 + \frac{1}{\alpha_ic_i}\lambda_i^2]e_i^2 - \varepsilon_{\eta,i}\eta_i, \quad (18)\end{aligned}$$

where

$$c_i := (1-\delta)(1-aN_i),\ \gamma_i := \sqrt{\frac{1}{a}N_i + \mu_i}, \quad (19)$$

and

$$\omega_i(\tau_i) := \begin{cases}\{1\}, & \text{when } \tau_i \in [0, \tau_{miet}^i) \\ [0,1] & \text{when } \tau_i = \tau_{miet}^i \\ \{0\}, & \text{when } \tau_i > \tau_{miet}^i.\end{cases} \quad (20)$$

Note that Ψ_i is set-valued because of ω_i. Constant τ_{miet}^i is given by

$$\tau_{miet}^i = -\frac{\sqrt{\alpha_ic_i}}{\gamma_i}\arctan\left(\frac{(\lambda_i^2-1)\sqrt{\alpha_ic_i}}{\lambda_i(\alpha_ic_i+1)}\right), \quad (21)$$

and, finally,

$$\eta_i^0(u_i, e_i, \eta_i, \tau_i) = \gamma_i\lambda_ie_i^2. \quad (22)$$

The constants $a \in (0, \frac{1}{N_i})$, $\alpha_i, \delta \in (0,1)$, $\varepsilon_{\eta,i} \in \mathbb{R}_{>0}$, $\mu_i \in (0, \gamma_i)$ and $0 < \lambda_i < 1$, for all $i \in \mathcal{V}$, are tuning parameters.

Let us remark that the function $\omega_i : \mathbb{R}_{\geqslant 0} \rightrightarrows [0,1]$, $i \in \mathcal{V}$, is defined such that the flow-map F is outer semicontinuous and thus that the hybrid system $\mathcal{H}$ complies with the hybrid basic conditions as presented in Assumption 6.5 of [14]. Let us also emphasize that the local control law as given in (5) and ETM as described in (6)-(7b), (18) and (22) indeed do not rely on numerical optimization methods and are therefore applicable to large-scale MASs.

Theorem 1. *The set* $\mathcal{A} := \{\xi \in \mathbb{X} \mid x_i = x_j \text{ for all } i, j \in \bar{N},\ e = 0,\ \eta = 0\} = \mathcal{A}_x \times \{0\} \times \{0\} \times \mathbb{R}^N_{\geq 0}$ *is uniformly globally asymptotically stable for the system* $\mathcal{H}$*, which implies asymptotic consensus. Moreover, it holds that* $t_{k+1}^i - t_k^i \geq \tau_{miet}^i > 0$ *for all* $i \in \bar{N}$ *and all* $k \in \mathbb{N}$ *implying non-Zenoness.*

The proof is omitted due to space limitations.

VI. Transmission delays

In this section, we present an important generalization of the ETM presented in the previous section so that it is applicable in presence of unknown, non-uniform, and time-varying transmission delays.

A. Unknown, non-uniform, and time-varying transmission delays

In case of non-uniform delays, a broadcast state measurement is not necessarily received at the same time by each connected agent. Hence, the delay can vary per edge/connection. For this purpose, we introduce the notation $\hat{x}_j^i$, which denotes the most recent state measurement of agent $\mathcal{A}_j$ received by agent $\mathcal{A}_i$, $i \in \mathcal{V}$, and $j \in \mathcal{V}_i^{\text{in}} \cup \{i\}$. Hence, typically, we have that $\hat{x}_j^i \neq \hat{x}_j^l$, for $j \in \mathcal{V}$ and $i, l \in \mathcal{V}_j^{\text{out}}$ with $i \neq l$. After a transmission is sent by agent

$\mathcal{A}_i$, $i \in \mathcal{V}$, at time t_k^i, the data is received by the agent(s) $\mathcal{A}_j$, $j \in \mathcal{V}_i^{\text{out}}$, after a communication delay of $\Delta_k^{i,j}$ time units, $k \in \mathbb{N}$. In other words, at time $t_k^i + \Delta_k^{i,j}$, $i \in \mathcal{V}, j \in \mathcal{V}_i^{\text{out}}, k \in \mathbb{N}$, the value of $\hat{x}_i^j$ is updated according to

$$\hat{x}_i^j((t_k^i + \Delta_k^{i,j})^+) = x_i(t_k^i), \tag{23}$$

for all $i \in \mathcal{V}$ and $j \in \mathcal{V}_i^{\text{out}} \cup \{i\}$. Since an agent $\mathcal{A}_i$, $i \in \mathcal{V}$, has direct access to its own state, we define $\Delta_k^{i,i} = 0$ for all $k \in \mathbb{N}$ (and thus $\hat{x}_i^i(t) = x_i(t_k^i)$, when $t \in (t_k^i, t_{k+1}^i]$, $k \in \mathbb{N}$).

Observe that the variables $\hat{x}_i^j$ for which $i \notin \mathcal{V}_j^{\text{in}}$ with $j \in \mathcal{V}$, i.e., $(i,j) \notin \mathcal{E}$, are non-existent in practice and thus in principle redundant. However, for the ease of notation, we still use these variables and assume that $\hat{x}_i^j(t) = 0$ for all $i \notin \mathcal{V}_j^{\text{in}}$ with $j \in \mathcal{V}$ and all $t \in \mathbb{R}_{\geqslant 0}$.

The delays $\Delta_k^{i,j}$, $i \in \mathcal{V}, j \in \mathcal{V}_i^{\text{out}}, k \in \mathbb{N}$, are assumed to be bounded from above by a (known) time-constant called the *maximally allowable delay* (MAD), which can be agent-dependent.

Assumption 1. *The transmission delays are bounded according to* $0 \leqslant \Delta_k^{i,j} \leqslant \tau_{mad}^i$, $i \in \mathcal{V}$, $j \in \mathcal{V}_i^{out}$ *for all* $k \in \mathbb{N}$, *where* τ_{mad}^i *denotes the* maximum allowable delay *for agent* $\mathcal{A}_i$, $i \in \mathcal{V}$.

Instead of (5), consider the control law

$$u_i = -\sum_{j \in \mathcal{V}_i^{\text{in}}} (\hat{x}_i^i - \hat{x}_j^i). \tag{24}$$

Moreover, we now define the network-induced errors as

$$e_i^j := \hat{x}_i^j - x_i, \tag{25}$$

$i \in \mathcal{V}$ and $j \in \mathcal{V}_i^{\text{out}}$. Hence, e_i^j, $i \in \mathcal{V}$, $j \in \mathcal{V}_i^{\text{out}}$, denotes the error present in the information $\hat{x}_i^j$ available at agent $\mathcal{A}_j$ regarding the state x_i of agent $\mathcal{A}_i$. A detailed description of the corresponding hybrid system as formalized in (8) is omitted here due to space limitations but is presented in [8].

B. Design and analysis

In the presence of transmission delays as described above, we need to modify the expression of the triggering mechanism parameters in (18)-(22) as follows

$$\tau_{miet}^i = -\frac{\sqrt{\alpha_i c_i}}{\gamma_i} \arctan\left(\frac{(\phi_{1,i}(0)(\lambda_i^2 - 1)\sqrt{\alpha_i c_i}}{\lambda_i\,(\alpha_i c_i + \phi_{1,i}(0))}\right) \tag{26}$$

with c_i and γ_i as in (19), and the time-constant τ_{mad}^i, $i \in \mathcal{V}$, such that

$$\frac{\phi_{1,i}(\tau_i)}{\lambda_i} \geqslant \phi_{0,i}(\tau_i), \text{ for all } \tau_i \in [0, \tau_{mad}^i], \tag{27}$$

where $\phi_{\ell,i}$, $\ell \in \{0,1\}$, evolves according to

$$\frac{d}{d\tau_i}\phi_{\ell,i} = -\frac{\gamma_i}{\lambda_i^\ell}\left(\frac{1}{\alpha_i c_i}\phi_{\ell,i}^2 + 1\right), \tag{28}$$

for some fixed initial conditions $\phi_{\ell,i}(0)$, $\ell \in \{0,1\}$, that satisfy $\frac{\phi_{1,i}(0)}{\lambda_i} \geqslant \phi_{0,i}(0) > \lambda_i\phi_{1,i}(0) > 0$, for all $i \in \mathcal{V}$ and $\ell \in \{0,1\}$. Moreover, we take the functions Ψ_i, and η_i^0, $i \in \mathcal{V}$, as

$$\begin{aligned}\Psi_i(u_i, e_i, \eta_i, \tau_i) &:= (1-\alpha_i)c_i u_i^2 \\ &- \omega_i(\tau_i)\gamma_i^2[1 + \frac{1}{\alpha_i c_i}\phi_{1,i}^2(0)\lambda_i^2]e_i^2 - \varepsilon_{\eta,i}\eta_i,\end{aligned} \tag{29}$$

and

$$\eta_i^0(u_i, e_i, \eta_i, \tau_i) = \gamma_i \phi_{1,i}(0)\lambda_i e_i^2, \tag{30}$$

respectively, where $e_i := e_i^i = \hat{x}_i^i - x_i$, $i \in \mathcal{V}$ with e_i^i as in (25).

As we will show in Section VII, (26)-(28) lead to intuitive $(\tau_{miet}^i, \tau_{mad}^i)$ curves that can be used to select appropriate values for λ_i, $\phi_{0,i}(0)$ and $\phi_{1,i}(0)$.

Theorem 2. *The set* $\mathcal{A} := \{\chi \in \mathbb{X}_{ext} \mid x_i = x_j \text{ for all } i,j \in \bar{N},\ e = 0,\ \eta = 0\}$ *is UGAS for the system* $\mathcal{H}$, *which implies asymptotic consensus. Moreover, it holds that* $t_{k+1}^i - t_k^i \geq \tau_{miet}^i > 0$ *for all* $i \in \bar{N}$ *and all* $k \in \mathbb{N}$ *implying non-Zenoness.*

Due to space limitations, the proof is omitted. Let us remark, however, that the result presented in this section is derived from ideas in Chapter 7 of [8], which relies on the construction of a genuinely new hybrid Lyapunov function.

VII. Numerical Example

In this section, we consider a MAS consisting of eight agents ($N = 8$) with single integrator dynamics that are connected according to the undirected communication graph $\mathcal{G}$ containing the undirected edges $(1,2)$, $(1,8)$, $(2,3)$, $(2,7)$, $(3,4)$, $(3,6)$, $(4,5)$, $(5,6)$ $(5,8)$ and $(7,8)$. The communication channels are subject to non-uniform varying delays. Hence, the local control law of each agent is as given in (24).

The ETMs are designed according to (6)-(7b) and (26)-(30) with tuning parameters $\delta = \mu_i = \varepsilon_{\eta,i} = 0.05$ and $\alpha_i = 0.5$, for all $i \in \mathcal{V}$. Moreover, we select $a = 0.1$ such that $a < \frac{1}{N_i}$ for all $i \in \mathcal{V}$. Given these tuning parameters, we obtain that $\gamma_i = 4.478$ and $c_i = 0.76$ for $i \in \mathcal{V}$ for which $N_i = 2$ (agent $\mathcal{A}_1$, $\mathcal{A}_4$, $\mathcal{A}_6$ and $\mathcal{A}_7$), and $\gamma_i = 5.482$ and $c_i = 0.665$ for which $N_i = 3$, $i \in \mathcal{V}$ (agent $\mathcal{A}_2$, $\mathcal{A}_3$, $\mathcal{A}_5$ and $\mathcal{A}_8$).

The variable λ_i allows to make trade-offs between τ_{miet}^i and τ_{mad}^i, $i \in \mathcal{V}$. To be more specific, τ_{miet}^i is computed according to (26) and τ_{mad}^i by solving (28) for various initial conditions, namely, $\phi_{0,i}(0) = \frac{1}{\lambda_i}$, with $\lambda_i \in (0,1)$, and $\phi_{1,i}(0) \in \left(\phi_{0,i}\lambda_i, \frac{\phi_{0,i}}{\lambda_i}\right)$, $i \in \mathcal{V}$, and consequently by determining the intersection of $\phi_{0,i}$ and $\frac{\phi_{1,i}}{\lambda_i}$. Figure 1 shows the tradeoff curves resulting from various values of λ_i for $\gamma_i = 4.478$ at the left side (corresponding to agent $\mathcal{A}_1$, $\mathcal{A}_4$, $\mathcal{A}_6$ and $\mathcal{A}_7$) and $\gamma_i = 5.482$ at the right side (agent $\mathcal{A}_2$, $\mathcal{A}_3$, $\mathcal{A}_4$ and $\mathcal{A}_8$). The blue circles in Figure 1 indicate the $(\tau_{miet}^i, \tau_{mad}^i)$-combination corresponding to $\phi_{0,i}(0) = 5$ and $\phi_{1,i}(0) = 2$, which are used in the simulations presented next. To be more specific, we take $(\tau_{miet}^i, \tau_{mad}^i) = (0.12, 0.016)$ for $i \in \mathcal{V}$ for which $N_i = 2$ (agent $\mathcal{A}_1$, $\mathcal{A}_4$, $\mathcal{A}_6$ and $\mathcal{A}_7$) and $(\tau_{miet}^i, \tau_{mad}^i) = (0.12, 0.016) = (0.09, 0.012)$

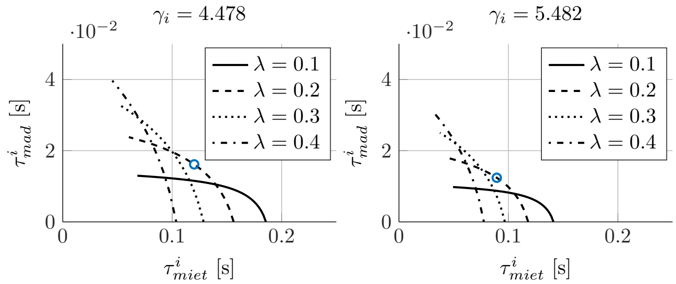

Fig. 1: Tradeoff curves that are obtained from (26) and (28) for $\phi_{0,i}(0) = \frac{1}{\lambda_i}$ and for various values of $\lambda_i \in (0,1)$ and $\phi_{1,i}(0) \in \left(\frac{\phi_{0,i}\gamma_{0,i}}{\gamma_{1,i}}, \frac{\phi_{0,i}\gamma_{0,i}}{\gamma_{1,i}\lambda_i^2}\right]$, $i \in \mathcal{V}$, corresponding to $\gamma_i = 4.478$ at the left side and $\gamma_i = 5.482$ at the right side

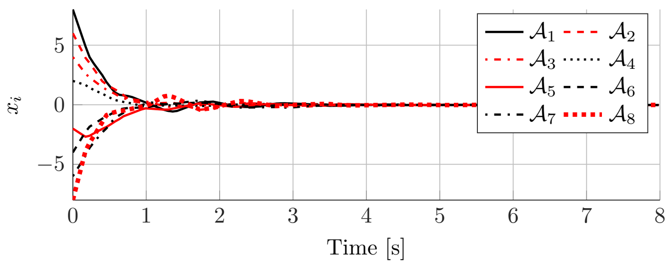

Fig. 2: Evolution of the states of the MAS described by (2), (23) and (24), the ETM described by (6) and (7) with initial condition $x(0) = (8, 6, 4, 2, -2, -4, -6, -98$.

for which $N_i = 3$, $i \in \mathcal{V}$ (agent $\mathcal{A}_2$, $\mathcal{A}_3$, $\mathcal{A}_5$ and $\mathcal{A}_8$). In Figure 2, the evolution of the states x_i, $i \in \mathcal{V}$, is shown for the initial condition $x(0) = [8, 6, 4, 2, -2, -4, -6, -8]^\top$ and in Figure Figure 3 the corresponding inter-event times. Observe that the inter-event times are significantly larger than the enforced minimum inter-event times. [2]

VIII. Conclusions

In this work, we presented a design framework for dynamic *event-triggered* control strategies for consensus seeking in multi-agents systems that result in asymptotic consensus and positive minimum inter-event times, and that can be design independently without the need for global information about the MAS. In addition, it is shown that the proposed event-triggering mechanisms can be modified such that the resulting MAS is robust to unknown, non-uniform, and time-varying transmission delays.

[2]The MATLAB code for this numerical example is available at https://github.com/victordolk/eventtriggered-consensus

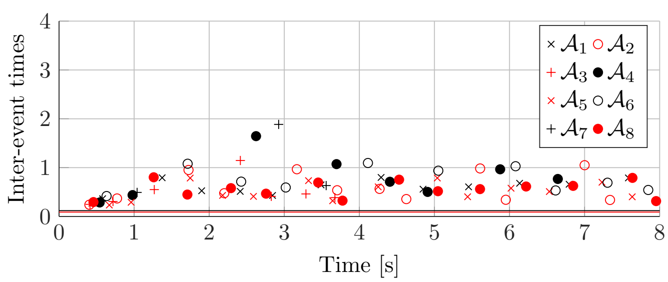

Fig. 3: Inter-event times generated by the ETM as given in (6) and (7). The solid lines represent the minimum inter-event times τ^i_{miet}.

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