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A hybrid MPC approach to the design of a Smart adaptive cruise controller

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Abstract— **In this paper we investigate the possibility of applying the hybrid Model Predictive Control (MPC) framework to solve a control problem regarding tracking of a moving vehicle. The study originates from the design of an adaptive cruise controller (ACC) of a Smart car, that aims to closely follow a reference trajectory transmitted by a leading vehicle. The physical behavior of the Smart and the constraints arising from the specifications related to safety and security issues make the hybrid MPC framework suitable for this task. An adaptation of the terminal cost and constraint set MPC approach, which is commonly used for fixed set-point regulation, is employed in order to achieve good tracking of a time-varying reference trajectory. The simulation results indicate the effectiveness of the developed hybrid MPC algorithm and the industrial feasibility with respect to on-line computation restrictions.**

I. INTRODUCTION

PieceWise Affine (PWA) systems are a class of hybrid systems that received an increased attention in the recent years from researchers active in many fields. PWA systems are equivalent, under certain mild assumptions [8], with several other relevant classes of hybrid systems, such as Mixed Logical Dynamical systems (MLD) [2], max-min-plus-scaling systems [5] or linear complementarity systems [15]. They also arise from the linear spline approximation of nonlinearities [14].

The application of several control techniques developed for linear or smooth non-linear systems to control hybrid systems is prohibited by the presence of integer variables, which yield complex numerical problems. Nevertheless, a considerable number of control methods were proposed in the literature. The results obtained by Bemporad and Morari in [2] showed that, under general conditions, a *Model Predictive Control* (MPC) based approach can be successfully used to control hybrid systems, by solving *on-line* a mixed integer optimization problem. In other approaches the control law may also be obtained *off-line* (see for instance [4]) by means of solving a parameterized mixed integer (linear or quadratic) programming problem, or computed via a max-min-plus-scaling [5] formulation. Properties like robustness [9], [11] or stability [12] were also investigated. The usual approach for guaranteeing stability in MPC, with respect to a fixed equilibrium (set-point) is the so-called terminal cost and constraint set method, e.g. see the survey [13] for an overview. This method uses the value function of the MPC cost as a candidate Lyapunov function for the closed-loop system and achieves stability via a particular terminal cost and an additional constraint on the terminal state, i.e. the predicted state at the end of the prediction horizon. In this paper we design an on-line hybrid MPC controller applied to a tracking problem. This study was motivated by the design of an ACC for an ordinary road vehicle (a Smart), whose target is to follow as good as possible a leading vehicle, in a highway environment. In order to meet realistic conditions several constraints on kinematic and dynamical entities are introduced, fulfilling safety, comfort and environmental issues. An adaptation of the terminal cost and constraint set method is employed with the purpose of achieving good tracking performance of a time-varying reference trajectory. The adaptation consists in using a time-varying terminal constraint set, which is calculated on-line as the Minkowski sum of a known set (computed off-line) and a future element of the reference trajectory, corresponding to the length of the prediction horizon.

The paper is organized as follows. We first describe the application and the corresponding constrained optimal control problem. This is transformed into a minimization problem with a mixed integer objective function. Then we present the procedure used to construct the terminal constraint set for the tracking problem. We show simulation results, illustrating the effectiveness of the methodology w.r.t. tracking and real-time performance.

II. MODEL AND CONTROL PROBLEM DESCRIPTION

Physical model of the Smart. In this section we describe the model of the experimental set-up that will be used in this paper. The aim of an Adaptive Cruise Controller (ACC) is to ensure a minimal separation between two vehicles, with one of the vehicles following the other. We assume that the front vehicle communicates its speed and position[1] to the rear vehicle, which has to track them as good as possible (see Figure 1.a). Hence, for the control design purpose, only the dynamics of the rear vehicle can be considered. The differential equation for positive velocity of the rear vehicle is:

$$m\ddot{s}(t) + c\dot{s}^2(t) + \mu\, mg = bu(t) \tag{1}$$

where $s(t)$ is the position and $bu(t)$ is the traction force, proportional to the normalized throttle/brake position $u(t)$,

[1]These can be measured with lasers and GPS.

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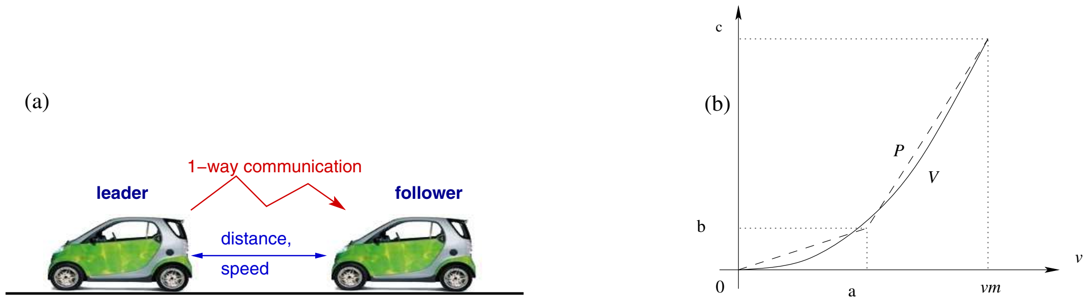

Fig. 1. *(a) ACC set-up, position x_1 and reference η_1, and (b) nonlinear to PWA approximation.*

TABLE I
Definitions and values of the entries of equation (1).

m	Mass of vehicle	800 kg
c	Viscous coefficient	0.5 kg/m
μ	Coulomb friction coefficient (dry asphalt)	0.01
b	Traction force	3700 N
g	Gravity acceleration	9.8 m/s^2

TABLE II
Values of the constraints.

$x_{1,\min}$	Minimum position	0 m
$x_{1,\max}$	Maximum position	2000 m
$x_{2,\min}$	Minimum velocity	5.0 m/s
$x_{2,\max}$	Maximum velocity	37.5 m/s
d_{safe}	Tracking tolerance	5.0 m
a_{acc}	Comfort acceleration	2.5 m/s^2
a_{dec}	Comfort deceleration	-1 m/s^2
ξ	Comfort jerk	2.0 m/s^3
$u_{\max}$	Maximum throttle/brake	1
Δ_u	Maximum throttle/brake variation	0.2
α	Switching velocity	18.75 m/s

considered as an input. The dissipative term $c\dot{s}^2(t) + \mu mg$ consists, respectively, of the *air drag* and the *ground-tire* static friction. Braking will be simulated by applying a negative throttle. Numerical values are listed in Table I. A state-space representation, $x = [s, \dot{s}]^{\mathrm{T}}$ (position and velocity), is: $\dot{x}(t) = f(x(t)) + Bu(t)$, with $f(x) = [x_2, -cx_2^2/m - \mu g]^{\mathrm{T}}$ and $B = [0, b/m]^{\mathrm{T}}$.

A least squares approximation (Figure 1.b) in $[0, x_{2,\max}]$ of the nonlinear friction curve $V(x_2) = cx_2^2$ leads to a PWA system, i.e.

$$\begin{cases} \dot{x}(t) = \mathcal{A}_1 x(t) + \mathcal{F}_1 + \mathcal{B}_1 u(t), & x_2(t) < \alpha \\ \dot{x}(t) = \mathcal{A}_2 x(t) + \mathcal{F}_2 + \mathcal{B}_2 u(t), & x_2(t) \geq \alpha, \end{cases} \tag{2}$$

where the matrices $\mathcal{A}_i, \mathcal{F}_i, \mathcal{B}_i$, $i = 1, 2$, are derived using the data shown in Figure 1.b[2] and Table I. A mode by mode discrete-time state-space representation (sampling time $T = 1s$, *zero order hold*) of (2) is given by

$$x(k+1) = A_i x(k) + B_i u(k) + F_i, \tag{3}$$

$i = 1, 2$ and the switching logics as in (2), with $A_1 = \begin{bmatrix} 1 & 0.97 \\ 0 & 0.99 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 0.98 \\ 0 & 0.96 \end{bmatrix}$, $B_1 = [2.31, 4.61]^{\mathrm{T}}$, $B_2 = [2.28, 4.54]^{\mathrm{T}}$, $F_1 = -[0.05, 0.10]^{\mathrm{T}}$ and $F_2 = [0.22, 0.44]^{\mathrm{T}}$.

Constraints. Safety, comfort and economy or environmental issues, as well as limitations on the model, constrain the behavior of the system. We consider limitations on the position, velocity, acceleration and jerk, and on the control input $u(k)$. More specifically, we have

$$\begin{gathered} x_1(k) \leq \eta_1(k) + d_{\text{safe}} \\ x_{2,\min} \leq x_2(k) \leq x_{2,\max} \\ a_{\text{dec}} T \leq x_2(k+1) - x_2(k) \leq a_{\text{acc}} T \\ |x_2(k+1) - 2x_2(k) + x_2(k-1)| \leq \xi T^2, \end{gathered} \tag{4}$$

for all k. The above equations express the operative range of the speed[3], the maximum tolerated overshoot d_{safe} of the position of the leading vehicle (see Figure 1.a), bounds on acceleration and jerk for comfort or security specifications. An additional *non-operational* constraint on the position[4], $x_{1,\min} \leq x_1(k) \leq x_{1,\max}$, is necessary to obtain a valid MLD model of the system, see the next section for details. We also consider limitations on the control input:

$$\begin{aligned} |u(k)| &\leq u_{\max} \\ |u(k+1) - u(k)| &\leq \Delta_u. \end{aligned} \tag{5}$$

Numerical values are listed in Table II.

III. Control Problem: A Hybrid MPC Approach

Given the PWA model of the rear vehicle we design the control action $u(k)$ to feed the engine/brake actuators, in order to satisfy (4), (5) and to track the front vehicle state η. We assume that at each sample step k a set of N_{p} predictions of the future reference state of the front vehicle are received by the rear vehicle[5]. Moreover, the rear vehicle measures with a suitable level of precision its current state $x(k)$ and keeps track of the previous control input $u(k-1)$ and state $x(k-1)$. In other words, at time k the on-board computer of the Smart calculates the action $u(k)$ on the basis of on-line and past measurements $\vartheta(k) =$

[2]For simplicity we only consider one breakpoint, i.e. a PWA system consisting of two operating modes. A finer approximation is also possible.

[3]A lower bound on x_2 validates the *ground-tire* friction model [7].

[4]This is not restrictive, as in the considered MPC set-up one can always reset the origin of the position measurements.

[5]This set-up requires the knowledge of the future reference vector, which is available in the case of prescheduled trajectories, or it may be predicted via reliable distribution models.

$[u(k-1), x(k-1)^{\mathrm{T}}, x(k)^{\mathrm{T}}, \eta(k+1)^{\mathrm{T}}, \ldots, \eta(k+N_{\mathrm{p}})^{\mathrm{T}}]^{\mathrm{T}}$, commonly denoted as the *parameter vector*.

A possible approach to the computation of the control action $u(k)$ under the given constraints and the on-line measurements is the hybrid MPC framework. This method has proved to be very efficient (see [2], [9], [12] to cite a few), in terms of trade-off between targeting good performances and dealing with strong model uncertainties and tight constraints, in several real-time applications.

To formulate the problem in a hybrid MPC framework we need to perform the following steps. First, we provide an equivalent MLD form [2] of the PWA model (3) subject to the constraints (4) and (5). The resulting MLD system will be used as the *prediction model* by the MPC algorithm. The MLD modeling framework is an alternative way to represent PWA models (see [2]) by means of auxiliary binary variables. Then we formally describe the MPC set-up and the corresponding constrained optimization problem. The MLD model is needed because it allows the conversion of the MPC optimization problem into a *mixed integer linear programming* (MILP) problem (an ℓ_1-norm based MPC cost is employed).

PWA to MLD conversion. The PWA system (3) is transformed into an MLD system by the introduction of a binary variable $\delta(k)$ [2]. The value of $\delta(k)$ equals 0 when the active mode in (3) is system 1 and $\delta(k) = 1$ when the active mode in (3) is system 2. Hence the new model of the system is:

$$x(k+1) = A_1 x(k) + L v(k) + F_1, \tag{6}$$

where $L = [A_2 - A_1 | B_2 - B_1 | F_2 - F_1 | B_1]$ and $v(k) = [z(k)^{\mathrm{T}}, y(k), \delta(k), u(k)]^{\mathrm{T}}$ (with $z(k) = x(k)\delta(k)$, $y(k) = u(k)\delta(k)$, $\delta(k) \in \{0, 1\}$) is the auxiliary *mixed logical* vector. To get rid of nonlinearity of variables $z(k), y(k)$ we introduce the constraints [2]:

$$\begin{gathered} x_{\min}\delta(k) \leq z(k) \leq x_{\max}\delta(k) \\ -x_{\max}(1-\delta(k)) \leq z(k) - x(k) \leq -x_{\min}(1-\delta(k)) \\ |y(k)| \leq u_{\max}\delta(k) \\ |y(k) - u(k)| \leq u_{\max}(1-\delta(k)). \end{gathered} \tag{7}$$

The switching condition leads to the constraints:

$$\begin{gathered} -\delta(k)(v_{\min} - \alpha) \leq x_2(k) - v_{\min} \\ \delta(k)(\alpha - v_{\max}) \leq -x_2(k) + \alpha. \end{gathered} \tag{8}$$

Optimal control problem. The control signal $u(k)$ is calculated by solving a constrained finite time optimal control problem in a receding horizon fashion. Consider now the following constrained optimization problem.

Problem 1: Let $N_{\mathrm{p}} \geq 1$ be given and $Q, Q_{N_{\mathrm{p}}}, R$ be assigned matrices of appropriate dimensions and full-column rank. Let $\vartheta(k)$ be the vector of parameters given at time k. We define the problem

$$\min_{\tilde{u}^{(N_{\mathrm{p}})}(k)} \Big\{ J(\vartheta(k), \tilde{u}^{(N_{\mathrm{p}})}(k)) \triangleq F(\varepsilon(k+N_{\mathrm{p}})) + \sum_{j=0}^{N_{\mathrm{p}}-1} L(\varepsilon(k+j), u(k+j)) \Big\}, \tag{9}$$

subject to (4)-(8), where $\varepsilon(k+j) = x(k+j) - \eta(k+j)$ is the *tracking error* and $\tilde{u}^{(N_{\mathrm{p}})}(k) = [u(k), \ldots, u(k+N_{\mathrm{p}}-1)]$ is the sequence of N_{p} control inputs. We define $F(\varepsilon(k+N_{\mathrm{p}})) \triangleq ||Q_{N_P}\varepsilon(k+N_{\mathrm{p}})||_1$ and $L(\varepsilon(k+j), u(k+j)) \triangleq ||Q\varepsilon(k+j)||_1 + ||Ru(k+j)||_1$. ■

Suppose that Problem 1 is feasible and let $\tilde{u}_{\mathrm{opt}}^{(N_{\mathrm{p}})}(k)$ denote its optimal solution. Then, the control action $u^*(k) \triangleq \big(\tilde{u}_{\mathrm{opt}}^{(N_{\mathrm{p}})}(k)\big)_1$ (i.e. the first element of the vector $u_{\mathrm{opt}}^{(N_{\mathrm{p}})}(k)$) is applied to the nonlinear model (1), in a *receding horizon* manner. Next, the set of parameters $\vartheta(k+1)$ is updated at time step $k+1$ and, a new optimal control problem is solved to obtain the new control action $u^*(k+1)$.

A shorter control horizon $N_{\mathrm{c}} < N_{\mathrm{p}}$ may also be used, i.e., $u(k+j) = u(k+N_{\mathrm{c}}-1)$, $j = N_{\mathrm{c}}, \ldots, N_{\mathrm{p}}-1$. This choice has the general advantage of reducing the number of variables and providing a smoother solution. Here we only consider the case $N_{\mathrm{p}} = N_{\mathrm{c}}$. The choice of the ℓ_1-norm is a valid trade-off between the complexity of the optimization problem and the quality of the solution. It allows the use of MILP solvers [1], [6], [10], more reliable than mixed integer quadratic programming solvers, and gives better performance than the one obtained via the ℓ_∞-norm [4].

At each time step k, Problem 1 can be converted into a MILP problem of the form

$$\begin{aligned} J^*(\vartheta(k)) &= \min_{\tilde{v}^{(N_{\mathrm{p}})}} c' \, \tilde{v}^{(N_{\mathrm{p}})} \\ \text{s.t.} \quad & E\tilde{v}^{(N_p)} \leq G + E_0\vartheta(k), \end{aligned} \tag{10}$$

which can be tackled using efficient solvers like `Cplex`.

IV. A TERMINAL CONSTRAINT SET APPROACH TO TRACKING

In general, the MPC control law computed as described in the previous section is not necessarily guaranteed to achieve asymptotic stability of the closed-loop error dynamics ε, not even in the case when $\eta(k) = x_e$ for all k (x_e denotes an equilibrium of the prediction model). For fixed-point regulation, a possible solution to guarantee stability is to use a particular terminal cost and to introduce a terminal constraint set in Problem 1 (see [13] for details). By using a terminal cost and constraint set in a time-varying reference tracking context, we propose in what follows a heuristic solution.

First, we follow off-line the usual steps performed for computing the terminal cost and constraint set in the case of a fixed setpoint. Let $h_{\text{aux}} : \mathbb{R}^n \to \mathbb{R}^m$ with $h_{\text{aux}}(0) = 0$ be an auxiliary state feedback control law, which is commonly employed in *terminal cost and constraint set* MPC [13]. In the PWA setting of system (3) we take this state feedback piecewise linear (PWL), i.e. $h_{\text{aux}}(x) := K_1 x$ when $x_2 < \alpha$ and $h_{\text{aux}}(x) := K_2 x$ when $x_2 \geq \alpha$, $K_{1,2} \in \mathbb{R}^{m \times n}$. Then, we require that the terminal cost and the auxiliary feedback controller h_{aux} satisfy

$$F((A_j + B_j K_j)x + F_j) - F(x) + L(x, K_j x) \leq 0, \tag{11}$$

for all x and all $j = 1, 2$. The terminal constraint set $\mathbb{X}_{N_{\mathrm{p}}}$ is taken as a positively invariant set [3] for system (3) in closed-loop with h_{aux}.

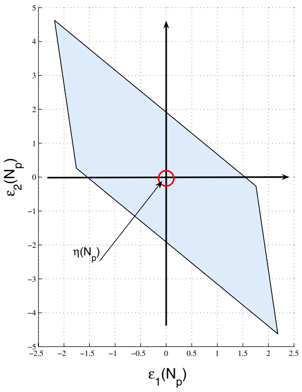

Fig. 2. *Positively invariant set, used as end point constraint, centered on the last point of the predicted reference trajectory.*

Recently, a method for finding a solution to the above inequality for ℓ_∞-norm based costs F and L was presented in [12]. Note that although the method of [12] is presented for the ℓ_∞-norm case, in fact, it can be applied to MPC cost functions defined using any p-norm, including the ℓ_1-norm. By applying the method of [12] we have obtained the terminal cost matrix $Q_{N_\mathrm{p}} = \begin{bmatrix} 4.58 & 0.45 \\ 5.14 & 4.15 \end{bmatrix}$ and the feedback gains $K_1 = \begin{bmatrix} -0.2417 & -0.3294 \end{bmatrix}$, $K_2 = \begin{bmatrix} -0.2245 & -0.3176 \end{bmatrix}$, which satisfy the condition (11) for the stage cost matrices $Q = \mathrm{diag}[0.8, 0.8]$ and $R = 0.01$, any x and $j = 1, 2$. The sublevel sets of the calculated terminal cost $F(x) = \|Q_{N_P} x\|_1$ are λ-contractive sets [12] and hence, positively invariant sets for system (3) in closed-loop with h_{aux}.

Next, let $\widetilde{\mathbb{X}}_{N_\mathrm{p}}$ be the maximal sublevel set of F contained in the set of states where the local PWL control law h_{aux} is admissible, i.e. it satisfies the imposed state and input constraints (see Figure 2). Note that the set $\widetilde{\mathbb{X}}_{N_\mathrm{p}}$ is a common positively invariant set for both dynamics 1 and 2 of system (3) in closed-loop with h_{aux}, due to the fact that F is a common ℓ_1-norm based (local) Lyapunov function for this closed-loop system.

The idea is to compute the terminal constraint set on-line as $\mathbb{X}_{N_\mathrm{p}}(k) = \{\eta(k + N_P)\} \oplus \widetilde{\mathbb{X}}_{N_\mathrm{p}}$, where $\oplus$ denotes the Minkowski sum of sets and $\eta(k+N_P)$ is known before hand, and to add the terminal constraint $x(k + N_P) \in \mathbb{X}_{N_\mathrm{p}}(k)$ to Problem 1. This results in a time-varying terminal constraint set, which changes along with the reference trajectory, see Figure 4. Although the reformulation of Problem 1 with the terminal cost and the terminal constraint set computed as described above does not necessarily guarantee asymptotic stability of the closed-loop error dynamics ε, the good tracking performance obtained in the simulations encourage us to further investigate the convergence properties of the hybrid MPC set-up developed for tracking.

V. Simulation results

We present the simulation results, obtained with the numerical values of the model given in Section II, and the terminal cost and constraint set calculated in Section IV. The equivalent MLD form of system (3) is used as the prediction model, and the control input is applied to the continuous-time nonlinear system (1). For the considered trajectory and initial conditions, a prediction horizon of $N_\mathrm{p} = 19$ was required to attain feasibility of the corresponding MPC optimization problem. The simulations have been carried out in `Matlab 7`, on the OS `Linux 2.4.22`, `INTEL pentium 4, 3GHz` processor. The MILP optimizations were performed with `ILOG Cplex` in `TOMLAB v5.1`.

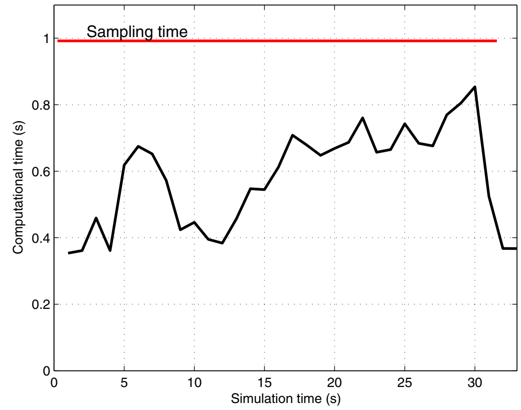

Fig. 6. *On-line computational time at each time step. Note that this is always lower than the sampling time.*

The initial values of the parameters collected in the vector ϑ are $u(0) = 0$, $x(0) = [0, 5]^\mathrm{T}$ and $x(-1) = [-5, 5.3]^\mathrm{T}$. The constraints are collected in Table II. The duration of the simulation is $t = 33\,s$. The tracking of the trajectory, the control input and the variation of the control input are depicted in Figure 3.

Additionally we report in Figure 4 a detail of the simulation that shows how the open-loop evolution of the predicted model enters the terminal set after $N_\mathrm{p} = 19$ time samples. This figure also shows how the terminal constraint set is updated on-line by simply centering it into the last element of the predicted reference trajectory. In Figure 5 we show that acceleration and jerk meet the given constraints. The mismatch between the hybrid prediction model and the nonlinear plant causes minor violations of the nominal constraint. This can be avoided by setting the constraint thresholds within a robust calculated margin. In Figure 6 we plot the on-line computational time history, which shows that the required calculations are always carried out well within the duration of the imposed sampling period ($T = 1\,s$), which is a necessary condition in all real-time applications.

VI. Conclusions

We have studied the problem of designing an adaptive cruise controller for a Smart. The number of constraints that arose from the realistic case study have led us to consider an MPC framework. The PWA prediction model of the system enabled us to cast the control problem in a MILP formulation. An adaptation of the terminal cost

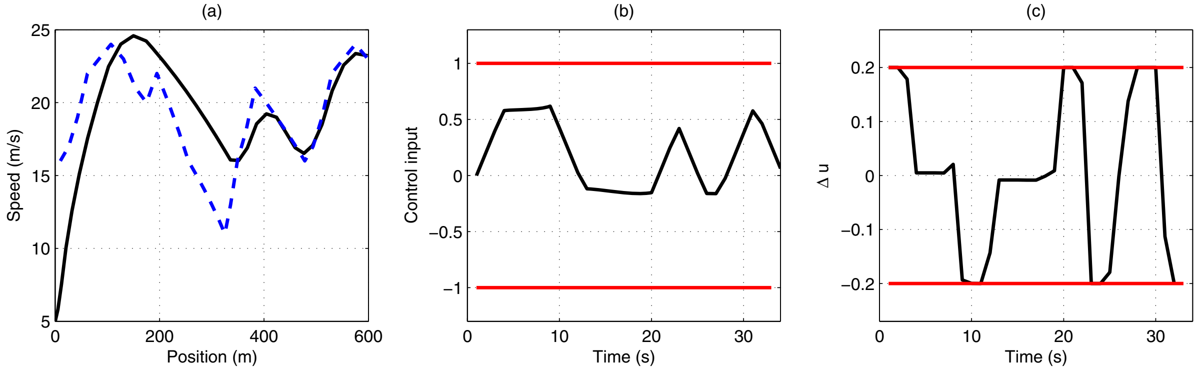

Fig. 3. *(a) State space trajectory (solid line) of the rear Smart tracking the reference (dashed line). (b) Optimal receding horizon control input and (c) Variation of the optimal control input.*

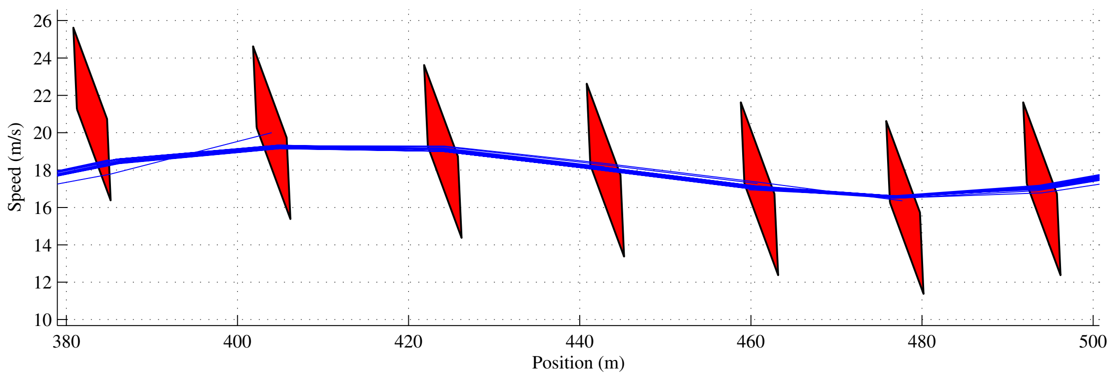

Fig. 4. *Zoom-in on the open-loop evolution of the predicted state (the blue thin line in the beginning of the simulation), which illustrates that the terminal set is reached, and plot of the terminal constraint set evolution as the reference changes.*

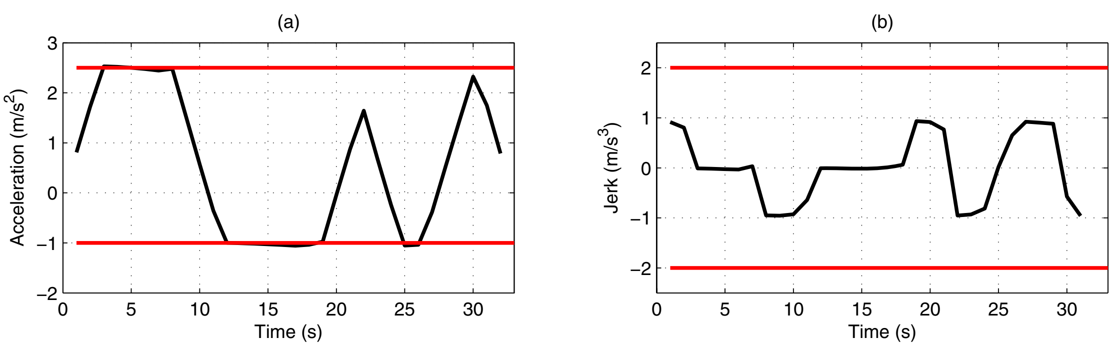

Fig. 5. *Behavior of the acceleration and jerk during the simulation described in Section V.*

and constraint set method has been employed to obtain a hybrid MPC scheme with good tracking performance. The successful simulation results, regarding both tracking and computational complexity, encourage us to pursue the theoretical aspects related to the convergence properties of the hybrid MPC set-up for tracking as well as the real-time implementation on a Smart car.

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