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Controller synthesis for networked control systems[☆]

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ABSTRACT

This paper presents a discrete-time model for networked control systems (NCSs) that incorporates all network phenomena: time-varying sampling intervals, packet dropouts and time-varying delays that may be both smaller and larger than the sampling interval. Based on this model, constructive LMI conditions for controller synthesis are derived, such that stabilizing state-feedback controllers can be designed. Besides the proposed controller synthesis conditions a comparison is made between the use of parameter-dependent Lyapunov functions and Lyapunov–Krasovskii functions for stability analysis. Several examples illustrate the effectiveness of the developed theory.

1. Introduction

The literature on modeling, analysis and controller design of networked control systems (NCSs) expanded rapidly over the last decade (Antsaklis & Baillieul, 2007; Tipsuwan & Chow, 2003; Zhang, Branicky, & Phillips, 2001). The use of networks offers many advantages such as low installation and maintenance costs, reduced system wiring (in the case of wireless networks) and increased flexibility of the system. However, from a control theory point of view, the presence of the network also introduces several disadvantages such as time-varying network-induced delays, aperiodic sampling or packet dropouts. To understand the impact of these network effects on control performance several models have been developed. Roughly speaking, these NCS models can be categorized into continuous-time and discrete-time models. A further discrimination can be given on the basis of which network phenomena they include.

In the continuous-time domain, Fridman, Seuret, and Richard (2004) applied a descriptor system approach to model the sampled-data dynamics of systems with varying sampling intervals in terms of (infinite-dimensional) delay differential equations (DDEs) and study their stability based on the Lyapunov–Krasovskii functional method. In Gao, Chen, and Lam (2008), Yu, Wang, and Chu (2005) and Yue, Han, and Peng (2004), this approach is used for the stability analysis of NCSs with time-varying delays and constant sampling intervals, using (linear) matrix inequality-based techniques. The recent results in Gao et al. (2008) also involve H_∞ controller designs based on linear matrix inequalities (LMIs). However, Mirkin (2007) showed that the use of such an approach for digital control systems neglects the piecewise constant nature of the control signal due to the zero-order-hold mechanism and that it introduces conservatism when exploiting such modeling for stability analysis. More specifically, the conservatism is introduced by the fact that the zero-order hold and delay jointly introduce a particular linearly increasing time-varying delay within each control update interval (sometimes indicated by the sawtooth behaviour

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of the delay), whereas in the modeling approach mentioned above it is replaced by an arbitrary bounded time-varying delay. Moreover, in Mirkin (2007) one indicated that less conservative stability conditions are obtained using a robust parametric modeling of the delay operator as proposed in Kao and Lincoln (2004). An alternative approach, proposed in Naghshtabrizi and Hespanha (2006) and Naghshtabrizi, Teel, and Hespanha (2008), is based on impulsive delay differential equations and does take into account the piecewise constant nature of the control signal due to the zero-order-hold mechanism and has also been shown by Mirkin (2007) to be less conservative than the descriptor approach. As also noted in Naghshtabrizi et al. (2008), the usage of infinite-dimensional DDE models and Lyapunov functionals to analyse the stability of essentially *finite-dimensional* sampled-data NCS does not seem to offer any advantage. The approach in Naghshtabrizi and Hespanha (2006) is able to deal simultaneously with time-varying delays and time-varying sampling intervals but does not explicitly include packet dropouts in the model (although they might be considered as variations in the sampling intervals or delays). Moreover, the stability analysis leads to bilinear matrix inequalities (BMIs), which are generally difficult to solve. As a consequence, for the moment no effective control synthesis results exist within this framework.

The majority of NCS models are discrete-time formulations based on the exact discretization of the continuous-time linear plant over a sample interval (see Cloosterman, van de Wouw, Heemels, & Nijmeijer, 2006; Felicioni & Junco, 2008; Fujioka, 2008; García-Rivera & Barreiro, 2007; Lin & Antsaklis, 2005; Nilsson, 1998; Sala, 2005; Wouw, Naghshtabrizi, Cloosterman, & Hespanha, 2007; Wang & Yang, 2008; Zhang et al., 2001, and the references therein). Such models avoid the problem of an infinite-dimensional state that is encountered in the continuous-time (DDE) models due to delays. Moreover, in these discrete-time models the piecewise constant nature of the control signal due to the zero-order hold is taking into account exactly. Additionally, it has been shown in van de Wouw et al. (2007), that for systems with aperiodic sampling and time-varying delay less than the sampling interval the use of discrete-time models for stability analysis gives less conservative characterization of stability than the use of (impulsive) delay differential equations. On the other hand van de Wouw et al. (2007) and Wouw, Naghshtabrizi, Cloosterman, and Hespanha (2010) show that the modeling in terms of impulsive difference equations is favorable for ISS gain analysis for perturbed NCS. Under simplified assumptions, such that the delay is a multiple of the sampling interval or it takes values in a finite set, the obtained models lead to switched linear systems and corresponding stability conditions can be applied (Lin & Antsaklis, 2005; Nikolakopoulos, Tzes, & Koutroulis, 2005; Xiao, Hassibi, & How, 2000; Zhang, Shi, Chen, & Huang, 2005). However, these models are not so realistic as in practice one typically encounters an infinite number of possible values for the delay. Moreover, more realistic models should take into account that the sampling periods might be aperiodic.

For systems with time-varying sampling intervals, Felicioni and Junco (2008), Fujioka (2008), Sala (2005) and van de Wouw et al. (2007) address the stability analysis and control design using a discrete-time model. In Felicioni and Junco (2008), discrete difference inclusions are obtained for the different values of the sampling interval and sufficient algebraic conditions for existence of quadratic Lyapunov function are derived based on the construction of a solvable Lie algebra. A different approach is given in Fujioka (2008) and Sala (2005), where the authors used the gridding of the set of possible sampling intervals to derive LMI-based stability conditions.

Several discrete-time approaches have been proposed for dealing with network-induced delays. In this context, using the exact discretization over a sampling period, the obtained model is generally a difference equation with time-varying delays in the input and unknown time-varying system matrices. When the variation of the delay is smaller than the sampling period, the analysis/control design problems can be addressed by using a lifted state vector and robust control methods for parametric uncertainties (Cloosterman et al., 2006; García-Rivera & Barreiro, 2007; Hetel, Daafouz, & Iung, 2006) or by applying the Lyapunov–Krasovskii function (LKF) approach (Pan, Marquez, & Chen, 2006; Wu & Chen, 2007; Xie & Wang, 2004; Yoo & Kwon, 2005) (for the LKF approach in discrete-time, see Fridman & Shaked, 2005). In this context, the main problems are the conservatism inherent to the use of upper boundings in the increment of the LKF and the reduced applicability of the results since they are able to deal only with delay variations smaller than the sampling interval. Generalising such models to the case of large delay variations, packet dropouts and time-varying sampling intervals is not a trivial task.

In the literature, two ways of modeling network-induced uncertainties (such as time-varying delays and sampling intervals and packet dropouts) can be distinguished. Firstly, in Fujioka (2008), García-Rivera and Barreiro (2007), Lin and Antsaklis (2005), Naghshtabrizi and Hespanha (2006), Nešić and Teel (2004), Zhang et al. (2001) and many other works, bounds are imposed on the delays, sampling intervals and the maximum number of subsequent dropouts. Secondly, in e.g. (Hespanha, Naghshtabrizi, & Xu, 2007; Montestruque & Antsaklis, 2004; Seiler & Sengupta, 2005; Sinopoli, Schenato, Franceschetti, Poolla, Jordan, & Sastry, 2004), a stochastic modeling approach is adopted. In this paper, we will adopt the first approach. Given bounds on delays, sampling intervals and subsequent dropouts, we will formulate stability conditions and constructive controller synthesis results independent of the probability distribution of the uncertain variables. So, such *robust* results also apply in the stochastic setting and can be seen as 'probability distribution-free' results for the stochastic case if the domain of the probability distribution function is bounded.

In the current paper, we propose a discrete-time NCS model that can deal simultaneously with packet dropouts and time-varying delays smaller and larger than a possibly time-varying sampling interval. This model is obtained using the exact discretization over a sampling interval and it takes into account also the complicated case in which the delay variations may be larger than the sampling interval. Moreover, the possibility of packet dropouts is modeled explicitly. Based on this model, controller synthesis conditions in terms of LMIs will be derived, using both a common quadratic and a parameter-dependent Lyapunov approach. Note that recently, in Hetel, Daafouz, and Iung (2008a), a simplified event-based discrete-time model has been proposed for taking into account the different implementation problems in digital control systems. This model is obtained using the systems representation at both sampling and actuation times. The advantage of the model presented in this paper in comparison to this event-based model is that it generally leads to a discrete-time representation of a smaller dimension. Moreover, it generalises several of the models that exist in the literature to the case in which all the network effects appear simultaneously. This enables the theoretical comparison with the existing approaches. A discussion on the stability characterization based on LKFs and on parameter-dependent Lyapunov functions (PDLF) will be given. This discussion is inspired by the results in Hetel, Daafouz, and Iung (2008b) in which a comparison between LKFs and Lyapunov functions for switched systems is presented in the case of difference equations with time-varying delays in the state. The approach in Hetel et al. (2008b) can deal only with delays that are a multiple of the sampling time, and therefore it does not apply to continuous-time systems as the NCS studied here. We show that the stability analysis based on the most general LFK of a quadratic type is always more conservative than the novel stability characterization presented here. This result applies to the context of NCS in which we are faced with an interaction between continuous-time systems and discrete-time controllers under different perturbing networked effects. In particular, we will show that the existence of general LKFs as used in the literature, implies also the existence of a Lyapunov function in our framework. It is important to note that all existing LKFs are a particular case of the one proposed in this paper and our approach allows much more freedom in the Lyapunov function than the typical LKFs adopted in the literature (Pan et al., 2006; Wu & Chen, 2007; Xie & Wang, 2004; Yoo & Kwon, 2005), which have repetitions of terms. Stated differently, the Lyapunov function that we consider corresponds to a general LKF for which LMI-based stability conditions never appeared in the literature before. In addition, it can formally be proven that our approach is never more conservative than the LKF approach. Next to the stability characterization, we will also present LMI-based synthesis techniques for feedback based on the state vector and on an augmented state vector that includes old inputs next to the system state.

In summary, the main contributions of the paper are as follows. Firstly, a model for NCSs including three network-induced uncertainties (large delays, time-varying sampling intervals and packet dropouts) is developed. Moreover, we present a procedure for the overapproximation of this model to arrive at a polytopic model suitable for stability analysis and controller synthesis. Secondly, we present a stability characterization for NCSs using (parameter-dependent) quadratic Lyapunov functions, which generalises stability characterizations using Lyapunov–Krasovskii functionals existing in the literature. Thirdly, next to LMI-based stability conditions, we provide a solution to the (structured) state feedback synthesis problem in terms of linear matrix inequalities for NCS models including all the above network-induced uncertainties.

This paper is structured as follows: In Section 2 we present our NCS model. Section 3 is dedicated to the theoretical comparison of stability characterizations. Section 4 presents LMI control design methods that are illustrated by numerical examples in Section 4. Section 5 closes with concluding remarks.

2. NCS modeling

In this section, the discrete-time description of a NCS including delays larger than the uncertain, and time-varying sampling interval and packet dropouts is presented. The NCS is depicted schematically in Fig. 1. It consists of a linear continuous-time plant

$$\dot{x}(t) = Ax(t) + Bu(t),$$

with $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$, and a discrete-time static time-invariant controller which are connected over a communication network that induces network delays (τ^{sc} and τ^{ca}). The state measurements ($y(t) = x(t)$) are sampled resulting in the sampling time instants s_k:

$$s_k = \sum_{i=0}^{k-1} h_i \quad \forall k \geq 1, \qquad s_0 = 0, \tag{1}$$

which are non-equidistantly spaced in time due to the time-varying sampling intervals $h_k > 0$. The sequence of sampling instants $s_0, s_1, s_2, \ldots$ is strictly increasing in the sense that $s_{k+1} > s_k$, for all $k \in \mathbb{N}$. We denote by $y_k := y(s_k)$ the kth sampled value of y and by u_k the control value corresponding to $y_k = x_k$. Packet drops may occur (see Fig. 1) and is modeled by the parameter m_k. This parameter denotes whether or not a packet is dropped:

$$m_k = \begin{cases} 0, & \text{if } y_k \text{ and } u_k \text{ are received} \\ 1, & \text{if } y_k \text{ and/or } u_k \text{ is lost.} \end{cases} \tag{2}$$

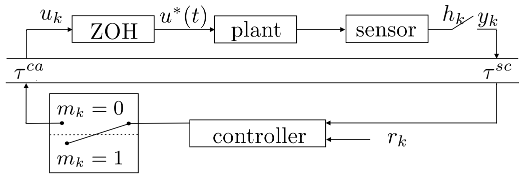

Fig. 1. Schematic overview of the NCS with variable sampling intervals, network delays and packet dropouts.

In (2), we make no distinction between packet dropouts that occur in the sensor-to-controller connection and the controller-to-actuator connection in the network. This can be justified by realizing that, for static controllers, the effect of the packet dropouts on the control updates implemented on the plant is the same in both cases. Indeed, for packet dropouts between the sensor and the controller no new control update is computed and thus no new control input is sent to the actuator. In the case of packet dropouts between the controller and the actuator no new control update is received by the actuator either. Finally, the zero-order-hold (ZOH) function (in Fig. 1) is applied to transform the discrete-time control input u_k to a continuous-time control input $u^*(t)$ being the actual actuation signal of the plant.

In the model, both the varying computation time (τ_k^c), needed to evaluate the controller, and the network-induced delays, i.e. the sensor-to-controller delay (τ_k^{sc}) and the controller-to-actuator delay (τ_k^{ca}), are taken into account. We assume that the sensor acts in a time-driven fashion (i.e. sampling occurs at the times s_k defined in (1)) and that both the controller and the actuator act in an event-driven fashion (i.e. responding instantaneously to newly arrived data). Furthermore, we consider that not all the data is used due to packet dropouts and message rejection, i.e. the effect that more recent control data is available before the older data is implemented and therefore the older data is neglected. Under these assumptions, all three delays can be captured by a single delay $\tau_k := \tau_k^{sc} + \tau_k^c + \tau_k^{ca}$, see also Nilsson, 1998; Zhang et al., 2001. To include these effects in the continuous-time model, let us define the parameter $k^*(t)$ that denotes the index of the most recent control input that is available at time t as $k^*(t) := \max\{k \in \mathbb{N} | s_k + \tau_k \leq t \wedge m_k = 0\}$. The continuous-time model of the plant of the NCS is then given by:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu^*(t) \\ u^*(t) &= u_{k^*(t)}, \end{aligned} \tag{3}$$

with $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$. Here, we assume that the most recent control input remains active in the plant if a packet is dropped.

We assume that the variation in the delays is bounded by $\tau_{\min}$ and $\tau_{\max}$, the variation in the sampling interval is bounded by $h_{\min}$ and $h_{\max}$ and that the number of subsequent packet dropouts is upper bounded by $\bar{\delta}$. The latter means that

$$\sum_{v=k-\bar{\delta}}^{k} m_v \leq \bar{\delta} \tag{4}$$

as this guarantees that from the control inputs $u_{k-\bar{\delta}}, u_{k-\bar{\delta}+1}, \ldots, u_k$ at least one is implemented. In summary, the class $\mathcal{S}$ of admissible sequences $\{(s_k, \tau_k, m_k)\}_{k\in\mathbb{N}}$ can be described as follows:

$$\mathcal{S} := \Big\{ \{(s_k, \tau_k, m_k)\}_{k\in\mathbb{N}} | \, h_{\min} \leq s_{k+1} - s_k \leq h_{\max}, \\ s_0 = 0, \ \tau_{\min} \leq \tau_k \leq \tau_{\max}, \ \sum_{v=k-\bar{\delta}}^{k} m_v \leq \bar{\delta}, \ \forall k \in \mathbb{N} \Big\}, \tag{5}$$

which includes variable sampling intervals, large delays, and packet dropouts.

Remark. In the modeling of the network-induced uncertainties, we impose bounds on the delays, sampling intervals and the maximum number of subsequent dropouts as was also done in Fujioka (2008), García-Rivera and Barreiro (2007), Lin and Antsaklis (2005), Naghshtabrizi and Hespanha (2006), Nešić and Teel (2004), Zhang et al. (2001) and many other works. Given such bounds, we will formulate stability conditions and constructive controller synthesis results independent of the probability distribution of the uncertain variables. So, such *robust* results also apply in the stochastic setting and can be seen as probability distribution-free results of the stochastic case if the domain of the probability distribution function is bounded.

Next, the general description of the continuous-time control input $u^*(t)$ in (3) is reformulated to indicate explicitly which control inputs u_k are active in the sampling interval $[s_k, s_{k+1})$. Such a reformulation is needed to derive the discrete-time NCS model, which will ultimately be employed in the controller synthesis methods.

Lemma 1. *Consider the continuous-time NCS as defined in (3) and the admissible sequences of sampling instants, delays, and packet dropouts as defined in (5). Define* $\underline{d} := \lfloor \frac{\tau_{\min}}{h_{\max}} \rfloor$*, the largest integer smaller than or equal to* $\frac{\tau_{\min}}{h_{\max}}$ *and* $\overline{d} := \lceil \frac{\tau_{\max}}{h_{\min}} \rceil$*, the smallest integer larger than or equal to* $\frac{\tau_{\max}}{h_{\min}}$*. Then, the control action* $u^*(t)$ *in the sampling interval* $[s_k, s_{k+1})$ *is described by*

$$u^*(t) = u_{k+j-\overline{d}-\overline{\delta}} \quad \text{for } t \in [s_k + t_j^k, s_k + t_{j+1}^k), \tag{6}$$

where the actuation instants $t_j^k \in [0, h_k]$ *are defined as:*

$$\begin{aligned} t_j^k = \min \Bigg\{ &\max \Bigg\{ 0, \tau_{k+j-\overline{d}-\overline{\delta}} - \sum_{l=k+j-\overline{\delta}-\overline{d}}^{k-1} h_l \Bigg\} + m_{k+j-\overline{d}-\overline{\delta}} h_{\max}, \\ &\max \Bigg\{ 0, \tau_{k+j-\overline{d}-\overline{\delta}+1} - \sum_{l=k+j+1-\overline{\delta}-\overline{d}}^{k-1} h_l \Bigg\} + m_{k+j-\overline{d}-\overline{\delta}+1} h_{\max}, \\ &\ldots, \max \Bigg\{ 0, \tau_{k-\underline{d}} - \sum_{l=k-\underline{d}}^{k-1} h_l \Bigg\} + m_{k-\underline{d}} h_{\max}, h_k \Bigg\}, \end{aligned} \tag{7}$$

with $t_j^k \leq t_{j+1}^k$ *and* $j \in \{0, 1, \ldots, \overline{d} + \overline{\delta} - \underline{d}\}$ *(see Fig. 2). Moreover,* $0 = t_0^k \leq t_1^k \leq \cdots \leq t_{\overline{d}+\overline{\delta}-\underline{d}}^k \leq t_{\overline{d}+\overline{\delta}-\underline{d}+1}^k := h_k$*.*

Proof. The proof is given in Appendix A. □

Based on Lemma 1 and $\sigma = \{(s_k, \tau_k, m_k)\}_{k\in\mathbb{N}} \in \mathcal{S}$, the discrete-time NCS model can be defined as:

$$x_{k+1} = e^{Ah_k} x_k + \sum_{j=0}^{\overline{d}+\overline{\delta}-\underline{d}} \int_{h_k - t_{j+1}^k}^{h_k - t_j^k} e^{As} ds B u_{k+j-\overline{d}-\overline{\delta}}, \tag{8}$$

with t_j^k as defined in Lemma 1. The minimum and maximum values of the t_j^k parameters are described in Lemma 2.

Lemma 2. *Consider the time instants* t_j^k *as defined in (7), where* $s_{k+j-\overline{d}-\overline{\delta}}$ *(with* $h_{k+j-\overline{d}-\overline{\delta}} = s_{k+j-\overline{d}-\overline{\delta}+1} - s_{k+j-\overline{d}-\overline{\delta}}$*),* $\tau_{k+j-\overline{d}-\overline{\delta}}$*, and* $m_{k+j-\overline{d}-\overline{\delta}}$ *are taken from the class* $\mathcal{S}$ *defined in (5). The minimum value of* $t_j^k, j \in \{0, 1, \ldots, \overline{d} + \overline{\delta} - \underline{d}\}$*, is given by*

$$t_{j,\min} = \begin{cases} \min\{\tau_{\min} - \underline{d} h_{\max}, h_{\min}\} & \text{if } j = \overline{d} + \overline{\delta} - \underline{d} \\ 0 & \text{if } 1 \leq j < \overline{d} + \overline{\delta} - \underline{d}, \end{cases} \tag{9}$$

and the maximum value of $t_j^k, j \in \{1, 2, \ldots, \overline{d} + \overline{\delta} - \underline{d}\}$*, is given by*

$$t_{j,\max} = \begin{cases} \min\{\tau_{\max} - (\overline{d} - j) h_{\min}, h_{\max}\} & \text{if } 1 \leq j \leq \overline{d} - \underline{d} \\ h_{\max} & \text{if } \overline{d} - \underline{d} + 1 \leq j \\ & \quad \leq \overline{d} + \overline{\delta} - \underline{d}. \end{cases} \tag{10}$$

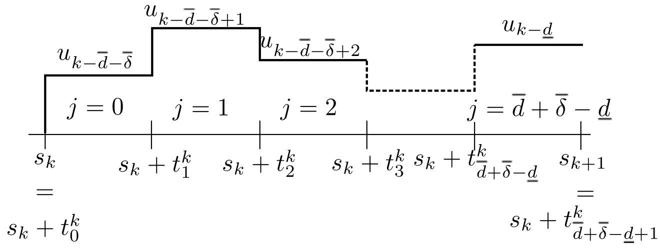

Fig. 2. Graphical interpretation of t_j^k.

Additionally, $t_0^k := 0$ *and* $t_{\overline{d}+\overline{\delta}-\underline{d}+1}^k := h_k$*, which gives for the minimum and maximum bound* $t_{\overline{d}+\overline{\delta}-\underline{d}+1} \in [h_{\min}, h_{\max}]$*.*

Proof. The proof can be derived based on Lemma 1 if the bounds on the delay, sampling interval and number of subsequent packet dropouts are taken into account. The interested reader is referred to Cloosterman (2008) for the detailed proof. □

Let θ_k denote the vector of uncertain parameters consisting of the sampling interval and the actuation instants:

$$\theta_k := (h_k, t_1^k, \ldots, t_{\overline{d}+\overline{\delta}-\underline{d}}^k). \tag{11}$$

The description of θ_k does not include $t_{\overline{d}+\overline{\delta}-\underline{d}+1}^k$, as $h_k = t_{\overline{d}+\overline{\delta}-\underline{d}+1}^k$, which is already included in θ_k. Moreover, t_0^k is not considered either, since it represents a constant term, $t_0^k = 0$. Using the fact that $h_k \in [h_{\min}, h_{\max}]$ and the bounds $t_{j,\min}, t_{j,\max}$ on the actuation instants given in (9) and (10), we can define the set

$$\begin{aligned} \Theta = \Big\{ &\theta_k \in \mathbb{R}^{\overline{d}+\overline{\delta}-\underline{d}+1} \,|\, h_k \in [h_{\min}, h_{\max}], \\ &t_j^k \in [t_{j,\min}, t_{j,\max}], \ 1 \leq j \leq \overline{d} + \overline{\delta} - \underline{d}, \\ &0 \leq t_1^k \leq \cdots \leq t_{\overline{d}+\overline{\delta}-\underline{d}}^k \leq h_k \Big\}. \end{aligned} \tag{12}$$

Note that this set does not depend on k. System (8) represents a discrete-time system with multiple delays in the input. Moreover, the system matrices are time-varying according to the uncertain parameters $\theta_k \in \Theta$. In the following section, we will show how to characterize the stability of this system based on LMIs and compare this to the Lyapunov–Krasovskii Function (LKF) approach.

3. Stability characterizations and relations with the LKF theory

In this section we discuss the stability characterization for the NCS (3) with a state feedback of the form

$$u_k = -\overline{K} x_k. \tag{13}$$

We can without loss of generality assume that $\overline{K}$ has a full row rank. When $\overline{K}$ does not have a full row rank, it is always possible to write the controller in the form

$$u_k = \begin{pmatrix} u_k^a \\ u_k^b \end{pmatrix} = \begin{pmatrix} I \\ G \end{pmatrix} K_a x_k = \begin{pmatrix} I \\ G \end{pmatrix} u_k^a,$$

where K_a has full row rank (possibly after a permutation of the inputs) and we obtain a model similar to (3) with K_a instead of $\overline{K}$ and $B \begin{pmatrix} I \\ G \end{pmatrix}$ instead of B that does satisfy the full row rank condition on the feedback gain.

To render the model (8) with the feedback (13) suitable for analysis, we consider an equivalent delay-free model, based on a lifted state vector

$$\xi_k = \begin{pmatrix} x_k^T & u_{k-1}^T & \ldots & u_{k-\overline{d}-\overline{\delta}}^T \end{pmatrix}^T$$

that includes past system inputs.

This leads to the lifted model

$$\xi_{k+1} = \tilde{A}_1(\theta_k)\xi_k, \tag{14}$$

where $\tilde{A}_1(\theta_k) =$

$$\begin{pmatrix} \Lambda(\theta_k) & \widetilde{M}_{\bar{d}+\bar{\delta}-1}(\theta_k) & \widetilde{M}_{\bar{d}+\bar{\delta}-2}(\theta_k) & \dots & \widetilde{M}_1(\theta_k) & \widetilde{M}_0(\theta_k) \\ -\overline{K} & 0 & 0 & \dots & 0 & 0 \\ 0 & I & 0 & \dots & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ \vdots & & \ddots & \ddots & 0 & 0 \\ 0 & \dots & \dots & 0 & I & 0 \end{pmatrix}$$

with $\Lambda(\theta_k) = e^{Ah_k} - \widetilde{M}_{\bar{d}+\bar{\delta}}(\theta_k)\overline{K}$ and

$$\widetilde{M}_j(\theta_k) = \begin{cases} \int_{h_k - t_{j+1}^k}^{h_k - t_j^k} e^{As} ds B & \text{if } 0 \le j \le \bar{d} + \bar{\delta} - \underline{d}, \\ 0 & \text{if } \bar{d} + \bar{\delta} - \underline{d} < j \le \bar{d} + \bar{\delta}. \end{cases} \tag{15}$$

The goal of this section is to prove that characterizing the stability of the closed-loop NCS (8) using the lifted model (14) and (parameter-dependent) quadratic Lyapunov functions is less (or, in the worst case, equally) conservative than the methods available in the literature based on discrete-time LKF.

In order to show this, we will use an alternative lifted state space model as an intermediate step in the proof. This model uses the state vector $\chi_k = \left(x_k^T \; x_{k-1}^T \; \dots \; x_{k-\bar{d}-\bar{\delta}}^T\right)^T$, i.e.

$$\chi_{k+1} = \tilde{A}_2(\theta_k)\chi_k, \tag{16}$$

where $\tilde{A}_2(\theta_k) =$

$$\begin{pmatrix} \Lambda(\theta_k) & -\widetilde{M}_{\bar{d}+\bar{\delta}-1}(\theta_k)\overline{K} & -\widetilde{M}_{\bar{d}+\bar{\delta}-2}(\theta_k)\overline{K} & \dots & -\widetilde{M}_0(\theta_k)\overline{K} \\ I & 0 & 0 & \dots & 0 \\ 0 & I & 0 & \dots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \dots & 0 & I & 0 \end{pmatrix}.$$

This second lifted model is important since it is easy to show that if there exists a LKF (even the most general LKF that can be obtained using quadratic terms), then there exists a parameter-dependent quadratic Lyapunov function for (16) as well. This relation will be described in detail at the end of the section. First we will show that the existence of a parameter-dependent Lyapunov function for (16) is equivalent to the existence of a parameter-dependent Lyapunov function for (14). This issue is relevant since it would formally prove that we can base the stability analysis for the NCS (8) with state feedback controller (13) on (14) without losing stability properties that could be obtained via (16). Note that the state dimension of ξ_k in (14) is smaller than the dimension of χ_k in (16), which clearly has modeling and numerical advantages.

3.1. Equivalence of stability characterizations for the two lifted models

Let us discuss the equivalence of the lifted models (16) and (14) with respect to stability and Lyapunov functions in more detail. Clearly, for a given *constant* parameter θ, the stability of (16) is equivalent to the stability of (14) and vice versa. Moreover, since for linear time-invariant systems the existence of a quadratic Lyapunov is a necessary and sufficient stability condition, there exists a quadratic Lyapunov function for (16) if and only if there exists one for (14) when θ is constant. However, assuming that there exists a quadratic Lyapunov function for one of the systems, (16) or (14), there is no constructive method available in the literature for deducing a Lyapunov function for the other one. We will provide such a constructive method, and moreover, we will even consider a more complicated problem as (16) and (14) are uncertain systems that vary over time as θ_k is changing. In this case, quadratic Lyapunov functions are known to be sufficient only for characterizing stability, not necessary. The question is now whether, in the time-varying uncertain case, the existence of a quadratic Lyapunov function for system (16) is equivalent to the existence of a quadratic Lyapunov function for (14). In Theorem 4, we will answer this question and we will show that there exists a quadratic-like Lyapunov function for system (14) if and only if there exists one for the alternative representation (16). To prove this result for any parameter-dependent quadratic Lyapunov function, the following lemma will be needed.

Lemma 3. *Consider the matrix $R \in \mathbb{R}^{q\times p}$ and the matrices $A(\theta) \in \mathbb{R}^{p\times p}$ that depend continuously on $\theta \in \Theta$, where $\Theta \subset \mathbb{R}^l$ is a compact set. Define the matrices*

$$\bar{A}(\theta) = \begin{pmatrix} A(\theta) & 0 \\ R & 0 \end{pmatrix} \in \mathbb{R}^{(p+q)\times(p+q)}, \tag{17}$$

for $\theta \in \Theta$. The following statements are equivalent:

- *There exist symmetric positive definite matrices $P(\theta) \in \mathbb{R}^{p+q\times p+q}$, $\theta \in \Theta$ such that*

$$\bar{A}(\theta_1)^T P(\theta_2)\bar{A}(\theta_1) - P(\theta_1) < 0, \quad \forall \theta_1, \theta_2 \in \Theta. \tag{18}$$

- *There exist symmetric positive definite matrices $Q(\theta) \in \mathbb{R}^{p\times p}$, $\theta \in \Theta$ such that*

$$A(\theta_1)^T Q(\theta_2) A(\theta_1) - Q(\theta_1) < 0, \quad \forall \theta_1, \theta_2 \in \Theta. \tag{19}$$

Moreover, there exists a common solution $P(\theta) = P > 0$, for all $\theta \in \Theta$ to (18) if and only if there exists a common solution $Q(\theta) = Q > 0$ to (19).

Proof. See Appendix B. □

Theorem 4. *Consider the NCS (8) with state feedback controller (13) and the two representations (14) and (16). The following statements are equivalent:*

- *There exist symmetric positive definite matrices $P(\theta)$, $\theta \in \Theta$ such that*

$$\tilde{A}_1^T(\theta_k) P(\theta_{k+1})\tilde{A}_1(\theta_k) - P(\theta_k) < 0, \tag{20}$$

  for all $\theta_k, \theta_{k+1} \in \Theta$, thus

$$V(\xi_k) = \xi_k^T P(\theta_k)\xi_k \tag{21}$$

  is a parameter-dependent Lyapunov function for system (14).

- *There exist symmetric positive definite matrices $Q(\theta)$, $\theta \in \Theta$ such that*

$$\tilde{A}_2^T(\theta_k) Q(\theta_{k+1})\tilde{A}_2(\theta_k) - Q(\theta_k) < 0, \tag{22}$$

  for all $\theta_k, \theta_{k+1} \in \Theta$, thus

$$V(\chi_k) = \chi_k^T Q(\theta_k)\chi_k \tag{23}$$

  is a parameter-dependent Lyapunov function for system (16). Moreover, system (14) has a common quadratic Lyapunov function $V(\xi_k) = \xi_k^T P \xi_k$ if and only if system (16) has a common quadratic Lyapunov function $V(\chi_k) = \chi_k^T Q \chi_k$.

Proof. Since the state feedback matrix $\overline{K}$ has full row rank there exists a matrix $S \in \mathbb{R}^{(n-m)\times n}$ such that the matrix $\begin{pmatrix} \overline{K} \\ S \end{pmatrix}$ is invertible.

Define the matrices $\tilde{A}_3(\theta_k) =$

$$\left(\begin{array}{c|cccc|c|c|c}
\Lambda(\theta_k) & \widetilde{M}_{\bar{d}+\bar{\delta}-1}(\theta_k) & \dots & \dots & \widetilde{M}_1(\theta_k) & \widetilde{M}_0(\theta_k) & 0 & 0 \\ \hline
-\overline{K} & 0 & \dots & \dots & 0 & 0 & 0 & 0 \\ \hline
0 & I & 0 & \dots & 0 & 0 & 0 & 0 \\
\vdots & 0 & \ddots & & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & & \ddots & 0 & \vdots & \vdots & \vdots \\
0 & 0 & \dots & 0 & I & 0 & 0 & 0 \\ \hline
S & 0 & \dots & \dots & 0 & 0 & 0 & 0 \\ \hline
0 & 0 & \dots & \dots & 0 & 0 & I & 0
\end{array}\right)$$

and $W =$

$$\left(\begin{array}{c|cccc|c}
I & 0 & \dots & \dots & 0 & 0 \\ \hline
0 & -\overline{K} & 0 & \dots & 0 & 0 \\
\vdots & 0 & \ddots & & \vdots & \vdots \\
\vdots & \vdots & & \ddots & 0 & \vdots \\
0 & 0 & \dots & 0 & -\overline{K} & 0 \\ \hline
0 & 0 & \dots & \dots & 0 & -\overline{K} \\ \hline
0 & S & 0 & \dots & 0 & 0 \\
\vdots & 0 & \ddots & & \vdots & \vdots \\
\vdots & \vdots & & \ddots & 0 & \vdots \\
0 & 0 & \dots & 0 & S & 0 \\ \hline
0 & 0 & \dots & \dots & 0 & S
\end{array}\right) \in \mathbb{R}^{(n+1)\cdot(\bar{d}+\bar{\delta})\times(n+1)\cdot(\bar{d}+\bar{\delta})}.$$

Notice that $\tilde{A}_3(\theta_k)W = W\tilde{A}_2(\theta_k) =$

$$\left(\begin{array}{c|cccc|c}
\Lambda(\theta_k) & -\widetilde{M}_{\bar{d}+\bar{\delta}-1}(\theta_k)\overline{K} & \dots & \dots & -\widetilde{M}_1(\theta_k)\overline{K} & -\widetilde{M}_0(\theta_k)\overline{K} \\ \hline
-\overline{K} & 0 & \dots & \dots & 0 & 0 \\ \hline
0 & -\overline{K} & 0 & \dots & 0 & 0 \\
\vdots & 0 & \ddots & & \vdots & \vdots \\
\vdots & \vdots & & \ddots & 0 & \vdots \\
0 & 0 & \dots & 0 & -\overline{K} & 0 \\ \hline
S & 0 & \dots & \dots & 0 & 0 \\ \hline
0 & S & 0 & \dots & 0 & 0 \\
\vdots & 0 & \ddots & & \vdots & \vdots \\
\vdots & \vdots & & \ddots & 0 & \vdots \\
0 & 0 & \dots & 0 & S & 0
\end{array}\right).$$

This implies that $\tilde{A}_3(\theta_k)$ is similar to $\tilde{A}_2(\theta_k)$. It is easy to show that (22) holds if and only if there exists symmetric positive definite matrices $\tilde{P}(\theta_k) = W^{-T}Q(\theta_k)W^{-1}$ such that

$$\tilde{A}_3^T(\theta_k)\tilde{P}(\theta_{k+1})\tilde{A}_3(\theta_k) - \tilde{P}(\theta_k) < 0.$$

Furthermore, notice that $\tilde{A}_3(\theta_k)$ can be expressed as

$$\tilde{A}_3(\theta_k) = \left(\begin{array}{c|c|c}
\tilde{A}_1(\theta_k) & 0 & 0 \\ \hline
S \mid 0 & 0 & 0 \\ \hline
0 & I & 0
\end{array}\right).$$

Then, apply Lemma 3 with

$$A(\theta) := \left(\begin{array}{c|c}
\tilde{A}_1(\theta_k) & 0 \\ \hline
S \mid 0 & 0
\end{array}\right) \quad \text{and} \quad R := \left(0 \mid I\right).$$

Next apply Lemma 3 again for

$$A(\theta) := \tilde{A}_1(\theta_k) \quad \text{and} \quad R := \left(S \mid 0\right)$$

in order to complete the proof. □

3.2. Relations with the Lyapunov–Krasovskii stability characterization

For discrete-time uncertain systems with delay in the input such as (8), several stability results exist based on Lyapunov–Krasovskii functions (LKFs). Using an adequate partition of the Lyapunov matrix

$$Q(\theta_k) = \begin{pmatrix}
Q^{0,0}(\theta_k) & Q^{0,1}(\theta_k) & \dots & Q^{0,\bar{d}+\bar{\delta}}(\theta_k) \\
Q^{0,1}(\theta_k) & Q^{1,1}(\theta_k) & \ddots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
Q^{0,\bar{d}+\bar{\delta}}(\theta_k) & \dots & \dots & Q^{\bar{d}+\bar{\delta},\bar{d}+\bar{\delta}}(\theta_k)
\end{pmatrix}, \tag{24}$$

it can be shown that the Lyapunov function (23) is equivalent to the LKF

$$V(x_k, \dots, x_{k-\bar{d}-\bar{\delta}}) = \sum_{i=0}^{\bar{d}+\bar{\delta}} \sum_{j=0}^{\bar{d}+\bar{\delta}} x_{k-i}^T Q^{i,j}(\theta_k) x_{k-j}, \tag{25}$$

which is the most general LKF that can be obtained using quadratic forms. Any of the quadratic LKFs found in the literature (see Pan et al., 2006; Wu & Chen, 2007; Xie & Wang, 2004; Yoo & Kwon, 2005) are a particular case of (25). As a consequence of Theorem 4, we know that there exists a Lyapunov function (23) for (16) if and only if there exists one of the form (21) for (14), i.e. if and only if the equations (20) are satisfied. Consequently, condition (20) represents a necessary and sufficient condition for the existence of the most general form of LKFs that can be obtained using quadratic terms as in (25). Hence, using a stability characterization based on the model (14) is less (or, in the worst case, equally) conservative than the stability analysis results based on quadratic LKF that are available in the literature (Pan et al., 2006; Wu & Chen, 2007; Xie & Wang, 2004; Yoo & Kwon, 2005).

In the next section, we will present a constructive LMI method for controller design using stability characterizations based on parameter-dependent Lyapunov functions such as in (21).

4. Controller synthesis

To render the model (8) suitable for controller synthesis, we rewrite it as:

$$\xi_{k+1} = \tilde{A}(\theta_k)\xi_k + \tilde{B}(\theta_k)u_k, \tag{26}$$

where

$$\tilde{A}(\theta_k) = \begin{pmatrix}
e^{Ah_k} & \widetilde{M}_{\bar{d}+\bar{\delta}-1}(\theta_k) & \widetilde{M}_{\bar{d}+\bar{\delta}-2}(\theta_k) & \dots & \widetilde{M}_0(\theta_k) \\
0 & 0 & 0 & \dots & 0 \\
0 & I & 0 & \dots & 0 \\
\vdots & & \ddots & & \vdots \\
0 & \dots & 0 & I & 0
\end{pmatrix},$$

$\tilde{B}(\theta_k) = \begin{pmatrix}\widetilde{M}_{\bar{d}+\bar{\delta}}^T(\theta_k) & I & 0 & \dots & 0\end{pmatrix}^T$. This model is equivalent to (14) when the input is a state feedback of the form (13).

Let us now design a static state feedback controller of the form (13). The main difficulty to synthesize a state feedback (13) is that it results in a structured control synthesis problem, i.e. we need to design a control law of the form

$$u_k = -K\xi_k \tag{27}$$

with a specific structure in the feedback gain matrix: $K = \begin{pmatrix}\overline{K} & 0_{m\times(\bar{d}+\bar{\delta})m}\end{pmatrix}$. A solution to this structured controller synthesis problem is to apply the approach presented in de Oliveira, Bernussou, and Geromel (1999). Moreover, such an approach allows for the use of a parameter-dependent Lyapunov function (Daafouz & Bernussou, 2001) that might result in less conservative controller synthesis results than the use of a common quadratic Lyapunov function.

Remark. From the control synthesis point of view, when dealing with a system such as (26), a natural alternative would be to design a state feedback controller of the form (27) using the full state ξ_k of the underlying model (26). However, from the point of view of the NCS (3), this is equivalent to using a dynamical controller of the form

$$u_k = -K_0 x_k - K_1 u_{k-1} \cdots - K_{\bar{d}+\bar{\delta}} u_{k-\bar{d}-\bar{\delta}}. \tag{28}$$

The use of such a dynamic control law requires a reconsideration of the assumption made earlier to lump all the delays τ_k^{sc}, τ_k^{c} and τ_k^{ca} in one parameter τ. Using a dynamic control law as in (28) actually leads to more restrictive assumptions on the network modeling setup as $y_k = x_k$ should always arrive at the controller after the moment that u_{k-1} is sent to the actuator, i.e. $s_k + \tau_k^{sc} > s_{k-1} + \tau_{k-1}^{sc} + \tau_{k-1}^{c}$ as otherwise special precautions are needed to handle out-of-order arrival of measured outputs resulting in longer delays. In addition, the adopted modeling setup and controller in (28) require that no packet dropouts occur between the sensors and the controller. Namely, in the case of a packet dropout between the sensor and controller, it is possible that $y_k = x_k$ does not arrive at the controller and thus u_k cannot be computed; furthermore the controller (28) cannot be updated beyond the k-th update. Therefore, a deadlock in the controller can occur and the worst case scenario would be not sending control updates at all to the actuator. Although modeling dropouts alternatively as prolongations of the sampling interval (see, e.g., the comparison in van Schendel, Donkers, Heemels, and van de Wouw (2010)) might alleviate these issues to some extent, dropouts in the channel between the controller and the actuators introduce similar complications in this case. We care to stress that a static state feedback as in (13) does not suffer from such problems and additional assumptions, as explicated in above, are not needed, which greatly enhances its applicability.

To derive the control synthesis conditions, the model (26) is rewritten using the real Jordan form of the continuous-time system matrix A. Basically, we express the state matrix $A = TJT^{-1}$ with J the real Jordan form, and T an invertible matrix. Next, we compute all the integrals in (15) using $e^{As} = Te^{Js}T^{-1}$ to obtain a model in which the uncertain parameters θ_k appear explicitly. This leads to a generic model of the form

$$\xi_{k+1} = \left(F_0 + \sum_{i=1}^{\zeta} \alpha_i(\theta_k) F_i\right) \xi_k + \left(G_0 + \sum_{i=1}^{\zeta} \alpha_i(\theta_k) G_i\right) u_k, \tag{29}$$

with θ_k defined in (11) and ζ the number of time-varying functions $\alpha_i(\cdot)$ given by $(\bar{d} + \bar{\delta} - \underline{d} + 1)\nu$, with $\nu \leq n$, where n is the dimension of the state vector x. We have $\nu = n$ when the geometric multiplicity of each distinct eigenvalue of A is equal to one and $\nu < n$ when the geometric multiplicity of an eigenvalue is larger than one. A typical function $\alpha_i(\theta_k)$ is of the form $e^{\lambda(h_k - t_j^k)}$, with λ a real eigenvalue of A, and of the form $e^{a(h_k - t_j^k)}\cos(b(h_k - t_j^k))$ or $e^{a(h_k - t_j^k)}\sin(b(h_k - t_j^k))$ when λ is a complex eigenvalue ($\lambda = a + bj$) of A. For more details on the use of the Jordan form, including the case that $\nu < n$ the reader is referred to Appendix B in Cloosterman (2008).

Using bounds on the uncertain parameters $\theta_k = (h_k, t_1^k, \ldots, t_{\bar{d}+\bar{\delta}-\underline{d}}^k)$ described by the set Θ in equation (12) this gives rise to the set of matrices

$$\mathcal{FG} = \left\{ \left(F_0 + \sum_{i=1}^{\zeta} \alpha_i(\theta_k) F_i, G_0 + \sum_{i=1}^{\zeta} \alpha_i(\theta_k) G_i\right) \middle| \theta_k \in \Theta \right\} \tag{30}$$

that contains all possible matrix combinations in (26) and (29). Based on this infinite set $\mathcal{FG}$ of matrices we will derive a stabilizing controller of the form (13) for the NCS (3). To overcome the infinite dimension of the set $\mathcal{FG}$ a convex overapproximation of the set is used. Denote the maximum and minimum value of $\alpha_i(\theta_k)$, respectively, by

$$\overline{\alpha}_i = \max_{\theta_k \in \Theta} \alpha_i(\theta_k), \qquad \underline{\alpha}_i = \min_{\theta_k \in \Theta} \alpha_i(\theta_k), \tag{31}$$

with Θ defined in (12). Then the set of matrices $\mathcal{FG}$, given in (30), is a subset of $co(\mathcal{H}_{FG})$ with

$$\mathcal{H}_{FG} = \left\{ \left(\left(F_0 + \sum_{i=1}^{\zeta} \alpha_i F_i\right), \left(G_0 + \sum_{i=1}^{\zeta} \alpha_i G_i\right) \right) : \alpha_i \in \{\underline{\alpha}_i, \overline{\alpha}_i\},\ i = 1, 2, \ldots, \zeta \right\}, \tag{32}$$

where 'co' denotes the convex hull.

We will also write the set of vertices $\mathcal{H}_{FG}$ as $\mathcal{H}_{FG} = \{(H_{F,j}, H_{G,j}) \mid j = 1, 2, \ldots, 2^{\zeta}\}$. Using this finite set of 2^{ζ} vertices, a finite number of LMI controller synthesis conditions is given for the state-feedback controller (13) in the following theorem.

Theorem 5. *Consider the NCS model (3), (6), (7), (13), and its discrete-time representation (26), (13) for sequences of sampling instants, delays, and packet dropouts $\sigma \in \mathcal{S}$ with $\mathcal{S}$ as in (5). Consider the equivalent representation (29) based on the Jordan form of A and the set of vertices $\mathcal{H}_{FG}$ defined in (32).*

If there exist symmetric positive definite matrices $Y_j \in \mathbb{R}^{(n+(\bar{d}+\bar{\delta})m)\times(n+(\bar{d}+\bar{\delta})m)}$, a matrix $\bar{Z} \in \mathbb{R}^{m\times n}$, matrices $X_j = \begin{pmatrix} \bar{X}_1 & 0 \\ \bar{X}_{2,j} & \bar{X}_{3,j} \end{pmatrix}$, with $\bar{X}_1 \in \mathbb{R}^{n\times n}$, $\bar{X}_{2,j} \in \mathbb{R}^{(\bar{d}+\bar{\delta})m\times n}$, $\bar{X}_{3,j} \in \mathbb{R}^{(\bar{d}+\bar{\delta})m\times(\bar{d}+\bar{\delta})m}$, $j = 1, 2, \ldots, 2^{\zeta}$, and a scalar $0 \leq \gamma < 1$ that satisfy

$$\begin{pmatrix} X_j + X_j^T - Y_j & X_j^T H_{F,j}^T - \begin{pmatrix}\bar{Z} & 0\end{pmatrix}^T H_{G,j}^T \\ H_{F,j} X_j - H_{G,j}\begin{pmatrix}\bar{Z} & 0\end{pmatrix} & (1-\gamma) Y_l \end{pmatrix} > 0, \tag{33}$$

for all $j, l \in \{1, 2, \ldots, 2^{\zeta}\}$, then the closed-loop NCS (3), (6), (7), (13) with $\bar{K} = \bar{Z}\bar{X}_1^{-1}$ is globally asymptotically stable.

Proof. To prove this theorem, we first note that, due to the convex overapproximation based on the uncertain parameters $\alpha_i(\cdot)$, it holds for all $\theta_k \in \Theta$ that $\left(\tilde{A}(\theta_k), \tilde{B}(\theta_k)\right) \in \mathcal{FG} \subset co(\mathcal{H}_{FG})$. Hence, for the stability of (26) and (29) with the state feedback controller (13), it is sufficient to prove stability of the system

$$\xi_{k+1} = \sum_{j=1}^{2^{\zeta}} \mu_j^k \left(H_{F,j} - H_{G,j} K\right) \xi_k, \tag{34}$$

where $K = \begin{pmatrix}\bar{K} & 0_{m\times(\bar{d}+\bar{\delta})m}\end{pmatrix}$ and $\mu_1^k, \mu_2^k, \ldots, \mu_{2^{\zeta}}^k \geq 0$, satisfy $\sum_{j=1}^{2^{\zeta}} \mu_j^k = 1$, for all $k \in \mathbb{N}$. Assume that the inequalities (33) hold. Using the fact that $\bar{K}\bar{X}_1 = \bar{Z}$ we have that

$$\begin{pmatrix}\bar{Z} & 0\end{pmatrix} = K \begin{pmatrix} \bar{X}_1 & 0 \\ \bar{X}_{2,j} & \bar{X}_{3,j} \end{pmatrix},$$

and thus we obtain:

$$\begin{pmatrix} X_j + X_j^T - Y_j & X_j^T (H_{F,j} - H_{G,j}K)^T \\ (H_{F,j} - H_{G,j}K) X_j & (1-\gamma) Y_l \end{pmatrix} > 0. \tag{35}$$

Applying Theorem 3 in Daafouz and Bernussou (2001) this inequality implies that the function

$$V(\xi_k) = \xi_k^T P(\mu_1^k, \mu_2^k, \ldots, \mu_{2^{\zeta}}^k) \xi_k, \tag{36}$$

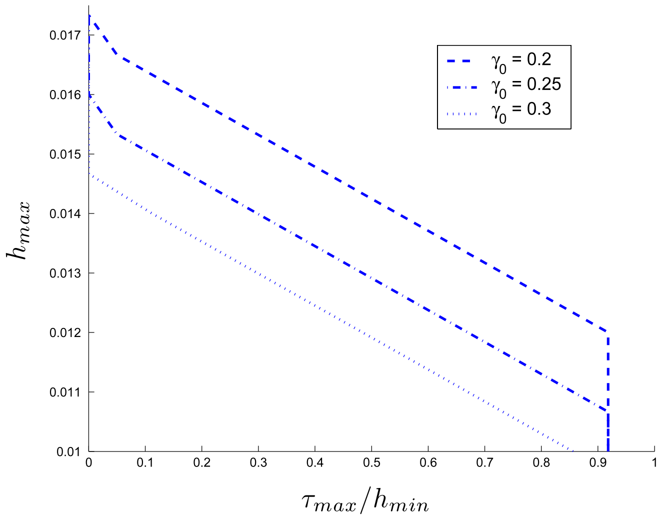

Fig. 3. Feasibility regions for different transient response specifications.

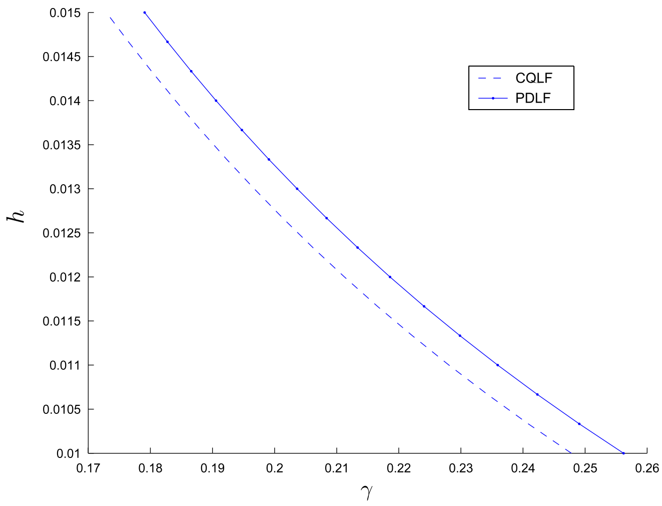

Fig. 4. Comparison between the CQLF and PDLF approaches.

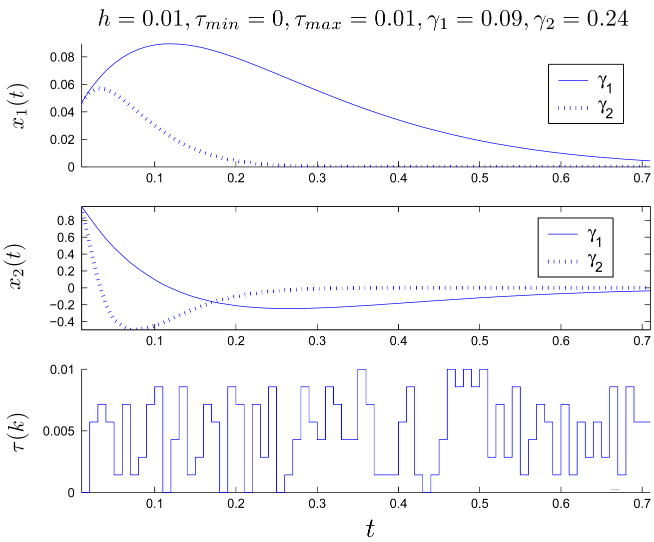

Fig. 5. Evolution from an arbitrary initial condition.

with $P(\mu_1^k, \mu_2^k, \dots, \mu_{2^\zeta}^k) = \sum_{j=1}^{2^\zeta} \mu_j^k P_j$ and $P_j = Y_j^{-1}$, is strictly decreasing along the trajectories of system (34). Consequently system (26), (13) is globally asymptotically stable. Using similar arguments as in Hetel et al. (2006) it can be shown that the intersample behaviour is stable as well and, consequently, that the NCS (3), (6), (7), (13) for all $\sigma \in \mathcal{S}$ is globally asymptotically stable. □

Remark. This theorem shows that (36) is a parameter-dependent Lyapunov function for the system (26) with the controller (13). Using the results from the previous section, this shows that if the LMIs (33) are satisfied they imply the existence of a LKF of the form (25). Notice that using this approach we avoid the conservative upper boundings in the difference of the LKF, which are usually encountered in the literature to arrive at LKF-based stability conditions in LMI form.

The case of a common quadratic Lyapunov function (CQLF) $V(\xi) = \xi_k^T P \xi_k$ is a particular case of this theorem by taking $Y_j = Y,\ \forall j = 1, \dots, 2^\zeta$, with $P = Y^{-1}$.

If one is still interested in using an extended state feedback (27) despite the mentioned disadvantages, then Theorem 5 can be modified by replacing the matrices X_j, $\forall i \neq j$ with a constant matrix X without a specific structure and using Z instead of $(\bar{Z}\ 0)$. The extended controller is obtained then by $K = ZX^{-1}$.

In this paper we adopted an overapproximation of the NCS model using the Jordan form and leading to (32). All the theory also applies if the overapproximation is obtained by other techniques (e.g. Fujioka, 2008; Hetel et al., 2006, or any other).

5. Illustrative examples

Consider a NCS represented by (3), with $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (double integrator). First, let us show the applicability of the presented theory for time-varying sampling intervals and delays. We consider $\bar{\delta} = 0$ and we analyse the feasibility of the LMI conditions in Theorem 5 for various values of $\tau_{\max}$, $h_{\max}$ and γ (keeping $h_{\min}$ fixed, i.e. $h_{\min} = 0.01$ s). In order to use the same continuous-time transient response specifications, the parameter γ used in the LMIs is scaled according to the different values of $h_{\max}$, i.e. we use $\gamma = 1 - (1 - \gamma_0)^{h_{\max}/h_{\min}}$ where γ_0 represents the value taken for $h_{\max} = h_{\min} = 0.01$ s. The delay is considered to be time-varying, and the LMIs are solved for $\tau_{\min} = 0.1h_{\min}$ and $\tau_{\max}$ in the interval $[0.1h_{\min}, 0.98h_{\min}]$. Note that in this case message rejection cannot occur since $\tau_{\max} < h_{\min}$. The tradeoff curves between transient performance (decay rate) and robustness versus uncertainties ($h_{\max}$, $\tau_{\max}$), i.e. the regions for which Theorem 5 provides a stabilizing state feedback, are depicted in Fig. 3. We can remark that the feasibility of LMIs is reduced as γ_0 increases. This is due to the fact that if the parameter γ_0 is increased, a faster transient response is required. As an example, for the bounds $h_{\min} = 0.01$ s and $h_{\max} = 0.014$ s on the sampling interval and time-varying delays characterized by $\tau_{\min} = 0.1h_{\min}$ and $\tau_{\max} = 0.6h_{\min}$, the stabilizing state-feedback controller with gain $K = \begin{pmatrix} 0.622 & 1.089 \end{pmatrix}$ is obtained using Theorem 5 with $\gamma_0 = 0$. For $\gamma_0 = 0.17$ a faster transient response is obtained with the controller gain $K = \begin{pmatrix} 164.837 & 22.64 \end{pmatrix}$.

Next, a comparison between the use of a common quadratic (CQLF) and of a parameter-dependent Lyapunov function (PDLF) is given in Fig. 4 for a constant sampling interval and time-varying delays characterized by $\tau_{\min} = 0$ and $\tau_{\max} = h$. The example illustrates the improvement of the transient response specification (γ) with the PDLF approach. A simulation is given in Fig. 5, for two different values of γ with $h_{\min} = h_{\max} = 0.01$ *s* and $\tau_{\max} = h_{\min}$. The controller gain used in this simulation has been obtained using parameter-dependent Lyapunov functions.

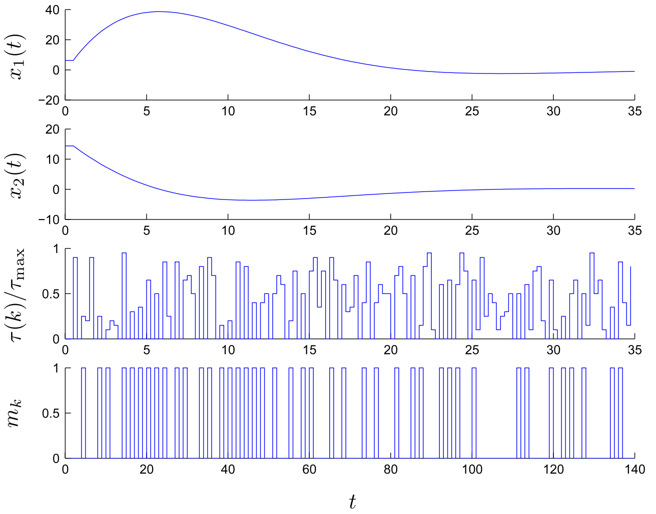

Fig. 6. Time response with delay and packet dropouts for $h = 0.25$, $\tau_{\max} = h$ and $\bar{\delta} = 1$.

Finally, we illustrate the situation with time delays larger than the constant sampling interval h. In this case $\tau_{\max} = 2.8\ h$ and $\tau_{\min} = 0$. Note that the same results hold also for the situation with packet dropouts $\bar{\delta} = 1$ and $\tau_{\max} = 1.8\ h$ or $\bar{\delta} = 2$ and $\tau_{\max} = 0.8\ h$. This is generally true for our result as long as $\bar{d} + \bar{\delta} = constant$. In this case a stabilizing controller can be found for sampling intervals up to $h = 0.55$ s (e.g. for $h = 0.55$ s a stabilizing controller is given by $K = (0.0363 \quad 0.2525)$). A simulation with both delay and packet dropouts using the latter controller gain is presented in Fig. 6 for $h = 0.25$, $\tau_{\max} = h$ and $\bar{\delta} = 1$.

6. Conclusions

A discrete-time NCS model, based on an exact discretization of the continuous-time system at the sampling instants, is presented. This model includes all relevant network phenomena: the presence of time-varying delays that may be larger than the sampling interval, message rejection, packet dropouts, and variations in the sampling interval. Next, a stability characterization based on parameter-dependent Lyapunov functions is proposed. It is shown theoretically that the stability characterization presented here is generally less conservative than the methods available in the literature based on LKF. Based on the developed model and on the proposed stability characterization, constructive state feedback synthesis conditions are derived in terms of linear matrix inequalities (LMIs). Simulations are presented that show the applicability and effectiveness of the obtained controller synthesis results.

Appendix A. Proof of Lemma 1

To prove that $u_{k-\bar{d}-\bar{\delta}}$ is the oldest control input that might be active during the sampling interval $[s_k, s_{k+1})$, we consider, firstly, the case without packet dropouts, and secondly, the case with packet dropouts. From the definition of $\bar{d}$ in Lemma 1, it follows that the control input $u_{k-\bar{d}}$ is always available at the plant before or exactly at s_k, if $u_{k-\bar{d}}$ is not dropped. To prove this, we use the relation $s_k = s_{k-\bar{d}} + \sum_{l=k-\bar{d}}^{k-1} h_l$, which provides the upper and lower bounds on s_k, given by $s_{k-\bar{d}} + \bar{d}h_{\min} \leq s_k \leq s_{k-\bar{d}} + \bar{d}h_{\max}$. Combining the lower bound on s_k and $s_{k-\bar{d}} + \tau_{k-\bar{d}} \leq s_{k-\bar{d}} + \tau_{\max}$ gives: $s_{k-\bar{d}} + \tau_{k-\bar{d}} \leq s_{k-\bar{d}} + \tau_{\max} \leq s_k - \bar{d}h_{\min} + \tau_{\max} \leq s_k$, due to the definition of $\bar{d} = \lceil \frac{\tau_{\max}}{h_{\min}} \rceil$. Hence, in case that the control input $u_{k-\bar{d}}$ is not dropped (i.e. $m_{k-\bar{d}} = 0$), it is available before or on s_k and no older control inputs $u_{k+j-\bar{d}}$, with $j < 0$ will be active in the sampling interval $[s_k, s_{k+1})$. To show that $u_{k-\bar{d}}$ can be active in the sampling interval $[s_k, s_{k+1})$, we need to show that $u_{k-\bar{d}+1}$ can become active after s_k, if no packets are dropped. To do so, note that $\bar{d} - 1 < \tau_{\max}/h_{\min} \leq \bar{d}$ and thus we have that $s_{k-\bar{d}+1} + \tau_{\max} > s_{k-\bar{d}+1} + (\bar{d} - 1)h_{\min}$. As the smallest value of $s_k = s_{k-\bar{d}+1} + \sum_{l=k-\bar{d}+1}^{k-1} h_l$ is equal to $s_{k-\bar{d}+1} + (\bar{d} - 1)h_{\min}$, and the largest implementation time of $u_{k-\bar{d}+1}$ is $s_{k-\bar{d}+1} + \tau_{\max}$, the previous inequality shows that $u_{k-\bar{d}+1}$ might be available for implementation (strictly) after s_k. As a consequence, in the case without dropouts, $u_{k-\bar{d}+1}$ can indeed be active in the sampling interval $[s_k, s_{k+1})$.

To prove that in the case of packet dropouts $u_{k-\bar{d}-\bar{\delta}}$ is the oldest control input that can possibly be active in $[s_k, s_{k+1})$, note that, from (4), it follows that at least one of the control inputs $u_{k-\bar{d}-\bar{\delta}}, u_{k-\bar{d}-\bar{\delta}+1}, \dots, u_{k-\bar{d}}$ is not lost. If $u_{k-\bar{d}+1}$ is indeed implemented after s_k (which is possible as just shown), then at least one of the inputs $u_{k-\bar{d}-\bar{\delta}}, u_{k-\bar{d}-\bar{\delta}+1}, \dots, u_{k-\bar{d}}$ will be active in the sampling interval $[s_k, s_{k+1})$. The fact that the maximum number of subsequent packet dropouts equals $\bar{\delta}$ implies that $u_{k-\bar{d}-\bar{\delta}}$ is the oldest control input that might be implemented in the sampling interval $[s_k, s_{k+1})$.

From the definition of $\underline{d}$ in Lemma 1, it follows that the input $u_{k-\underline{d}}$ represents the most recent control input that might be implemented during the sampling interval $[s_k, s_{k+1})$. To prove this, consider the smallest time at which $u_{k-\underline{d}}$ might be implemented that is given by $s_{k-\underline{d}} + \tau_{\min}$. Based on the definition of $\underline{d}$, which gives that $\tau_{\min} < (\underline{d} + 1)h_{\max}$, we can conclude that $s_{k-\underline{d}} + \tau_{\min} < s_{k-\underline{d}} + (\underline{d} + 1)h_{\max}$. Combining this with the tight bounds on s_{k+1}, given by:

$$s_{k-\underline{d}} + (\underline{d} + 1)h_{\min} \leq s_{k+1} \leq s_{k-\underline{d}} + (\underline{d} + 1)h_{\max}$$

yields that it might hold that $s_{k-\underline{d}} + \tau_{\min} \leq s_{k+1}$ as s_{k+1} may attain the value $s_{k-\underline{d}} + (\underline{d} + 1)h_{\max}$. Consequently, $u_{k-\underline{d}}$ might be implemented before s_{k+1}.

To show that $u_{k-\underline{d}}$ is the most recent data that can be active in $[s_k, s_{k+1})$, we prove that more recent control inputs always arrive after s_{k+1}. Consider $j > \bar{d} + \bar{\delta} - \underline{d}$. Then, we have that $s_{k+j-\bar{d}-\bar{\delta}} + \tau_{\min}$ is the earliest time at which $u_{k+j-\bar{d}-\bar{\delta}}$ might be implemented. To determine if this moment may occur before s_{k+1}, consider the upper bound on s_{k+1}, in terms of $s_{k+j-\bar{d}-\bar{\delta}}$, given by $s_{k+1} \leq s_{k+j-\bar{d}-\bar{\delta}} + (-j + \bar{d} + \bar{\delta} + 1)h_{\max}$ for $j > \bar{d} + \bar{\delta} - \underline{d}$. However, for $j > \bar{d} + \bar{\delta} - \underline{d}$

$$s_{k+j-\bar{d}-\bar{\delta}} + \tau_{\min} \geq s_{k+j-\bar{d}-\bar{\delta}} + (-j + \bar{d} + \bar{\delta} + 1)h_{\max} \geq s_{k+1},$$

due to the definition of $\underline{d} = \lfloor \frac{\tau_{\min}}{h_{\max}} \rfloor$. This proves that $u_{k-\underline{d}}$ is indeed the most recent control input that can be active in the sampling interval $[s_k, s_{k+1})$.

So far we proved that $u_{k+j-\bar{d}-\bar{\delta}}$, $j \in \{0, 1, \dots, \bar{d} + \bar{\delta} - \underline{d}\}$ are the only control values that can be implemented in $[s_k, s_{k+1})$. Now, the times t_j^k with $j \in \{0, 1, \dots, \bar{d} + \bar{\delta} - \underline{d}\}$ will be constructed in such a manner that $[s_k + t_j^k, s_k + t_{j+1}^k)$ is the time interval in which the control input $u_{k+j-\bar{d}-\bar{\delta}}$ is active in the sampling interval $[s_k, s_{k+1})$. The time $t_{\bar{d}+\bar{\delta}-\underline{d}}^k$ (being the starting time of $u_{k-\underline{d}}$ in the interval $[s_k, s_{k+1}]$) is given by

$$t_{\bar{d}+\bar{\delta}-\underline{d}}^k = \min\left[h_k, \tau_{k-\underline{d}} - \sum_{l=k-\underline{d}}^{k-1} h_l + m_{k-\underline{d}} h_{\max}\right]. \tag{A.1}$$

Indeed, if $m_{k-\underline{d}} = 0$, then $s_k + \tau_{k-\underline{d}} - \sum_{l=k-\underline{d}}^{k-1} h_l$ is the time at which $u_{k-\underline{d}}$ is available at the plant. If $\tau_{k-\underline{d}} - \sum_{l=k-\underline{d}}^{k-1} h_l > h_k$, then $u_{k-\underline{d}}$ might be active after s_{k+1}, but not in $[s_k, s_{k+1})$. Since we are only interested in the interval $[s_k, s_{k+1})$, we take the minimum of this value and h_k in (A.1). Note that, by the definition of $\underline{d}$,

$\tau_{k-\underline{d}} - \sum_{l=k-\underline{d}}^{k-1} h_l \geq 0$. Finally, if $u_{k-\underline{d}}$ is dropped, i.e. $m_{k-\underline{d}} = 1$, then the expression in (A.1) gives h_k, which means that the input $u_{k-\underline{d}}$ is not used in $[s_k, s_{k+1})$.

Next, as $u_{k-\underline{d}-1}$ can only be active before $u_{k-\underline{d}}$ is available, $t^k_{\bar{d}+\bar{\delta}-\underline{d}-1}$ is given by

$$t^k_{\bar{d}+\bar{\delta}-\underline{d}-1} = \min\left[t^k_{\bar{d}+\bar{\delta}-\underline{d}}, \max\left\{0, \tau_{k-\underline{d}-1} - \sum_{l=k-\underline{d}-1}^{k-1} h_l\right\} + m_{k-\underline{d}-1}h_{\max}\right]. \quad \text{(A.2)}$$

Similarly to $t^k_{\bar{d}+\bar{\delta}-\underline{d}}$, if $\max\{0, \tau_{k-\underline{d}-1} - \sum_{l=k-\underline{d}-1}^{k-1} h_l\} + m_{k-\underline{d}-1}h_{\max} \in [0, t^k_{\bar{d}+\bar{\delta}-\underline{d}})$, then $s_k + \tau_{k-\underline{d}-1} - \sum_{l=k-\underline{d}-1}^{k-1} h_l$ is the time at which $u_{k-\underline{d}-1}$ is available at the plant. In case $\tau_{k-\underline{d}-1} - \sum_{l=k-\underline{d}}^{k-1} h_l < 0$, then $u_{k-\underline{d}-1}$ might be active before s_k. Since, we are only interested, here, in the interval $[s_k, s_{k+1})$, we take the maximum of this value and zero in (A.2). For the other values of t^k_j, the recursion can be derived similarly, which leads to

$$t^k_j = \min\left[t^k_{j+1}, \max\left\{0, \tau_{k+j-\bar{d}-\bar{\delta}} - \sum_{l=k+j-\bar{d}-\bar{\delta}}^{k-1} h_l\right\} + m_{k+j-\bar{d}-\bar{\delta}}h_{\max}\right],$$

for $0 \leq j \leq \bar{d} + \bar{\delta} - \underline{d}$, $m_{k+j-\bar{d}-\bar{\delta}}$ satisfying (4), and with $t^k_{\bar{d}+\bar{\delta}-\underline{d}+1} := h_k$. The evaluation of this recursive relation yields the explicit characterization of (7).

Appendix B. Proof of Lemma 3

Suppose that (18) holds for some matrices $P^T(\theta) = P(\theta) > 0, \forall\theta \in \Theta$. Decompose the matrices as follows:

$$P(\theta) = \begin{pmatrix} P_1(\theta) & P_2(\theta) \\ P_2^T(\theta) & P_3(\theta) \end{pmatrix}$$

in accordance with the matrix $\bar{A}(\theta)$. By expanding (18) we obtain for all $\theta_1, \theta_2 \in \Theta$ that

$$\begin{pmatrix} \begin{bmatrix} A^T(\theta_1)P_1(\theta_2)A(\theta_1) - P_1(\theta_1) + R^TP_2(\theta_2)A(\theta_1) \\ +A^T(\theta_1)P_2(\theta_2)R + R^TP_3(\theta_2)R \end{bmatrix} & -P_2(\theta_1) \\ -P_2^T(\theta_1) & -P_3(\theta_1) \end{pmatrix} < 0.$$

This is equivalent (using the Schur complement lemma) to

$$A^T(\theta_1)P_1(\theta_2)A(\theta_1) - P_1(\theta_1) + R^TP_2(\theta_2)A(\theta_1) + A^T(\theta_1)P_2(\theta_2)R + R^TP_3(\theta_2)R + P_2(\theta_1)P_3^{-1}(\theta_1)P_2^T(\theta_1) < 0.$$

Adding and subtracting

$$A^T(\theta_1)P_2(\theta_2)P_3^{-1}(\theta_2)P_2^T(\theta_2)A(\theta_1)$$

to the previous inequality implies for all $\theta_1, \theta_2 \in \Theta$ that

$$A^T(\theta_1)Q(\theta_2)A(\theta_1) - Q(\theta_1) + W(\theta_1, \theta_2) < 0,$$

where

$$Q(\theta) = P_1(\theta) - P_2(\theta)P_3^{-1}(\theta)P_2^T(\theta), \quad \forall\, \theta \in \Theta \quad \text{(B.1)}$$

and

$$W(\theta_1, \theta_2) = (P_2(\theta_2)A(\theta_1) + P_3(\theta_2)R)^T \times P_3^{-1}(\theta_2) \times (P_2(\theta_2)A(\theta_1) + P_3(\theta_2)R).$$

As $W(\theta_1, \theta_2) \geq 0$ and $Q(\theta) > 0$ (since $P_3(\theta) > 0$ and $Q(\theta)$ is the Schur complement of $P(\theta)$), clearly $Q(\theta), \theta \in \Theta$, satisfy condition (19). Notice that when the matrices $P(\theta)$ are constant, i.e.

$$P(\theta) = P = \begin{pmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{pmatrix}, \quad \forall\theta \in \Theta,$$

the corresponding matrices $Q(\theta)$ as in (B.1) that satisfy (19) are constant as well:

$$Q(\theta) = Q = P_1 - P_2P_3^{-1}P_2^T, \quad \theta \in \Theta.$$

To prove the converse, assume that (19) holds. Then, due to the continuity of A with respect to θ and to the compactness of Θ, there exists $\epsilon > 0$ such that for all $\theta_1, \theta_2 \in \Theta$

$$\begin{pmatrix} A^T(\theta_1)Q(\theta_2)A(\theta_1) - Q(\theta_1) + \epsilon R^TR & 0 \\ 0 & -\epsilon I \end{pmatrix} < 0.$$

This inequality shows that the matrices $P(\theta)$ defined as

$$P(\theta) = \begin{pmatrix} Q(\theta) & 0 \\ 0 & \epsilon I \end{pmatrix} > 0, \quad \theta \in \Theta$$

satisfy (18).

Clearly, when $Q(\theta) = Q,\ \theta \in \Theta$, the common matrix

$$P(\theta) = P = \begin{pmatrix} Q & 0 \\ 0 & \epsilon I \end{pmatrix} > 0, \quad \theta \in \Theta$$

satisfies the inequality (18), which completes the proof.

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