A Control Lyapunov Approach to Predictive Control of Hybrid Systems

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Abstract. In this paper we consider the stabilization of hybrid systems with both continuous and discrete dynamics via predictive control. To deal with the presence of discrete dynamics we adopt a "hybrid" control Lyapunov function approach, which consists of using two different functions. A Lyapunov-like function is designed to ensure finite-time convergence of the discrete state to a target value, while asymptotic stability of the continuous state is guaranteed via a classical local control Lyapunov function. We show that by combining these two functions in a proper manner it is no longer necessary that the control Lyapunov function for the continuous dynamics decreases at each time step. This leads to a significant reduction of conservativeness in contrast with classical Lyapunov based predictive control. Furthermore, the proposed approach also leads to a reduction of the horizon length needed for recursive feasibility with respect to standard predictive control approaches.

1 Introduction

One of the central problems in hybrid systems is the regulation to a desired operating point along with the optimization of a performance criterion. A solution to this problem that can successfully deal with the combination of discrete and continuous dynamics is provided by the model predictive control (MPC) methodology, as illustrated in [1]. One of the major challenges signaled in [1] is constrained stabilization of both continuous and discrete dynamics (in terms of convergence to a desired equilibrium). The solution to this problem presented in [1] consisted in using a terminal equality constraint for both continuous and

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discrete states. This result was further relaxed in [2, 3] towards using a terminal inequality constraint on the continuous states. However, such relaxations were achieved in the absence of discrete states. In addition, a relatively long prediction horizon was still required for feasibility of the terminal constraints, which resulted in a high computational burden. This is a drawback, as discrete dynamics are indeed one of the fundamental features of hybrid dynamical systems (see e.g. [4, 5, 6]). Discrete dynamics can be used to represent robot tasks and sequences of operations in industrial batch processes, or computer program executions in embedded and software-enabled control systems. Therefore, predictive controllers that stabilize hybrid systems with both continuous and discrete dynamics, preferably in a numerically efficient way, are needed. While we consider stabilization of hybrid systems with discrete dynamics, stability analysis for the autonomous case was considered among the others in [6] (continuous time) and [4] (discrete time).

In this paper we build a framework for predictive control of hybrid systems with discrete dynamics (HSDD, for short), as opposed to piecewise affine systems used in [2, 3], where there are no discrete dynamics, but only a discrete static mapping. Rather than using a terminal constraint setup [7], we employ a hybrid version of the classical control Lyapunov function (CLF) notion [8] to ensure convergence and stability. For the synthesis of CLFs via optimization (including predictive control) we refer to [9, 10, 11]. However, for a general hybrid system it may be too restrictive to enforce a global control Lyapunov function defined over the continuous state only. Instead of using a global CLF defined over the continuous state only, we adopt a "hybrid" CLF defined over the whole hybrid system state and constituted of two functions. A control Lyapunov-like function for the discrete state (*discrete state CLF*) ensures finite-time convergence of the discrete state to the desired value, while asymptotic stability of the continuous state when the discrete state is reached is guaranteed by a standard local CLF (*continuous state CLF*). The main innovations are the combination of the global discrete state CLF and of the local continuous state CLF to address stabilization of hybrid systems, and the construction of the function related to the discrete dynamics, that is defined in terms of the graph distance from the current to the target discrete state over the graph associated with the finite state machine that describes the discrete dynamics. This function is required to decrease over a finite horizon, which is lower bounded by the horizon needed to perform a single transition to a discrete state "closer" to the target, and that is shorter than the horizon needed to reach the target state. Thus, the proposed strategy requires in general a shorter horizon for feasibility with respect to the approach of [1]. In contrast, the CLF associated with the continuous state is allowed *not to* decrease at each time step, until the target discrete state is reached.

The remainder of the paper is organized as follows. Section 2 presents basic definitions and notations, and the system model is defined in Section 3. Section 4 deals with the construction of the hybrid CLF, while Section 5 presents an algorithm that implements the hybrid CLF in a receding horizon control strategy. Conclusions are summarized in Section 6.

2 Preliminaries and Notation

Let $\mathbb{R}$, $\mathbb{R}_+$, $\mathbb{Z}$ and $\mathbb{Z}_+$ denote the field of real numbers, the set of non-negative reals, the set of integers and the set of non-negative integers, respectively. We use the notation $\mathbb{Z}_{\geq c_1}$ and $\mathbb{Z}_{(c_1,c_2]}$ (and similarly with $\mathbb{R}$) to denote the sets $\{k \in \mathbb{Z}_+ \mid k \geq c_1\}$ and $\{k \in \mathbb{Z}_+ \mid c_1 < k \leq c_2\}$, respectively, for some $c_1, c_2 \in \mathbb{Z}$, $c_1 < c_2$. For a set $\mathcal{S} \subseteq \mathbb{R}^n$, we denote by $\text{int}(\mathcal{S})$ its interior. By $\mathbf{0}$ and $\mathbf{1}$ we denote vectors/matrices of appropriate dimensions entirely composed of 0 and 1, respectively. The domains of the discrete state and of the discrete input are the symbolic sets $\mathcal{E} = \{\epsilon_1, \ldots, \epsilon_{n_b}\}$ and $\mathcal{E}_u = \{\epsilon_{u_1}, \ldots, \epsilon_{u_{m_b}}\}$, respectively. The Hölder p-norm of a vector $x \in \mathbb{R}^n$ is defined as $\|x\|_p \triangleq (|[x]_1|^p + \ldots + |[x]_n|^p)^{\frac{1}{p}}$, if $p \in \mathbb{Z}_{[1,\infty)}$ and $\|x\|_\infty \triangleq \max_{i=1,\ldots,n} |[x]_i|$, where $[x]_i$, $i = 1, \ldots, n$, is the i-th component of x and $|\cdot|$ is the absolute value. Let $\|\cdot\|$ denote an arbitrary p-norm.

Given a system $x(k+1) = \phi(x(k), u(k))$, an initial state $x(0)$ and an input sequence $\mathbf{u}_N = \{u(0), \ldots, u(N-1)\}$, $N \in \mathbb{Z}_{\geq 1}$, we use $\mathbf{x} = \{x(0), \ldots, x(N)\}$ to denote the sequence of states obtained by applying from the initial state $x(0)$ the input sequence $\mathbf{u}_N$. Furthermore, let $\phi^i(x(0), \mathbf{u}_i) \triangleq x(i)$, with $\phi^0(x(0), \mathbf{u}_0) \triangleq x(0)$. With some abuse of notation, when useful for clarity, we will separate the discrete and the continuous arguments of a function $f(x,u)$, i.e., given $x = [x_c^T \ x_b^T]^T$, $u = [u_c^T \ u_b^T]^T$, where x_c, u_c are the continuous components of state and input, respectively, and x_b, u_b are the discrete components of state and input, respectively, $f(x_c, x_b, u_c, u_b) \triangleq f(x,u)$.

A function $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ *belongs to class* $\mathcal{K}$ if it is continuous, strictly increasing and $\varphi(0) = 0$. It *belongs to class* $\mathcal{K}_\infty$ if $\varphi \in \mathcal{K}$ and $\varphi(s) \to \infty$ when $s \to \infty$. A function $\beta : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ *belongs to class* $\mathcal{KL}$ if for each $k \in \mathbb{R}_+$, $\beta(\cdot, k) \in \mathcal{K}$ and for each $s \in \mathbb{R}_+$, $\beta(s, \cdot)$ is non-increasing and $\lim_{k\to\infty} \beta(s,k) = 0$.

2.1 Stability Notions

Consider the discrete-time nonlinear system described by the difference inclusion

$$x(k+1) \in \Phi_c(x(k)), \quad k \in \mathbb{Z}_+, \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state at the discrete-time instant k. The mapping $\Phi_c : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ is an arbitrary nonlinear, possibly discontinuous, set-valued function. We assume that the origin is an equilibrium in (1), i.e. $\Phi_c(0) = \{0\}$.

Definition 1. *We call a set* $\mathcal{P} \subseteq \mathbb{R}^n$ positively invariant (PI) *for system* (1) *if for all* $x \in \mathcal{P}$ *it holds that* $\Phi_c(x) \subseteq \mathcal{P}$.

Next, we state a regional version of the global asymptotic stability property presented in [12, Chapter 4] along with sufficient stabilization conditions.

Definition 2. *Let* $\mathbb{X}$ *with* $0 \in \text{int}(\mathbb{X})$ *be a subset of* $\mathbb{R}^n$. *We call system* (1) asymptotically stable (AS) in $\mathbb{X}$ *if there exists a* $\mathcal{KL}$*-function* $\beta(\cdot,\cdot)$ *such that, for each* $x(0) \in \mathbb{X}$, *it holds that all corresponding state trajectories of* (1) *satisfy* $\|x(k)\| \leq \beta(\|x(0)\|, k)$, $\forall k \in \mathbb{Z}_{\geq 1}$.

Theorem 1. *Let* $\mathbb{X}$ *be a PI set for* (1)*, with* $0 \in \text{int}(\mathbb{X})$*. Furthermore, let* $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$*,* $\rho \in \mathbb{R}_{[0,1)}$ *and let* $V : \mathbb{R}^n \to \mathbb{R}_+$ *be a function such that:*

$$\alpha_1(\|x\|) \le V(x) \le \alpha_2(\|x\|) \tag{2a}$$

$$V(x^+) \le \rho V(x) \tag{2b}$$

for all $x \in \mathbb{X}$ *and all* $x^+ \in \Phi_c(x)$*. Then the system* (1) *is AS in* $\mathbb{X}$*.*

The proof of Theorem 1 is similar in nature to the proof given in [12, 11] by replacing the difference *equation* with the difference *inclusion* as in (1) and is omitted here for brevity. We call a function $V(\cdot)$ that satisfies the hypothesis of Theorem 1 a *Lyapunov function.*

Consider the system with discrete dynamics

$$x(k+1) = \Phi_b(x(k)), \qquad k \in \mathbb{Z}_+, \tag{3}$$

where $\mathcal{E} = \{\epsilon_1, \ldots, \epsilon_{n_b}\}$ is a symbolic set, $x(k) \in \mathcal{E}$ is the state and $\Phi_b : \mathcal{E} \to \mathcal{E}$ is an arbitrary function.

Definition 3. *Let* $x^e \in \mathcal{E}$ *denote a desired target state. We call system* (3) *convergent (with respect to* x^e*) if for all* $x(0) \in \mathcal{E}$ *there exists a* $\bar{k}(x(0)) \in \mathbb{Z}_{\ge 1}$ *such that* $\Phi_b(x(k)) = x^e$ *for all* $k \in \mathbb{Z}_{\ge \bar{k}(x(0))}$*.*

Consider now the discrete-time system described by the difference equation

$$x(k+1) = \phi_c(x(k), u(k)), \tag{4}$$

where $x(k) \in \mathbb{X} \subseteq \mathbb{R}^n$ is the state, $u(k) \in \mathbb{U} \subseteq \mathbb{R}^m$ is the control input at the discrete-time instant k, and $\phi_c : \mathbb{X} \times \mathbb{U} \to \mathbb{R}^n$ is an arbitrary nonlinear function, possibly discontinuous. We assume $0 \in \text{int}(\mathbb{X})$, $0 \in \text{int}(\mathbb{U})$, and $\phi_c(0, 0) = 0$.

Definition 4. *A function* $V : \mathbb{R}^n \to \mathbb{R}_+$ *that satisfies* (2a) *and for which there exists* $\rho \in \mathbb{R}_{[0,1)}$ *such that* $\forall x \in \mathbb{X}$*, there exists a control input* $u \in \mathbb{U}$ *for which*

$$V(\phi_c(x, u)) \le \rho V(x),$$

is called a Control Lyapunov function (CLF) for system (4)*.*

2.2 Graph Notions

We consider systems with discrete state dynamics defined by a finite state machine, and we employ a directed graph representation for the finite state machine. We introduce here some graph theory notions, that will be used to define the CLF-like function related to the discrete dynamics.

Let $G = (V, E, C)$ be a labelled directed graph, where $V = \{v_1, \ldots, v_s\}$ is the set of *nodes*, and $E \subseteq (V \times V)$ is the set of edges. We indicate by $e_{ij} = (v_i, v_j)$ the edge from node v_i to node v_j, and the label function C associates a positive value to each existing edge, i.e. $C : E \to \mathbb{R}_+$, $C(e_{ij}) = c_{ij}$, and $c_{ii} = 0$, for $i \in \mathbb{Z}_{[1,s]}$. An often employed definition of distance between elements of discrete sets is the *discrete distance* [6], that is $d(v_i, v_j) = 0$ if $i = j$, and $d(v_i, v_j) = 1$, otherwise. Even though this is a proper distance function definition[1], it is not very useful for control problems, since all the discrete states (except the target state itself) appear to be equally far from the target discrete state, and there is no concept of progress with respect to getting "closer" to the target. Thus, we consider a different notion of distance, the *graph distance*.

Definition 5. *Given a directed graph $G = (V, E, C)$, the one-step distance from v_i to v_j, $d_{v_i v_j} \in \mathbb{Z}_+$, is $d_{v_i v_j} = c_{ij}$, if $e_{ij} \in E$, and $d_{v_i v_j} = \infty$, otherwise.*

Definition 6. *Given a directed graph $G = (V, E, C)$, a graph path that starts at v_r and ends at v_s is a sequence of vertices $\tau = \{\nu^{(1)}, \ldots, \nu^{(\ell)}\}$, where $\nu^{(j)} \in V$, for $j \in \mathbb{Z}_{[1,\ell]}$, $(\nu^{(j)}, \nu^{(j+1)}) \in E$, for $j \in \mathbb{Z}_{[1,\ell-1]}$, and $\nu^{(1)} = v_r$, $\nu^{(\ell)} = v_s$. The length of the path is $\mathcal{L}(\tau) \triangleq \sum_{j=1}^{\ell-1} d_{\nu^{(j)} \nu^{(j+1)}}$.*

Definition 7. *Given a directed graph $G = (V, E, C)$, the graph distance between $v_r, v_s \in V$, is the length of the shortest path between them, $d(v_r, v_s) \triangleq \min_{\tau \in \mathcal{T}_{r,s}} \mathcal{L}(\tau)$, where $\mathcal{T}_{r,s}$ is the set of graph paths from v_r to v_s. In the case there is no path between v_r and v_s, $d(v_r, v_s) \triangleq \infty$.*

The graph distance is a proper distance function on undirected graphs, but it lacks the symmetry property on directed graphs, since in general $d(v_i, v_j) \neq d(v_j, v_i)$. Hence, it is a *pseudo-distance*. However, as we will see, this does not affect the problem we consider. In this paper we use $c_{ij} = 1$, $\forall (v_i, v_j) \in E$, that recovers the distance on unlabelled graphs. Once the one-step distances are known, the graph distance $d(v_i, v_j)$, for all $v_i, v_j \in V$ can be computed offline, for instance through a graph search based on Dijkstra's algorithm [13].

3 Reference Model and Problem Formulation

We consider a hybrid dynamical system, with both continuous and discrete states and inputs. The system dynamics are defined by

$$x(k+1) = \begin{bmatrix} x_c(k+1) \\ x_b(k+1) \end{bmatrix} = \begin{bmatrix} \phi_c(x(k), u(k)) \\ \phi_b(x(k), u(k)) \end{bmatrix} = \phi(x(k), u(k)), \tag{5}$$

where $\phi : \mathbb{X} \times \mathbb{U} \to \mathbb{X}$, $\mathbb{X} \subseteq \mathbb{R}^n \times \mathcal{E}$, $\mathbb{U} \subseteq \mathbb{R}^m \times \mathcal{E}_u$ is an arbitrary mapping, $\phi_c(\cdot)$ is the continuous state update function, and $\phi_b(\cdot)$ is the discrete state update function. The sets $\mathbb{X}$ and $\mathbb{U}$ represent physical constraints on state and input vectors, and are assumed to be bounded. Here, the state and input constraints are independent from each other.

Remark 1. $\mathbb{X} = \bigcup_{\epsilon_i \in \mathcal{E}} \mathcal{X}_h(\epsilon_i)$ where $\mathcal{X}_h(\epsilon_i) \triangleq \{x \in \mathbb{X} : x_b = \epsilon_i\}$. The set $\mathcal{X}_c(\epsilon_i) \triangleq \{x_c \in \mathbb{R}^n : \left[\begin{smallmatrix} x_c \\ \epsilon_i \end{smallmatrix}\right] \in \mathbb{X}\}$ is the set of continuous states compatible with discrete state ϵ_i. In [6], $\mathcal{X}_c(\epsilon_i)$ is referred to as the *domain of discrete state* ϵ_i.

[1] Such a definition has nonnegativity, symmetry, and triangle inequality properties.

We assume that the discrete state update function can be represented in the form of a finite state machine, with states $\{\epsilon_1, \ldots, \epsilon_{n_b}\}$ and with transitions $T \subseteq \mathcal{E} \times \mathcal{E}$. Such a finite state machine can be represented as a directed graph where $V \equiv \mathcal{E}$ and $E \equiv T$. The discrete state transitions are affected by the system input and by the continuous state. At time $k \in \mathbb{Z}_+$, if *(i)* $x_b(k) = \epsilon_i$ and *(ii)* $\mathcal{G}_{ij}(x_c, u) = 1$, where the functions $\mathcal{G}_{ij} : \mathcal{X}_c(\epsilon_i) \times \mathbb{U} \to \{0, 1\}$, $i, j = 1, \ldots, n_b$, are called *transition guards*, then $x_b(k+1) = \epsilon_j$. In (5), the transition guards are embedded in $\phi_b(\cdot)$. The discrete state affects the continuous dynamics, since x_b modifies the vector field $\phi_c(\cdot)$. We assume that (5) is well posed, i.e., for any $(x, u) \in \mathbb{X} \times \mathbb{U}$, $\phi(x, u)$ is uniquely defined.

An example of a fairly general class of hybrid systems that can be modeled by (5) is the class of discrete hybrid automata [14]. Consider now an autonomous version of system (5), i.e.

$$x(k+1) = \begin{bmatrix} x_c(k+1) \\ x_b(k+1) \end{bmatrix} = \begin{bmatrix} \Phi_c(x(k)) \\ \Phi_b(x(k)) \end{bmatrix} = \Phi(x(k)). \tag{6}$$

Let $x^e = [x_c^{eT} \; x_b^{eT}]^T$ be an equilibrium for (6) (i.e. $\Phi(x^e) = x^e$) and for any $\mathcal{X} \subseteq \mathcal{X}_h(x_b^e)$ define $\bar{\mathcal{X}}_c \triangleq \{x_c \in \mathbb{R}^n : \left[\begin{smallmatrix} x_c \\ x_b^e \end{smallmatrix}\right] \in \mathcal{X}\}$, $\bar{\mathcal{X}}_c \subseteq \mathcal{X}_c(x_b^e)$.

Definition 8. *The hybrid system* (6) *is called* globally asymptotically stable *if there exists a positively invariant set* $\mathcal{X} \subseteq \mathcal{X}_h(x_b^e)$ *for* (6) *with* $x^e \in \mathcal{X}$ *such that for any initial state, the system state converges to* $\mathcal{X}$ *in finite time, and the continuous state dynamics* $x_c(k+1) = \Phi_c(x_c(k), x_b^e)$ *is asymptotically stable in* $\bar{\mathcal{X}}_c$ *(with respect to* $x_c^e \in \mathrm{int}(\bar{\mathcal{X}}_c)$*).*

Definition 8 combines global convergence of the discrete dynamics with local AS in $\bar{\mathcal{X}}_c$ of the continuous dynamics, to obtain a stability-like property for HSDD. This property includes global convergence to x_c^e. The problem considered in this paper can be formulated as follows.

Problem 1. Feedback Control Design Problem: Given a desired equilibrium $x^e \in \mathbb{X}$ for system (5) with steady-state input $u^e \in \mathbb{U}$ (i.e, $\phi(x^e, u^e) = x^e$), synthesize a control law $u(k) = \pi(x(k))$ such that $u(k) \in \mathbb{U}$, $x(k) \in \mathbb{X}$, $\forall k \in \mathbb{Z}_+$ and HSDD in closed-loop with $u(k) = \pi(x(k))$ is globally asymptotically stable in the sense of Definition 8.

In this paper we employ the CLF framework in combination with predictive control to obtain a solution to Problem 1. Let $V(\cdot)$ be a CLF for the continuous dynamics $x_c(k+1) = \phi_c(x_c(k), x_b, u_c(k), u_b(k))$, for all $x_b \in \mathcal{E}$. Then, according to Theorem 1 and Definition 8, it is sufficient to have a feasible control input $u(k)$ at each time $k \in \mathbb{Z}_+$ such that the discrete dynamics $\phi_b(x_c(k), x_b(k), u_c(k), u_b(k))$ converge in finite time to x_b^e and remains there. However, asking the CLF for the continuous dynamics to decrease at each time step $k \in \mathbb{Z}_+$ for all the values of x_b can be overconservative in the hybrid system setting, and it may collide with the objective of steering the discrete state to the target value. This is what may happen for instance in a system with hysteresis such as the one in [5, Example 3.1].

Even for simple hybrid systems it may be impossible to obtain a continuous state CLF on the whole hybrid state space. Rather, to achieve stability, it would be sufficient to keep the continuous state trajectory bounded while the discrete state converges, and have a local continuous state CLF only for the dynamics associated to the target discrete state. To design a control law that yields such a closed-loop behavior, we propose a "hybrid" CLF consisting of two CLF-like functions that depend on each other. More precisely, instead of using a single standard CLF, we exploit the hybrid structure of the problem which consists of two objectives: (i) the convergence to the target discrete state; (ii) the stabilization of the continuous state while keeping the discrete state at its target value. These objectives are consistent with the existence of two functions, namely $\psi : \mathbb{X} \times \mathbb{U}^N \to \mathbb{Z}_+$ and $V : \mathbb{R}^n \to \mathbb{R}_+$, that satisfy

$$\begin{cases} \psi(x(k), \mathbf{u}_N(k)) < \psi(x(k-1), \mathbf{u}_N(k-1)), \\ V(\phi_c(x(k), u(k))) \leq \rho V(x_c(k)) + M_c \end{cases} \quad \text{if } x_b(k) \neq x_b^e \tag{7a}$$

$$\begin{cases} \psi(x(k), \mathbf{u}_N(k)) \leq \psi(x(k-1), \mathbf{u}_N(k-1)), \\ V(\phi_c(x(k), u(k))) \leq \rho V(x_c(k)), \end{cases} \quad \text{if } x_b(k) = x_b^e \tag{7b}$$

where $\rho \in \mathbb{R}_{[0,1)}$, $M_c \in \mathbb{R}_+$, $V(\cdot)$ is a control Lyapunov function only on the set $\mathcal{P}(x_b^e) = \{x_c \in \mathbb{R}^n : \left[\begin{smallmatrix} x_c \\ x_b^e \end{smallmatrix}\right] \in \mathbb{X},\ \exists u \in \mathbb{U},\ \phi_b(x, u) = x_b\}$, and $\psi(\cdot)$ is a CLF-like function that enforces convergence of the discrete state.

4 Synthesis and Properties of the Hybrid CLF

In what follows we show how to synthesize the functions in (7).

4.1 Control Lyapunov Function on the Continuous State

We first design the local CLF on the continuous state according to (7). Let $V : \mathbb{R}^n \to \mathbb{R}_+$ be a CLF for dynamics $x(k+1) = \phi_c(x_c(k), u_c(k), x_b^e, u_b^e)$ in the set $\mathcal{X}_c(x_b^e)$ accordingly to Definition 4. Such a CLF is a relaxed version of a global CLF, since (2b) must hold only in $\mathcal{X}_c(x_b^e)$.

Assumption 1. *For all $x \in \mathcal{X}_h(x^e)$ there exists an input $u \in \mathbb{U}$ such that $V(\phi_c(x, u)) \leq \rho V(x_c)$, and $\phi(x, u) \in \mathcal{X}_h(x^e)$.*

Assumption 1 states that there exists a feasible control law according to the CLF which makes the set $\mathcal{X}_h(x^e)$ positively invariant for the closed-loop dynamics. This only requires that when the discrete state reaches the target, it remains there, which is in general less conservative than requiring the invariance of a generic set $\mathcal{X} \subseteq \mathbb{X} = \bigcup_{\epsilon_i \in \mathcal{E}} \mathcal{X}_h(\epsilon_i)$ containing different dynamics and different discrete states. Assumption 1 can be guaranteed a priori by constraining the continuous state to lie in a pre-defined set of the state space. The tools developed in [11] can be used for this purpose.

4.2 Control Lyapunov-Like Function on the Discrete State

Given the finite state machine representation of function $\phi_b(\cdot)$, we can associate to it a directed graph, where the node v_j is associated to the state ϵ_j. Hence, the distance from ϵ_j to x_b^e can be computed as the distance from v_j to the node corresponding to x_b^e, as defined in Section 2.2. Given a state x, an input sequence $\mathbf{u}_N$, and a target discrete state x_b^e define

$$\psi(x, \mathbf{u}_N) \triangleq \sum_{j=1}^{N} d(\phi_b^j(x, \mathbf{u}_j), x_b^e). \tag{8}$$

Let $h_{x_b^e}(\cdot)$ be the indicator function of the target discrete state, i.e., $h_{x_b^e}(x_b^e) = 1$, and $h_{x_b^e}(x_b) = 0$, for $x_b \neq x_b^e$. For any $k \in \mathbb{Z}_+$ define the *cumulative discrete distance contraction* (CDDC)

$$\psi(x(k), \mathbf{u}_N(k)) - \psi(x(k-1), \mathbf{u}_N(k-1)) \leq -1 + h_{x_b^e}(x_b(k)). \tag{9}$$

Constraint (9) is obtained as a relaxation of the discrete distance contraction constraint $d(\phi_b(x(k), u), x_b^e) < d(x_b(k), x_b^e)$, that requires that at each step the distance to the target discrete state decreases. In general, this constraint will generate infeasibility, because it is not possible that the discrete distance decreases at every step. CDDC (9) relaxes the discrete distance contraction constraint requiring that the sum of the discrete distances along the predicted trajectory at time k is smaller than the one along the predicted trajectory at time $k-1$. The effect of the indicator function $h_{x_b^e}(\cdot)$ is to further relax the constraint when $x_b(k) = x_b^e$. Constraint (9) forces the cumulative discrete distance to decrease until $x_b(k) = x_b^e$ implementing the first inequality of (7a), after which $\psi(x(k), \mathbf{u}_N(k))$ is kept constant. The value $\psi(x(-1), \mathbf{u}_N(-1))$ initializes (9). To achieve feasibility it can be set to a large number, while a more efficient initialization value is proposed later in this section. In what follows we use the shorthand notation $\psi(i) \triangleq \psi(x(i), \mathbf{u}_N(i))$, for $i \in \mathbb{Z}_+$. We show that under suitable assumptions this constraint is feasible and steers the discrete state to the target value in finite time.

Assumption 2. *Given a target state $x^e \in \mathbb{X}$ and given any discrete initial state $x_b = \epsilon_i \neq x_b^e$, there exists a finite value $n_i \in \mathbb{Z}_+$ such that for any initial state $x \in \mathcal{X}_h(\epsilon_i)$, there exists $\epsilon_j \in \mathcal{E}$ and an input sequence $\mathbf{u}_{\ell_i} \in \mathbb{U}^{\ell_i}$, such that $\ell_i \leq n_i$, $\phi^h(x, \mathbf{u}_h) \in \mathbb{X}$, $\phi_b^h(x, \mathbf{u}_h) = \epsilon_i$, $h \in \mathbb{Z}_{[1,\ell_i-1]}$, and $\phi_b^{\ell_i}(x, \mathbf{u}_{\ell_i}) = \epsilon_j$, where $d(\epsilon_j, x_b^e) < d(\epsilon_i, x_b^e)$.*

Definition 9. *Given $x_b = \epsilon_i \neq x_b^e$, the minimum discrete distance progress horizon $\bar{n}_i \in \mathbb{Z}_+$ for discrete state ϵ_i is the minimum value for which Assumption 2 holds for discrete state ϵ_i. For $x_b^e = \epsilon_{\bar{\imath}}$, define $\bar{n}_{\bar{\imath}} \triangleq 1$.*

Assumption 2 requires that given any valid discrete state different from the target, for any continuous state in its domain, there exists a feasible trajectory that brings the discrete state closer to the target discrete state in n_i time steps, which is a sort of finite-time discrete reachability property. The minimum length of input sequences that guarantees such a property is the *minimum discrete progress horizon*, the minimum horizon needed to see the discrete state approaching the target. A (possibly over-approximated) value of $\bar{n}_i$ can be computed by offline reachability analysis.

Note that Assumption 2 requires the existence of a horizon such that the discrete state gets closer to the target discrete state. This is in general less conservative (and requires a shorter horizon) than the condition in [1], which requires the existence of a (finite) horizon such that the target state is reached.

Assumption 3. *For any* $x \in \mathcal{X}_h(x_b^e)$*, the set* $\{u \in \mathcal{U}_f(x) : \phi_b(x,u) = x_b^e\}$ *is non-empty, where* $\mathcal{U}_f(x) = \{u \in \mathbb{U} : \phi(x,u) \in \mathbb{X}\}$*.*

Assumption 2 and 3 are reachability assumptions. Assumption 2 is a finite-time discrete reachability. Assumption 3 requires that for any state value in which the discrete state is at the target, there exists a feasible input that keeps the discrete state constant.

Lemma 1. *Under Assumptions 2 and 3, given* $x(k) \in \mathbb{X}$ *and a target discrete state* x_b^e*, for any* $M \in \mathbb{Z}_+$*, there exists* $\mathbf{u}_M \in \mathbb{U}^M$*, such that* $\phi^i(x(k), \mathbf{u}_i) \in \mathbb{X}$*, for* $i \in \mathbb{Z}_{[1,M]}$*, and* $d(\phi_b^{i+1}(x(k), \mathbf{u}_{i+1}), x_b^e) \leq d(\phi_b^i(x(k), \mathbf{u}_i), x_b^e)$*, for* $i \in \mathbb{Z}_{[0,M-1]}$*.*

Proof. Consider the case $x(k) \in \mathcal{X}_h(x_b^e)$. By Assumption 3 there exists $u \in \mathcal{U}_f(x)$ such that $x_b(k+1) = \phi_b(x(k), u(k)) = x_b^e$, hence $d(x_b(k+1), x_b^e) = d(x_b(k), x_b^e)$. Consider the case $x(k) \notin \mathcal{X}_h(x_b^e)$. By Definition 7 and Assumption 2 there exists an input sequence $\mathbf{u}_\ell \in \mathbb{U}^\ell$ such that $\phi_b^i(x(k), \mathbf{u}_i) = x_b(k)$, for $i \in \mathbb{Z}_{[0,\ell-1]}$, and $d(\phi_b^\ell(x(k), \mathbf{u}_\ell), x_b^e) < d(x_b(k), x_b^e)$. In the case $\phi_b^\ell(x(k), \mathbf{u}_\ell) \neq x_b^e$ the procedure can be repeated. In the case $\phi_b^\ell(x(k), \mathbf{u}_\ell) = x_b^e$, we have already proven that there exists $u \in \mathcal{U}_f(x)$ that keeps the discrete state at the target. □

Consider now the system formed by (5) in closed loop with a control law obtained by solving a finite horizon (N) optimization problem (e.g., as done in predictive control) with the constraint (9) added. Suppose that at each time step a sequence of N inputs is computed and only the first element of the sequence is applied to system (5). We show next that constraint (9) is initially feasible and remains feasible at all future time instants under suitable assumptions. The complete control algorithm will be defined later in Section 5, as the following result does not depend on it.

Theorem 2. *Suppose Assumptions 2 and 3 hold. Given a target discrete state* x_b^e *and any initial state* $x(0) \in \mathbb{X}$*, let the prediction horizon be* $N \geq \max_{\{i:\epsilon_i \in \mathcal{E}\}} \bar{n}_i$*, and* $\psi(-1) \triangleq N d(x_b(0), x_b^e)$*. Then, (i) constraint (9) is feasible for all* $k \in \mathbb{Z}_+$*, and (ii) there exists a finite* $\bar{k} \in \mathbb{Z}_+$ *such that* $x_b(k) = x_b^e$*,* $\forall k \in \mathbb{Z}_{\geq \bar{k}}$*.*

Proof. (*i*). Assumption 3 guarantees the constraint feasibility for $x_b(k) = x_b^e$. Due to $N \geq \max_{i:\epsilon_i \in \mathcal{E}} \bar{n}_i$ and by Assumption 2, we have that at $k = 0$ there exists a feasible input sequence $\mathbf{u}_N(0)$ and an index $\bar{\jmath} \in \mathbb{Z}_{[1,N]}$ such that for $j \in \mathbb{Z}_{[0,\bar{\jmath}-1]}$, $\phi_b^j(x(0), \mathbf{u}_j(0)) = x_b(0)$, $d(\phi_b^{\bar{\jmath}}(x(0), \mathbf{u}_j(0)), x_b^e) < d(x_b(0), x_b^e)$ and for $j \in \mathbb{Z}_{[\bar{\jmath}+1,N]}$, $d(\phi_b^j(x(0), \mathbf{u}_j(0)), x_b^e) \leq d(\phi_b^{\bar{\jmath}}(x(0), \mathbf{u}_j(0)), x_b^e)$, by Lemma 1. Hence, constraint (9) is feasible at $k = 0$.

For $k \geq 1$, we have $\psi(k-1) = \sum_{j=1}^{N} d(\phi_b^j(x(k-1), \mathbf{u}_j(k-1)), x_b^e)$ and $x_b(k) = \phi_b^1(x(k-1), \mathbf{u}_1(k-1))$. By Lemma 1, there exists $\mathbf{u}_N(k)$ such that

$$\psi(k) \leq \psi(k-1) - d(x_b(k), x_b^e) + d(\phi_b^N(x(k-1), \mathbf{u}_N(k-1)), x_b^e).$$

If $d(\phi_b(x(k-1), \mathbf{u}_N(k-1)), x_b^e) < d(\phi_b^1(x(k-1), \mathbf{u}_1(k-1)), x_b^e)$, (9) is feasible. If for all $j \in \mathbb{Z}_{[1,N]}$, $\phi_b^j(x(k-1), \mathbf{u}_j(k-1)) = x_b(k-1)$, then by the choice of N, there exists $\mathbf{u}_{\bar{\jmath}}(k) \in \mathbb{U}^{\bar{\jmath}}$, $\bar{\jmath} \leq N$, such that, $d(\phi_b^{\bar{\jmath}}(x(k), \mathbf{u}_{\bar{\jmath}}(k)), x_b^e) < d(\phi_b^1(x(k-1), \mathbf{u}_1(k-1)), x_b^e)$. If needed, the input sequence can be extended to length N enforcing $d(\phi_b^j(x(k), \mathbf{u}_j(k)), x_b^e) \leq d(\phi_b^{\bar{\jmath}}(x(k), \mathbf{u}_{\bar{\jmath}}(k)), x_b^e)$, for $j \in \mathbb{Z}_{[\bar{\jmath}+1,N]}$, as guaranteed by Lemma 1. Such an input sequence is feasible with respect to (9).

(ii). We prove that there exists $\bar{k}$ such that $x_b(\bar{k}) = x_b^e$ by contradiction. Suppose $x_b(k) \neq x_b^e$, for $k \in \mathbb{Z}_+$ and note that by definition $\psi(\cdot) \geq 0$. From (i), for all k such that $x_b(k) \neq x_b^e$, $\Delta\psi(k) \triangleq \psi(k) - \psi(k-1) \leq -1$. Hence,

$$\psi(k) = \psi(0) + \sum_{j=1}^{k} \Delta\psi(j) \leq \psi(0) - k.$$

Thus, $0 \leq \lim_{k\to\infty} \psi(k) \leq \lim_{k\to\infty} \psi(0) - k$. Since $\psi(0)$ is finite, we reached a contradiction. For $k \geq \bar{k}$, (9) guarantees that $x_b(k) = x_b(\bar{k})$. □

5 Stabilizing Predictive Control of HSDD

We propose now a mixed integer linear formulation of the hybrid CLF constraints (7) which can be included in an optimization problem. For computational purposes, we model the discrete state and input of the hybrid system by Boolean vectors. In detail, $u_b \in \{0,1\}^{m_b}$, and we model the discrete state by *one-hot encoding*, i.e., $[x_b]_i \in \{0,1\}$ and $\sum_i [x_b]_i = 1$. In this way, the symbolic variable ϵ_j is represented by the j^{th} unitary vector of $\mathbb{R}^{n_b}$, the vector entirely composed of 0, except for the j^{th} coordinate, which is 1. As a consequence $\mathcal{E}$ is the set of the unitary vectors on $\mathbb{R}^{n_b}$, where n_b is the cardinality of $\mathcal{E}$.

Consider first the function $\psi(\cdot)$. For a given x_b^e, for all $x_b \in \mathcal{E}$, we have

$$d(x_b, x_b^e) = D_{x_b^e}^T x_b, \tag{10}$$

where $D_{x_b^e} \in \mathbb{Z}_+^{n_b}$ is a vector whose i^{th} component is equal to the graph distance from $x_b = \epsilon_i$ to x_b^e. The indicator function $h_{x_b^e}(\cdot)$ can be expressed by

$$h_{x_b^e}(x_b) = \left(1 - \frac{1}{2} H^T \cdot (x_b - x_b^e)\right), \tag{11}$$

where[2] $H = \left(\sum_{j=1, j\neq i}^{n_b} \epsilon_j\right) - \epsilon_i$. Since $H^T \cdot (x_b - x_b^e) = 0$, if $x_b = x_b^e$, and $H^T \cdot (x_b - x_b^e) = 2$, otherwise, (11) implements the desired indicator function.

[2] There are several other definitions of H that obtain the desired behavior. However, we find (11) the most intuitive one.

As a result, via (10) and (11) constraint (9) can be formulated as a set of mixed-integer linear inequalities.

In order to also implement the constraints involving the local CLF on the continuous state via mixed-integer linear inequalities we consider a Lyapunov function defined using the infinity norm, i.e. $V(x_c) = \|Px_c\|_\infty$ for some $P \in \mathbb{R}^{p\times n}$ with full column rank. The constraint on the CLF can be expressed as

$$V(\phi_c^1(x(k)), \mathbf{u}_1) \leq \rho V(x_c(k)) + \frac{M_c}{2} H^T(x_b(k) - x_b^e) \tag{12}$$

where $\rho \in \mathbb{R}_{[0,1)}$, $M_c \in \mathbb{R}_{>0}$ and the rightmost term is responsible for relaxing the constraint when $x_b \neq x_b^e$. Note that, since $\mathbb{X}$ is bounded, we can set $M_c = \max_{x\in\mathbb{X}} V(x_c)$. In this way, constraint (12) can be formulated as a set of mixed-integer linear constraints that ensure that x_c remains bounded when $x_b \neq x_b^e$, while, when $x_b = x_b^e$, ensure that $V(\cdot)$ is a CLF restricted to $\mathcal{X}_c(x_b^e)$.

Given $x(k)$, let $\Gamma(x(k)) = \{\mathbf{u}_N \in \mathbb{U}^N : \ \phi^i(x(k), \mathbf{u}_i) \in \mathbb{X},\ i \in \mathbb{Z}_{[1,N]},\ (9), (12)\}$ be the set of feasible input sequences, and $\gamma(x(k)) = \{u(0) \in \mathbb{U} :\ \mathbf{u}_N \in \Gamma(x(k))\}$. The system obtained by (5) in closed-loop with $\gamma(x(k))$ is described, with some abuse of notation, by the difference inclusion

$$x(k+1) \in \phi(x(k), \gamma(x(k))) \triangleq \{\phi(x(k), u) : u \in \gamma(x(k))\}. \tag{13}$$

Theorem 3. *Suppose Assumption 1 and the assumptions of Theorem 2 hold, and set* $M_c = \max_{x\in\mathbb{X}} V(x_c)$*. Then, the closed-loop system described by* (13) *is asymptotically stable in the sense of Definition 8.*

Proof. By Theorem 2 for any sequence $\{u(k)\}_{k=0}^{\infty}$, where $u(k) \in \gamma(x(k))$ for all $k \in \mathbb{Z}_+$, there exists $\bar{k} \in \mathbb{Z}_+$ such that for all $k \geq \bar{k}$, $x_b(k) = x_b^e$. By assumption $\mathbb{X}$ is bounded, hence $M_c = \max_{x\in\mathbb{X}} V(x)$ is finite. Thus, during the time interval $k \in \mathbb{Z}_{[0,\bar{k}-1]}$, since $x_c(k) \in \mathbb{X}$, constraint (12) is satisfied. For $k \geq \bar{k}$ constraint (9) is still feasible by Theorem 2 and it ensures that the discrete state remains at the target. In this case, recursive feasibility of (12) is guaranteed by Assumption 1 and thus, $V(\cdot)$ satisfies inequality (2b) for all $k \geq \bar{k}$. Hence, from Theorem 1 we obtain asymptotic stability of the continuous dynamics in the set $\mathcal{X}_c(x_b^e)$, and the result follows from Definition 8, with $\mathcal{X} = \mathcal{X}_h(x_b^e)$. □

As a consequence of Definition 8, Theorem 3 guarantees convergence to the desired equilibrium for any initial state. Even though the stabilizing properties established in Theorem 3 are guaranteed for any feasible control input, not just for the optimal one, a cost function can be considered to select a $u(k) \in \gamma(x(k))$ that optimizes performance. We introduce now an optimization-based receding horizon control strategy for system (5). Notice that there is no need to keep ρ and M_c fixed in (12). Instead, we will optimize over these two variables which results in improved convergence of the continuous state, when allowed by the condition on the discrete state. Let ρ be the constant that satisfies (12), choose $\bar{\rho} \in \mathbb{R}_{[\rho,1)}$ and let $\eta \in \mathbb{R}_{[0,\bar{\rho}]}$ and $M \in \mathbb{R}_+$ be two additional variables that play the role of ρ and M_c, respectively. Consider the cost function

$$J(x, \mathbf{u}_N, M, \eta) \triangleq w_\eta \eta + w_M M + F(\phi^N(x, \mathbf{u}_N)) + \sum_{i=0}^{N-1} L(\phi^i(x, \mathbf{u}_i), u(i)), \tag{14}$$

where $F(\cdot)$ and $L(\cdot)$ denote suitable terminal and stage costs, respectively, as in standard MPC [7]. The term $w_\eta \eta$, where $w_\eta \in \mathbb{R}_+$, optimizes the reduction of the CLF, while $w_M M$, where $w_M \in \mathbb{R}_{>0}$ penalizes the relaxation of (12) for $x_b(k) \neq x_b^e$. Whenever $M = 0$ the continuous state evolves satisfying (2b).

Algorithm 1. *(Receding Horizon Control of HSDD)*
Initialization. *Set $k = 0$, measure $x(0) \in \mathbb{X}$ and set $\psi(k-1) = N\, D_{x_b^e}^T x_b(0)$.*
Step 1. *Solve the optimization problem*

$$\min_{\mathbf{u}_N, M, \eta} J(x(k), \mathbf{u}_N, M, \eta) \tag{15a}$$

$$\text{s.t.: } \mathbf{u}_N \in \mathbb{U}^N,\ \phi^i(x, \mathbf{u}_i) \in \mathbb{X},\ i \in \mathbb{Z}_{[1,N]} \tag{15b}$$

$$M \geq 0,\ \eta \in \mathbb{R}_{[0,\bar{\rho}]} \tag{15c}$$

$$V(\phi_c^1(x(k)), \mathbf{u}_1) \leq \eta V(x_c(k)) + \frac{M}{2} H^T (x_b(k) - x_b^e) \tag{15d}$$

$$\sum_{i=1}^{N} D_{x_b^e}^T \phi^i(x, \mathbf{u}_i) \leq \psi(k-1) - 1 + \left(1 - \frac{1}{2} H^T (x_b - x_b^e)\right). \tag{15e}$$

Step 2. *Let $\bar{\mathbf{u}}_N = \{\bar{u}(0), \ldots, \bar{u}(N)\}$ be a feasible solution of problem* (15) *obtained by minimizing with respect to* (15a) *(possibly, but not necessarily, the optimal one). Set $u(k) = \bar{u}(0)$, and $\psi(k) = \sum_{i=1}^{N} D_{x_b^e}^T \phi^i(x, \bar{\mathbf{u}}_i)$.*
Step 3. *Set $k \leftarrow k+1$, measure $x(k)$ and go to Step* 1.

Algorithm 1 implements the constraints on the hybrid CLF and minimizes the performance criterion (14). Minimization of the cost (14) ensures finite values of M, and $\eta \leq \bar{\rho} < 1$. Thus, the result of Theorem 3 still applies.

From a computational point of view, constraint (15d) can be formulated as a set of mixed integer linear inequalities as shown in [11]. Furthermore, as (14) is linear in M and η, since $x(k)$ is known (measured) at each step $k \in \mathbb{Z}_+$, if the system dynamics (5) can be described by mixed-integer linear inequalities (e.g., DHA [14]) and $L(\cdot)$, $F(\cdot)$ are linear (quadratic) functions of their arguments, then (15) can be formulated as a mixed integer linear (quadratic) program.

It is worth to point out that, according to Theorem 3, it is not necessary to attain the globally optimal solution in the optimization problem defined in Step 1 of Algorithm 1 to guarantee stability of the resulting closed-loop system. Rather, stability is ensured for any feasible solution.

5.1 Simulation Example

We present a simple example that illustrates the proposed control strategy. Consider a system with: one continuous state, $x_c \in [-5, 35]$, four discrete states $x_b \in \{\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4\}$, one continuous input $u_c \in [-2.2, 2.2]$ and one discrete input $u_b \in \{0, 1\}$. As a consequence, $\mathbb{U} = [-2.2, 2.2]$, and $\mathbb{X} \subseteq [-5, 35] \times \{\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4\}$, where in particular $\mathcal{X}_c(\epsilon_4) = [-5, 6]$. The automaton describing the discrete dynamics of the example system is reported in Figure 1. The continuous dynamics are $x(k+1) = A_i x(k) + B_i u(k)$, if $x_b = \epsilon_i$, where $(A_1, B_1) = (1.09, 1)$, $(A_2, B_2) = (0.75, 0.8)$, $(A_3, B_3) = (0.92, 0.75)$, and $(A_4, B_4) = (1.1, 0.7)$.

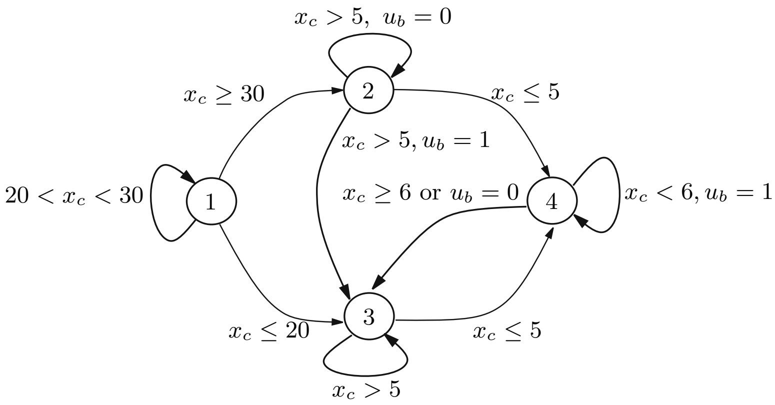
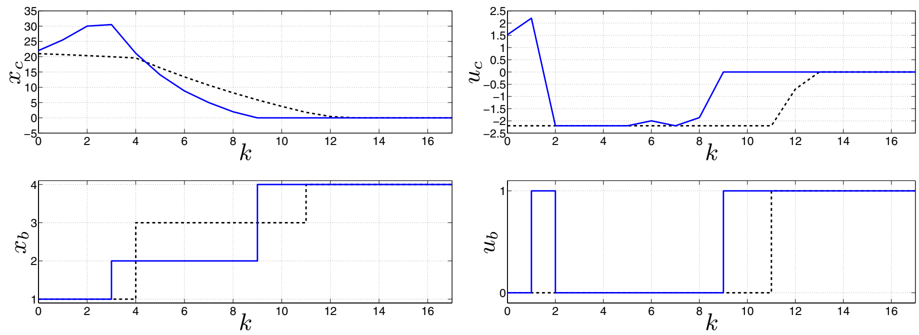


Fig. 1. Automaton describing the discrete dynamics of the example system

The desired equilibrium is $x_c^e = 0$, $x_b^e = \epsilon_4$ for a steady state input $u_c^e = 0$, $u_b^e = 1$. We implemented problem (15) where $L(x,u) = \|Q(x - x^e)\|_\infty + \|R(u - u^e)\|_\infty$, $Q = \left[\begin{smallmatrix}1 & 0\\ 0 & 1\end{smallmatrix}\right]$, $R = \left[\begin{smallmatrix}0.1 & 0\\ 0 & 0.1\end{smallmatrix}\right]$, $\bar{\rho} = 0.85$, $N = 7$ accordingly to Assumption 2, and $V(x_c) = \|x_c\|_\infty$ as the CLF for the continuous state. The HSDD was implemented as a discrete hybrid automaton using the tools in [14], and problem (15) was formulated as a mixed-integer linear program.

Figure 2 reports the simulation results. The dashed line reports the simulation for the case $x(0) = [22\ \ \epsilon_1]^T$. Note that in this simulation the CLF inequality (2b) holds at every step. The case for $x(0) = [23\ \ \epsilon_1]^T$ is reported as solid lines. Note that there is a discontinuity with respect to the initial conditions, and that in this case the CLF is not monotonically decreasing along the whole trajectory. This is accordingly to (12), since we require the continuous state CLF to decrease only on the set $\mathcal{X}_c(\epsilon_1)$. Moreover, for the same setup the optimization problem formulated as in [1] is infeasible (i.e., it requires a longer horizon).



(a) State Evolution (b) Input Evolution

Fig. 2. Simulation results for $x_c(0) = 23$ (solid lines) and $x_c(0) = 22$ (dashed lines)

6 Conclusions

We have studied the stabilization of hybrid systems with both discrete and continuous dynamics using predictive control based on control Lyapunov functions. We have introduced a hybrid control Lyapunov function constituted of a control Lyapunov-like function that guarantees convergence of the discrete dynamics and of a local control Lyapunov function on the continuous dynamics. The proposed controller is less conservative and less computationally demanding compared to standard predictive control, and it guarantees stability for all the feasible solutions of the optimization problem, not just for the optimal one.

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