On the Time-stepping Methods for Linear Passive Networks with Ideal Diodes

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Abstract

Simulation of switching networks is a problem that has been studied extensively in circuit theory [1, 2, 5, 11, 12, 15, 18, 25]. Roughly speaking, there are two main approaches, namely event-tracking (see e.g. [1, 15]) and time-stepping methods (see [2, 11, 12, 18] for electrical networks and [14, 16, 17, 22, 24] for unilaterally constrained mechanical systems with friction phenomena). Having a hybrid systems point of view (see for instance [21]), event-tracking methods are based on the idea of solving corresponding DAEs of the current circuit topology (called 'mode' in the hybrid systems terminology), monitoring possible changes of circuit topology (mode transition), and (if necessary) determining the exact time (event time) instant of the change of topology and the next topology. Time-stepping methods differ from this scheme by regarding the whole system as a collection of differential equations with constraints and trying to approximate the solutions of these differential equations with constraints. As a consequnce of this point of view, there is no need to locate exact event times. However, the convergence of the approximations in a suitable sense has to be guaranteed. Since the methods seem to work well in practice, the question of convergence is usually neglected in the literature. It is the objective of this paper to provide a rigorous basis for the use of time-stepping methods in the simulation of circuits with state events.

In [7] (see also [3]) the meaning of a transient true solution to the dynamical network model with ideal diodes has already been established. Using techniques borrowed from the theory of linear complementarity systems (LCS) [8, 9, 13, 19, 20], existence and uniqueness of solutions have been proven under mild conditions. Moreover, several regularity properties have been shown from which this paper will benefit.

The particular time-stepping method that we will study here is based on the well-known backward Euler scheme and has been described, for instance, in [2, 11, 12] for electrical networks. Similar methods have been used in a mechanical context in [14, 16, 17, 22, 24]. The advantage of the method is that it is straightforward to implement and many algorithms (e.g. Lemke's algorithm [4], Katzenelson's algorithm [10] and others [12]) are available to solve the one-step problems consisting of linear complementarity problems (LCPs).

In [11] the use of a time-stepping method based on the backward Euler scheme (or higher order linear multistep integration methods [6] like the trapezoidal rule) has already been proposed for the class of linear complementarity systems, i.e., linear time-invariant dynamical systems coupled with ideal diode characteristics (complementarity conditions). By an example, it will be shown that the method is not suited for the general class of linear complementarity systems. This example indicates also, that although the method has proven itself in practice, one should not indiscriminately apply it to general discontinuous dynamical systems.

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Convergence problems of time-stepping methods for mechanical systems subject to unilateral constraints or friction have been studied by Stewart [22,23]. He shows that for a broad class of nonlinear constrained mechanical systems there always exists a *subsequence* of approximating time functions that converge to a real solution of the mechanical model. However, the convergence of the complete sequence has not been shown in [22, 23]. The conditions used in [22, 23] do not cover electrical networks containing ideal diodes, which form the subject of this paper. Specifically, we will show that for the class of discontinuous dynamical systems consisting of linear electrical passive circuits with ideal diodes the backward Euler time-stepping method is consistent. To be specific, we prove that the whole sequence (and not only a subsequence) of the approximating time functions converge to the real transient solution of the network model, when the step size decreases to zero. Although the results are written down here for networks containing ideal diodes (internally controlled switches) only, externally controlled switches can easily be included without destroying the convergence proof. The results presented here form a justification of the backward Euler time-stepping scheme in the field of switched electrical networks. Such a justification seems required considering the problems that might occur due to changing configurations of the network, the possibility of Dirac impulses and the discontinuities of the system's variables.

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