Riccati-Based Design of Event-Triggered Controllers for Linear Systems With Delays

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***Abstract*—In event-triggered control (ETC) systems, the measured state or output of the plant is sent to the controller at so-called *event times*. In many ETC systems, these event times are generated based on a static function of the current state or output measurement of the system and its sampled-and-held version that is available to the controller. Hence, the event-generator does not include any dynamics of its own. In contrast, *dynamic* event-generators trigger events based on additional dynamic variables, whose dynamics depend on the state or output of the system. In this paper, we propose new static and dynamic continuous event-generators (which require continuous measuring of the plant output) and periodic event-generators (which only require periodic sampling of the plant output) for linear control systems with communication delays. All event-generators we propose lead to closed-loop systems which are globally exponentially stable with a guaranteed decay rate, $\mathcal{L}_2$-stable with a guaranteed $\mathcal{L}_2$-gain, and have a guaranteed positive minimum inter-event time. By using new Riccati-based analysis tools tailored to linear systems, the conservatism in our decay rate and $\mathcal{L}_2$-gain estimates is small. The dynamic event-generators even further reduce this conservatism, and as a result typically generate significantly fewer events than their static counterparts, while guaranteeing the same control performance. The benefits of these new event-generators are demonstrated via two numerical examples.**

***Index Terms*—Linear matrix inequalities, linear systems, networked control systems, riccati equations.**

I. Introduction

IN MOST digital control systems, the measured output of the plant is periodically transmitted to the controller, regardless of the state the system is in. This possibly leads to a waste of (e.g., computation, communication, and energy) resources, as many of the transmissions are actually not necessary to achieve the desired performance guarantees. To mitigate this potential waste of valuable resources, many *event-triggered control* (ETC) strategies have been proposed, which generate the transmission times based on a triggering condition involving the current state or output measurement of the plant and the most recently transmitted measurement data, see, e.g., [1]–[5] and the references therein. This brings a *feedback* mechanism into the sampling and communication process, such that measurement data are only transmitted to the controller when needed in order to guarantee the required stability and performance properties of the system.

ETC strategies can be divided into *static* and *dynamic* strategies, or into *continuous* and *periodic* strategies. In static ETC strategies, events are based on a static function of the current state or output measurement and its sampled-and-held version that is available to the controller (using, e.g., a zero-order-hold, a first-order-hold, or even a model-based-hold function). In contrast, in dynamic ETC strategies, events are based on an additional dynamic variable with dynamics that depend on the state or output of the system. Continuous ETC (CETC) strategies require continuous measuring of the plant output (which is sometimes difficult to implement on digital platforms), while periodic event-triggered control (PETC) strategies only require periodic sampling of the plant output.

Static CETC strategies have been proposed in [1], [2], [4], [6]–[9]. However, the event-generators in these works that lead to asymptotic stability of the CETC system can typically also lead to Zeno behavior (an infinite number of events in finite time) in the presence (and sometimes even in the absence) of disturbances [10], [11], and those that do not exhibit Zeno behavior only lead to practical stability and not to asymptotic stability [10]. To prevent Zeno behavior, static CETC strategies with waiting times (also called "time regularization") have been proposed in, e.g., [5], [12]–[17].

Dynamic CETC strategies for nonlinear systems have been proposed in [18]–[21]. In [19], a dynamic strategy is used to extend the inter-event times compared to a time-triggered system, and in [18], [20], and [21], dynamic strategies are used to extend the inter-event times compared to static CETC systems. In these works, the guaranteed control performance and minimum inter-event time of the proposed dynamic CETC system are identical to its static (or time-triggered) counterpart, while it is demonstrated that the average number of events in the dynamic CETC system is typically much smaller.

Manuscript received November 11, 2016; revised November 14, 2016 and March 16, 2017; accepted May 30, 2017. Date of publication June 7, 2017; date of current version December 27, 2017. This work was supported by the research programmes "Wireless control systems: A new frontier in automation" with Project 11382 and "Integrated design approach for safety-critical real-time automotive systems" with Project 12698, which are (partly) financed by the Netherlands Organisation for Scientific Research. Recommended by Associate Editor P. Bolzern. *(Corresponding author: Dominicus Paulus Borgers.)*

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Digital Object Identifier 10.1109/TAC.2017.2713047

While the results above show that for CETC systems, the inclusion of a dynamic variable in the event-generator can clearly lead to a significant further reduction of the consumption of communication and energy resources, there are currently no *dynamic* PETC strategies available in literature. Indeed, only static PETC strategies have been proposed, see, e.g., [3], [5], [22]–[27] for linear systems and [28]–[30] for nonlinear systems.

In this paper, we provide new (static and dynamic) CETC and PETC strategies tailored to linear systems with (varying) communication delays. We are able to guarantee a positive minimum inter-event time by design, and we provide tight estimates of the $\mathcal{L}_2$-gains and exponential decay rates of the resulting closed-loop systems, by making use of analysis tools specific to the domain of linear systems. These analysis tools are based on Lyapunov/storage functions exploiting matrix Riccati differential equations and computationally friendly semidefinite programming, using ideas from [22]. In addition, we also provide even less conservative conditions for global exponential stability (GES) and $\mathcal{L}_2$-stability based on piecewise quadratic Lyapunov functionals. Based on these conditions, we are able to provide tradeoffs of guaranteed control performance versus minimum and average inter-event times. Interestingly, for identical control performance guarantees, the dynamic ETC strategies produce significantly larger average inter-event times than their static counterparts, and hence, require much less communication. These results, based on exploiting the linearity of the underlying plants and controllers, provide significantly better results than the application of the results obtained for nonlinear systems [20]. In fact, to the best of our knowledge, the proposed continuous event-generators are the least conservative in literature, and we are the first to propose dynamic periodic event-generators. Preliminary results have appeared in [17] and [31], in which we did not consider communication delays. Moreover, the conditions provided in [17] and [31] are significantly more conservative in guaranteeing GES and $\mathcal{L}_2$-stability than the novel conditions provided here.

This paper is organized as follows. In Section II, we present the control setup that we consider in this paper, for which we present our new static and dynamic CETC strategies in Section III, and our new static and dynamic PETC strategies in Section IV. In both sections, we provide tight bounds on the $\mathcal{L}_2$-gains and decay rates of the resulting closed-loop systems. In Section V, we provide even tighter bounds based on less conservative conditions using state-space partitioning and piecewise quadratic Lyapunov/storage functionals. Finally, we demonstrate our results in Section VI and provide concluding remarks in Section VII. All proofs are given in the appendix.

A. Notation

For a vector $x \in \mathbb{R}^{n_x}$, we denote by $|x| := \sqrt{x^\top x}$ its Euclidean norm. For a symmetric matrix $A \in \mathbb{R}^{n\times n}$, we denote by $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ its maximum and minimum eigenvalue, respectively. For a matrix $P \in \mathbb{R}^{n\times n}$, we write $P \succ 0\,(P \succeq 0)$ if P is symmetric and positive (semi-)definite, and $P \prec 0\,(P \preceq 0)$ if P is symmetric and negative (semi-)definite. By I and O, we denote the identity and zero matrix of appropriate dimensions, respectively. For a measurable signal $w : \mathbb{R}_{\geqslant 0} \to \mathbb{R}^{n_w}$, we write $w \in \mathcal{L}_2$ if $\|w\|_{\mathcal{L}_2} < \infty$, where $\|w\|_{\mathcal{L}_2} := \left(\int_0^\infty |w(t)|^2 \mathrm{d}t\right)^{1/2}$ denotes its $\mathcal{L}_2$-norm, and we write $w \in \mathcal{L}_\infty$ if $\|w\|_{\mathcal{L}_\infty} < \infty$, where $\|w\|_{\mathcal{L}_\infty} := \operatorname{ess\,sup}_t |w(t)|$ denotes its $\mathcal{L}_\infty$-norm. By $\mathbb{N}$ we denote the set of natural numbers including zero, i.e., $\mathbb{N} := \{0, 1, 2, \dots\}$. A function $\gamma : \mathbb{R}_{\geqslant 0} \to \mathbb{R}_{\geqslant 0}$ is a $\mathcal{K}$-function if it is continuous, strictly increasing and $\gamma(0) = 0$, and a $\mathcal{K}_\infty$-function if it is a $\mathcal{K}$-function and, in addition, $\gamma(s) \to \infty$ as $s \to \infty$. A function $\beta : \mathbb{R}_{\geqslant 0} \times \mathbb{R}_{\geqslant 0} \to \mathbb{R}_{\geqslant 0}$ is a $\mathcal{KL}$-function if for each fixed $t \in \mathbb{R}_{\geqslant 0}$ the function $\beta(\cdot, t)$ is a $\mathcal{K}$-function and for each fixed $s \in \mathbb{R}_{\geqslant 0}$, $\beta(s, t)$ is decreasing in t and $\beta(s, t) \to 0$ as $t \to \infty$. For vectors $x_i \in \mathbb{R}^{n_i}$, $i \in \{1, 2, \dots, N\}$, we denote by $(x_1, x_2, \dots, x_N)$ the vector $[x_1^\top x_2^\top \cdots x_N^\top]^\top \in \mathbb{R}^n$ with $n = \sum_{i=1}^N n_i$. For a vector $y \in \mathbb{R}^n$, we write $y \geqslant 0$ if $y_i \geqslant 0$ for all $i \in \{1, 2, \dots, n\}$. For brevity, we sometimes write symmetric matrices of the form $\left[\begin{smallmatrix} A & B \\ B^\top & C \end{smallmatrix}\right]$ as $\left[\begin{smallmatrix} A & B \\ \star & C \end{smallmatrix}\right]$ or $\left[\begin{smallmatrix} A & \star \\ B^\top & C \end{smallmatrix}\right]$. For a left-continuous signal $f : \mathbb{R}_{\geqslant 0} \to \mathbb{R}^n$ and $t \in \mathbb{R}_{\geqslant 0}$, we use $f(t^+)$ to denote the limit $f(t^+) = \lim_{s\to t, s>t} f(s)$.

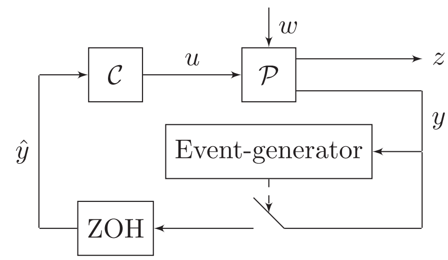

Fig. 1. ETC setup considered in this paper.

II. Control Setup

In this paper, we consider the ETC setup of Fig. 1, in which the plant $\mathcal{P}$ is given by

$$\mathcal{P} : \begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t} x_p(t) = A_p x_p(t) + B_p u(t) + B_{pw} w(t) \\ y(t) = C_y x_p(t) + D_y u(t) \\ z(t) = C_z x_p(t) + D_z u(t) + D_{zw} w(t) \end{cases} \tag{1}$$

and the controller $\mathcal{C}$ is given by

$$\mathcal{C} : \begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t} x_c(t) = A_c x_c(t) + B_c \hat{y}(t) \\ u(t) = C_u x_c(t) + D_u \hat{y}(t). \end{cases} \tag{2}$$

Here, $x_p(t) \in \mathbb{R}^{n_{x_p}}$ denotes the state of the plant $\mathcal{P}$, $y(t) \in \mathbb{R}^{n_y}$ its measured output, $z(t) \in \mathbb{R}^{n_z}$ the performance output, and $w(t) \in \mathbb{R}^{n_w}$ the disturbance at time $t \in \mathbb{R}_{\geqslant 0}$. Furthermore, $x_c(t) \in \mathbb{R}^{n_{x_c}}$ denotes the state of the controller $\mathcal{C}$, $u(t) \in \mathbb{R}^{n_u}$ is the control input, $\hat{y}(t) \in \mathbb{R}^{n_y}$ denotes the output that is available at the controller, given by

$$\hat{y}(t) = y(t_k),\ t \in (t_k + \tau_k, t_{k+1} + \tau_{k+1}] \tag{3}$$

where the sequence $\{t_k\}_{k\in\mathbb{N}}$ satisfying

$$t_0 = 0,\ t_{k+1} - t_k \geqslant h \tag{4}$$

denotes the event (or transmission) times with $h \in \mathbb{R}_{>0}$ the minimum inter-event time, and where the sequence $\{\tau_k\}_{k\in\mathbb{N}}$ with $\tau_k \in \mathcal{D} := \{d_1, d_2, \ldots, d_{n_d}\}$ for all $k \in \mathbb{N}$ denotes the communication delays. The set $\mathcal{D}$ contains the $n_d \in \mathbb{N}$ possible delays, $d_i \in \mathbb{R}_{\geqslant 0}$, and satisfies the following small-delay assumption.

Assumption II.1: $\mathcal{D} := \{d_1, d_2, \ldots, d_{n_d}\} \subset [0, h]$.

By making use of Assumption II.1, we ensure that each data packet has arrived at its destination before a new transmission is triggered.

In Sections III–V, we will propose a number of methods to generate the transmission times (4), such that the closed-loop system is globally exponentially stable with decay rate ρ and $\mathcal{L}_2$-stable with $\mathcal{L}_2$-gain θ.

III. Main Results for the CETC Case

In this section, we will propose *continuous* event-generators, which require continuous measuring of the plant output y. First, in Section III-A, we will propose a *static* continuous event-generator, and in Section III-B, we will propose a *dynamic* continuous event-generator, which generates the transmission times based on an additional dynamic variable $\eta \in \mathbb{R}$ that is included in the event-generator.

To describe the closed-loop system, we first have to introduce a number of variables, inspired by, e.g., the works [20], [32]. We introduce the memory variable $s \in \mathbb{R}^{n_y}$, the timer $\tau \in \mathbb{R}_{\geqslant 0}$, the integer $\kappa \in \mathbb{N}$, and the boolean $l \in \{0, 1\}$. The role of these variables will be explained below. Finally, we define $\zeta := (y, s) \in \mathbb{R}^{2n_y}$ and the state $\xi := (x_p, x_c, \hat{y}, s) \in \mathbb{R}^{n_\xi}$ with $n_\xi = n_{x_p} + n_{x_c} + n_y + n_y$.

The dynamic variable $\eta \in \mathbb{R}$ that will be included in the event-generator will evolve according to

$$\frac{\mathrm{d}}{\mathrm{d}t}\eta(t) = \Psi(o(t)), \qquad t \in \mathbb{R}_{\geqslant 0} \setminus \{t_k\}_{k\in\mathbb{N}} \tag{5a}$$

$$\eta(t^+) = \eta_T(o(t)), \qquad t \in \{t_k\}_{k\in\mathbb{N}} \tag{5b}$$

where the signal $o : \mathbb{R}_{\geqslant 0} \to \mathbb{R}^{2n_y} \times \mathbb{R}_{\geqslant 0} \times \mathbb{N} \times \{0, 1\} \times \mathbb{R}$ given by

$$o(t) := (\zeta(t), \tau(t), \kappa(t), l(t), \eta(t)) \tag{6}$$

is the information that is available to the continuous event-generator, and where the functions $\Psi : \mathbb{R}^{2n_y} \times \mathbb{R}_{\geqslant 0} \times \mathbb{N} \times \{0, 1\} \times \mathbb{R} \to \mathbb{R}$ and $\eta_T : \mathbb{R}^{2n_y} \times \mathbb{R}_{\geqslant 0} \times \mathbb{N} \times \{0, 1\} \times \mathbb{R} \to \mathbb{R}$ are to be designed.

Now, we can write the closed-loop system as the impulsive system [33]

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} \xi(t) \\ \tau(t) \\ \kappa(t) \\ l(t) \\ \eta(t) \end{bmatrix} = \begin{bmatrix} A\xi(t) + Bw(t) \\ 1 \\ 0 \\ 0 \\ \Psi(o(t)) \end{bmatrix}, \quad \begin{matrix} t \in \mathbb{R}_{\geqslant 0} \\ t \notin \{t_k\}_{k\in\mathbb{N}} \\ t \notin \{t_k + \tau_k\}_{k\in\mathbb{N}} \end{matrix} \tag{7a}$$

$$\begin{bmatrix} \xi(t^+) \\ \tau(t^+) \\ \kappa(t^+) \\ l(t^+) \\ \eta(t^+) \end{bmatrix} = \begin{bmatrix} J_0\xi(t) \\ 0 \\ \kappa(t) \\ 1 \\ \eta_T(o(t)) \end{bmatrix}, \quad \begin{matrix} l = 0 \\ t \in \{t_k\}_{k\in\mathbb{N}} \end{matrix} \tag{7b}$$

$$\begin{bmatrix} \xi(t^+) \\ \tau(t^+) \\ \kappa(t^+) \\ l(t^+) \\ \eta(t^+) \end{bmatrix} = \begin{bmatrix} J_1\xi(t) \\ \tau(t) \\ \kappa(t) + 1 \\ 0 \\ \eta(t) \end{bmatrix}, \quad \begin{matrix} l = 1 \\ t \in \{t_k + \tau_k\}_{k\in\mathbb{N}} \end{matrix} \tag{7c}$$

$$z(t) = C\xi(t) + Dw(t) \tag{7d}$$

where

$$A = \begin{bmatrix} A_p & B_pC_u & B_pD_u & O \\ O & A_c & B_c & O \\ O & O & O & O \\ O & O & O & O \end{bmatrix}, \; B = \begin{bmatrix} B_{pw} \\ O \\ O \\ O \end{bmatrix}$$

$$C = \begin{bmatrix} C_z & D_zC_u & D_zD_u & O \end{bmatrix}, \; D = D_{zw}$$

$$J_0 = \begin{bmatrix} I & O & O & O \\ O & I & O & O \\ O & O & I & O \\ C_y & D_yC_u & D_yD_u & O \end{bmatrix}, \text{ and}$$

$$J_1 = \begin{bmatrix} I & O & O & O \\ O & I & O & O \\ O & O & O & I \\ O & O & O & I \end{bmatrix}.$$

In this model, the memory variable $s \in \mathbb{R}^{n_y}$ stores the value of $y(t_k)$ that has been transmitted to the controller (and which will arrive at the update time $t_k + \tau_k$), the timer $\tau \in \mathbb{R}_{\geqslant 0}$ keeps track of the time that has elapsed since the latest transmission, the integer $\kappa \in \mathbb{N}$ is used to count the number of (received) transmissions, and the boolean $l \in \{0, 1\}$ indicates whether the next jump is a transmission (when $l = 0$) or an update (when $l = 1$). Furthermore, (7b) models the jumps at transmission times t_k, $k \in \mathbb{N}$, (7c) models the jumps at update times $t_k + \tau_k$, $k \in \mathbb{N}$ (when the transmitted data arrives at the controller), and (7a) models the flow in between transmissions and updates.

The sequence of transmission times $\{t_k\}_{k\in\mathbb{N}}$ will be generated by the dynamic continuous event-generator

$$t_0 = 0, \; t_{k+1} = \inf\{t \geqslant t_k + h \mid \eta(t) \leqslant 0 \wedge \zeta^\top(t)Q\zeta(t) \geqslant 0\} \tag{8}$$

where the timer threshold $h \in \mathbb{R}_{\geqslant 0}$ and the matrix $Q \in \mathbb{R}^{2n_y \times 2n_y}$ are design variables, in addition to the functions Ψ and η_T in (5). The time threshold h acts as a *waiting time* or *time regularization*, and ensures that (4) holds.

With the model (5), (8) we can also capture *static* event-generators by choosing $\eta(0) = 0$ and

$$\Psi(o) = 0 \tag{9a}$$

$$\eta_T(o) = 0 \tag{9b}$$

for all $o \in \mathbb{R}^{2n_y} \times \mathbb{R}_{\geqslant 0} \times \mathbb{N} \times \{0,1\} \times \mathbb{R}$, as then we have that $\eta(t) = 0$ for all $t \in \mathbb{R}_{\geqslant 0}$, and the dynamic event-generator (8) reduces to the static event-generator

$$t_0 = 0,\ t_{k+1} = \inf\{t \geqslant t_k + h \mid \zeta^\top(t) Q \zeta(t) \geqslant 0\} \tag{10}$$

which only has h and Q as design parameters.

While tuning h is straightforward as it is a scalar, choosing a suitable Q is more difficult. However, a possible design for Q can be derived from [2] and is given by

$$Q = \begin{bmatrix} (1-\sigma^2)I & -I \\ -I & I \end{bmatrix} \tag{11}$$

with $\sigma \in (0,1)$. With this choice of Q, (10) reduces to

$$t_0 = 0,\ t_{k+1} = \inf\{t \geqslant t_k + h \mid |s(t) - y(t)|^2 \geqslant \sigma^2 |y(t)|^2\}$$

which can be seen as the event-generator proposed in [2] with waiting time h.

In case $h = 0$, the setup (7), (10) can exhibit Zeno behavior in the presence of disturbances, as shown in [10]. Therefore, we often take $h > 0$, which leads to static event-triggered controllers with time regularization, see, e.g., [5], [13]. Other control setups and other choices of Q are also possible, see e.g., [22].

We will consider the following two notions of stability.

Definition III.1: The CETC system (7)–(8) is said to be input-to-state exponentially stable (ISES), if there exist functions $\alpha \in \mathcal{K}_\infty$, $\beta \in \mathcal{KL}$, and scalars $c > 0$, $\gamma > 0$, and $\rho > 0$ such that for any initial condition $\xi(0) = \xi_0 \in \mathbb{R}^{n_\xi}$, $\tau(0) = 0$, $\kappa(0) = 0$, $l(0) = 0$, $\eta(0) = 0$, and any sequence of delays $\{\tau_k\}_{k\in\mathbb{N}}$ with $\tau_k \in \mathcal{D}$ for all $k \in \mathbb{N}$, all corresponding solutions to (7)–(8) with $w \in \mathcal{L}_\infty$ satisfy $|\xi(t)| \leqslant ce^{-\rho t}|\xi_0| + \gamma\|w\|_{\mathcal{L}_\infty}$ and $|\eta(t)| \leqslant \beta(|\xi_0|, t) + \alpha(\|w\|_{\mathcal{L}_\infty})$ for all $t \in \mathbb{R}_{\geqslant 0}$. In this case, we call ρ a (lower bound on the) decay rate.

Note that our definition of ISES is a variation of the one in [34], which uses the "max" formulation. Moreover, we only require exponential decay of the state variable ξ, as we are mainly interested in the control performance regarding the plant and controller states, which are captured in ξ. In addition, we require that η stays bounded by a $\mathcal{KL}$-function for practical implementability. We do not put any constraint on the variables τ, κ, and l, as these are only used for modeling purposes. In case $\|w\|_{\mathcal{L}_\infty} = 0$, we have $|\xi(t)| \leqslant ce^{-\rho t}|\xi_0|$ and $|\eta(t)| \leqslant \beta(|\xi_0|, t)$, and thus, ISES implies GES in the absence of disturbances.

Definition III.2: The CETC system (7)–(8) is said to have an $\mathcal{L}_2$-gain from w to z smaller than or equal to θ, if there exists a function $\delta \in \mathcal{K}_\infty$ such that for any initial condition $\xi(0) = \xi_0 \in \mathbb{R}^{n_\xi}$, $\tau(0) = 0$, $\kappa(0) = 0$, $l(0) = 0$, $\eta(0) = 0$, and any sequence of delays $\{\tau_k\}_{k\in\mathbb{N}}$ with $\tau_k \in \mathcal{D}$ for all $k \in \mathbb{N}$, all corresponding solutions to (7)–(8) with $w \in \mathcal{L}_2$ satisfy $\|z\|_{\mathcal{L}_2} \leqslant \delta(|\xi_0|) + \theta\|w\|_{\mathcal{L}_2}$.

Before proceeding, we introduce the matrix $Y \in \mathbb{R}^{2n_y \times n_\xi}$

$$Y := \begin{bmatrix} C_y & D_y C_u & D_y D_u & O \\ O & O & O & I \end{bmatrix} \tag{12}$$

such that $\zeta = (y, s) = Y\xi$, and the transformation matrix $T \in \mathbb{R}^{n_\xi \times (n_{x_p} + n_{x_c} + n_y)}$

$$T := \begin{bmatrix} I & O & O \\ O & I & O \\ O & O & I \\ O & O & I \end{bmatrix} \tag{13}$$

such that

$$T \begin{bmatrix} x_p \\ x_c \\ s \end{bmatrix} = \begin{bmatrix} x_p \\ x_c \\ s \\ s \end{bmatrix}. \tag{14}$$

A. Static CETC

Before designing the dynamics of (5) and analyzing ISES and $\mathcal{L}_2$-stability of the system (7) with the dynamic event-generator (8), we will first consider *static* continuous event-generators with time regularization of the form (10).

To analyze ISES and $\mathcal{L}_2$-stability of the static CETC system (7) with (9) and (10), we will use the Lyapunov/storage function U given by

$$U(\xi, \tau, \kappa, l, \eta) = V(\xi, \tau, \kappa, l) + \eta \tag{15}$$

with V given by

$$V(\xi, \tau, \kappa, l) = \begin{cases} \xi^\top P_1^{\tau_\kappa}(\tau)\xi, & \tau \in [0, \tau_\kappa] \text{ and } l = 1 \\ \xi^\top P_0(\tau)\xi, & \tau \in [\tau_\kappa, h] \text{ and } l = 0 \\ \xi^\top P_0(h)\xi, & \tau \in [h, \infty) \text{ and } l = 0. \end{cases} \tag{16}$$

Here, $P_0 : [0, h] \to \mathbb{R}^{n_\xi \times n_\xi}$ is a continuously differentiable function with $P_0(\tau) \succ 0$ for $\tau \in [0, h]$, and for all $d \in \mathcal{D}$, $P_1^d : [0, d] \to \mathbb{R}^{n_\xi \times n_\xi}$ is a continuously differentiable function with $P_1^d(\tau) \succ 0$ for $\tau \in [0, d]$. The functions P_0, P_1^d, $d \in \mathcal{D}$, will be chosen such that (15) becomes a storage function [35]–[37] for the CETC system (7) with (9) and (10), with the supply rate $\theta^{-2} z^\top z - w^\top w$ and decay rate 2ρ. In particular, we will select the functions P_0, P_1^d, $d \in \mathcal{D}$, to satisfy the Riccati differential equations

$$\frac{\mathrm{d}}{\mathrm{d}\tau} P_0(\tau) = R(P_0(\tau)) \tag{17}$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau} P_1^d(\tau) = R(P_1^d(\tau)),\ d \in \mathcal{D} \tag{18}$$

where R denotes the Riccati operator

$$\begin{aligned} R(P) = &-A^\top P - PA - Y^\top N_F Y - 2\rho P - \theta^{-2} C^\top C \\ &- (PB + \theta^{-2} C^\top D) M (B^\top P + \theta^{-2} D^\top C). \end{aligned} \tag{19}$$

Here, $M := (I - \theta^{-2} D^\top D)^{-1}$ is assumed to exist and to be positive definite, which corresponds to $\theta^2 > \lambda_{\max}(D^\top D)$, and

$N_F \in \mathbb{R}^{2n_y \times 2n_y}$, $N_F \succeq 0$, is an arbitrary matrix, which we will use as a design parameter in Section III-B.

Note that V given by (16) depends on the delay τ_κ of the current transmission, and thus, the Lyapunov/storage function U is parametrized by the delay sequence $\{\tau_k\}_{k\in\mathbb{N}}$. However, as we will see below, based on this parametrized function U we are able to guarantee ISES and $\mathcal{L}_2$-stability for *any* sequence of delays $\{\tau_k\}_{k\in\mathbb{N}}$ with $\tau_k \in \mathcal{D}$ and $\mathcal{D}$ satisfying Assumption II.1. For ease of notation, we will not make this dependence on $\{\tau_k\}_{k\in\mathbb{N}}$ explicit in (15) and (16).

In order to find the explicit expressions for the functions P_0, P_1^d, $d \in \mathcal{D}$, we introduce the Hamiltonian matrix

$$H := \begin{bmatrix} A+\rho I+\theta^{-2}BMD^\top C & BMB^\top \\ -C^\top LC - Y^\top N_F Y & -\left(A+\rho I+\theta^{-2}BMD^\top C\right)^\top \end{bmatrix}$$

in which $L := (\theta^2 I - DD^\top)^{-1}$, and we define the matrix exponential

$$F(\tau) := e^{-H\tau} = \begin{bmatrix} F_{11}(\tau) & F_{12}(\tau) \\ F_{21}(\tau) & F_{22}(\tau) \end{bmatrix}. \quad (20)$$

To guarantee that solutions to $\frac{\mathrm{d}}{\mathrm{d}\tau}P_0(\tau) = R(P_0(\tau))$ are well defined on $[0, h]$ and that solutions to $\frac{\mathrm{d}}{\mathrm{d}\tau}P_1^d(\tau) = R(P_1^d(\tau))$ are well defined on $[0, d]$, $d \in \mathcal{D}$, we will make use of the following assumption, see also [22].

Assumption III.3: $F_{11}(\tau)$ is invertible for all $\tau \in [0, h]$.

Assumption III.3 can always be satisfied by choosing h sufficiently small, as $F_{11}(0) = I$ and F_{11} is a continuous function. Note that larger h can be allowed by reducing ρ or increasing θ.

In the delay-free case with periodic sampling and $\rho = 0$, the $\mathcal{L}_2$-gain θ of the system (7) can be determined *exactly* (so without conservatism) by using a lifting-based approach as in, e.g., [25], [38]. These works also require that Assumption III.3 (or an equivalent thereof) holds. Moreover, if an $\mathcal{L}_2$-gain θ cannot be achieved with periodic sampling with sampling period h and without communication delays, this will also not be possible with event-triggered sampling with minimum inter-event time h. Hence, Assumption III.3 is not restrictive in that sense.

The function $P_0 : [0, h] \to \mathbb{R}^{n_\xi \times n_\xi}$ is now explicitly defined for $\tau \in [0, h]$ by

$$P_0(\tau) = (F_{21}(h-\tau) + F_{22}(h-\tau)P_0(h)) \\ (F_{11}(h-\tau) + F_{12}(h-\tau)P_0(h))^{-1} \quad (21)$$

and the functions $P_1^d : [0, d] \to \mathbb{R}^{n_\xi \times n_\xi}$, $d \in \mathcal{D}$, are now explicitly defined for $\tau \in [0, d]$ by

$$P_1^d(\tau) = \left(F_{21}(d-\tau) + F_{22}(d-\tau)P_1^d(d)\right) \\ \left(F_{11}(d-\tau) + F_{12}(d-\tau)P_1^d(d)\right)^{-1} \quad (22)$$

where $P_0(h), P_1^d(d) \succ 0$, $d \in \mathcal{D}$, are to be selected. See [22], [39] for further details.

Before stating the next theorem, let us introduce the notation $P_{00} = P_0(0)$, $P_{0d} = P_0(d)$, $P_{0h} = P_0(h)$, $P_{10}^d = P_1^d(0)$, $P_{1d}^d = P_1^d(d)$, the functions

$$G_0(\tau) := F_{11}(\tau)^{-\top} P_{0h} F_{11}(\tau)^{-1} + F_{21}(\tau)F_{11}(\tau)^{-1} \quad (23)$$

for $\tau \in [0, h]$,

$$G_1^d(\tau) := F_{11}(\tau)^{-\top} P_{1d}^d F_{11}(\tau)^{-1} + F_{21}(\tau)F_{11}(\tau)^{-1} \quad (24)$$

for $\tau \in [0, d]$, $d \in \mathcal{D}$, and finally a matrix function $S : [0, h] \to \mathbb{R}^{n_\xi \times n_\xi}$ that satisfies $S(\tau)S(\tau)^\top := -F_{11}(\tau)^{-1}F_{12}(\tau)$ for $\tau \in [0, h]$. A matrix $S(\tau)$ exists under Assumption III.3, because this assumption guarantees that the matrix $-F_{11}(\tau)^{-1}F_{12}(\tau)$ is positive semidefinite [22].

Theorem III.4: If Assumption II.1 holds, and there exist matrices $N_F, N_T, N_N \in \mathbb{R}^{2n_y \times 2n_y}$ with $N_F, N_T, N_N \succeq 0$, $P_{0h}, P_{1d}^d \in \mathbb{R}^{n_\xi \times n_\xi}$ with $P_{0h}, P_{1d}^d \succ 0$, and scalars $\theta, \rho, \beta, \mu^d \in \mathbb{R}_{\geqslant 0}$, $d \in \mathcal{D}$, such that Assumption III.3 holds and the matrix inequalities

$$\begin{bmatrix} T^\top(A^\top P_{0h} + P_{0h}A + Y^\top(N_N - \beta Q)Y)T & \star \\ B^\top P_{0h} T & O \end{bmatrix} \\ \preceq \begin{bmatrix} T^\top(-2\rho P_{0h} - \theta^{-2}C^\top C)T & -\theta^{-2}T^\top C^\top D \\ \star & I - \theta^{-2}D^\top D \end{bmatrix} \quad (25)$$

$$\begin{bmatrix} J_0^\top G_1^d(d) J_0 & J_0^\top F_{11}(d)^{-\top} P_{1d}^d S(d) \\ \star & S(d)^\top P_{1d}^d S(d) \end{bmatrix} \\ \prec \begin{bmatrix} P_{0h} - Y^\top(N_T + \mu^d Q)Y & O \\ O & I \end{bmatrix} \quad (26)$$

and

$$\begin{bmatrix} J_1^\top G_0(h-d) J_1 & J_1^\top F_{11}(h-d)^{-\top} P_{0h} S(h-d) \\ \star & S(h-d)^\top P_{0h} S(h-d) \end{bmatrix} \\ \prec \begin{bmatrix} P_{1d}^d & O \\ O & I \end{bmatrix} \quad (27)$$

hold for all $d \in \mathcal{D}$, then the static CETC system (7) with (9) and (10) is ISES with decay rate ρ, and has an $\mathcal{L}_2$-gain from w to z smaller than or equal to θ.

Inequalities (25), (26), and (27) depend nonlinearly on the parameters N_F, ρ, and θ. However, once these parameters are *fixed*, also the matrices M, L, $F_{ij}(\tau)$, and $S(\tau)$ become fixed matrices. The matrices $G_0(\tau)$ and $G_1^d(\tau)$, $d \in \mathcal{D}$, then only depend *linearly* on the matrices P_{0h} and P_{10}^d, $d \in \mathcal{D}$. Hence, inequalities (25), (26), and (27) then become linear matrix inequalities (LMIs), and the parameters P_{0h}, P_{10}^d, N_N, N_T, β, and μ^d, $d \in \mathcal{D}$, can be synthesized numerically via semidefinite programming (e.g., using Yalmip/SeDuMi [40] in MATLAB).

The $\mathcal{L}_2$-gain estimate θ can be optimized via bisection when N_F and ρ are fixed. Although the optimization is nonconvex and we should expect to find local optima, good results can be found with proper initial estimates. The same holds for the decay rate ρ when N_F and θ are fixed.

As the term $-Y^\top N_F Y$ in (19) leads to an extra decrease $-\zeta^\top N_F \zeta$ in V during flow (7a) with $\tau \in [0, h]$, an increase in N_F will typically lead to an increase in θ or a decrease in ρ. Hence, to analyze ISES and $\mathcal{L}_2$-stability of the static CETC system (7) with (9) and (10), it is often best to choose $N_F = O$. However, for dynamic CETC it can sometimes be useful to choose $N_F \neq O$, as we will see in Section III-B.

B. Dynamic CETC

In this section, we present our design for a *dynamic* continuous event-generator, which follows from the analysis in Section III-A. The idea is as follows. In Section III-A, the function V (and, hence, also the Lyapunov/storage function U) has an "extra" decrease $-\zeta^\top N_F \zeta$ during flow (7a) with $\tau \in [0, h]$, and an "extra" decrease $-\zeta^\top (N_N - \beta Q)\zeta$ during flow (7a) with $\tau \in [h, \infty)$ and $\zeta^\top Q \zeta \leqslant 0$. Additionally, V is often strictly decreasing along jumps (7b), while we only require that U is nonincreasing along jumps [37]. To get even less conservative results, we can store this "unnecessary" decrease of V in the dynamic variable η (as much as possible), which acts as a buffer. When a transmission is necessary according to the static event-generator (i.e., when $\tau \in [h, \infty)$ and $\zeta^\top Q \zeta \geqslant 0$) the term $-\zeta^\top (N_N - \beta Q)\zeta$ can become positive, in which case the function V will start to increase with the excess amount $-\zeta^\top (N_N - \beta Q)\zeta$ if we do not transmit. However, as long as $\eta > 0$, we can compensate for this excess increase in V by reducing η, and thus, we can postpone the transmission until the buffer η becomes empty ($\eta = 0$). As a result, the conservatism in the stability analysis is reduced, and the same $\mathcal{L}_2$-gain and decay rate can be guaranteed with typically less transmissions, as will also be demonstrated by a numerical example in Section VI-A. In this way, our design will lead to a dynamic CETC system with *the same* $\mathcal{L}_2$-gain θ and decay rate ρ as the static CETC system (7) with (9) and (10), but with a significant reduction in the number of transmissions.

We select the flow dynamics (5a) of η as

$$\Psi(o) = \begin{cases} -2\rho\eta + \zeta^\top N_F \zeta, & \tau \in [0, h) \quad (28a) \\ -2\rho\eta + \zeta^\top (N_N - \beta Q)\zeta, & \tau \in [h, \infty). \quad (28b) \end{cases}$$

For the function η_T, we provide the following two different designs:

1) State-based dynamic CETC:

$$\eta_T(o) = \eta + \min_{d \in \mathcal{D}} \xi^\top \left(P_{0h} - J_0^\top P_{10}^d J_0\right) \xi. \quad (29)$$

2) Output-based dynamic CETC:

$$\eta_T(o) = \eta + \zeta^\top N_T \zeta. \quad (30)$$

Here, the scalars ρ and β, and the matrices N_F, N_T, N_N, $P_{0h}, P_{10}^d \in \mathbb{R}^{n_\xi \times n_\xi}$, $d \in \mathcal{D}$, follow from the stability analysis of the static CETC system in Theorem III.4.

The first design requires that the full state $\xi(t_k)$ is known to the event-generator at transmission time t_k. This is the case when Assumption II.1 holds and $y = (x_p, x_c)$ (e.g., when $\mathcal{C}$ is a static state-feedback controller in which case $y = x_p$ and $n_{x_c} = 0$), as then $\hat{y}(t_k) = s(t_k)$, $k \in \mathbb{N}$, and $\xi(t_k) = T\zeta(t_k)$. Note that when the set of possible delays $\mathcal{D}$ contains only a single element, a copy of the controller could be included in the event-generator in order to track the controller state x_c. In case $\mathcal{D}$ contains multiple elements, this is not possible, as the exact input $\hat{y}$ to the controller is then unknown.

The second design is more conservative, but can also be used in case the event-generator does not have access to the full state ξ. Hence, this choice can be used for output-based dynamic CETC.

Theorem III.5: If the conditions of Theorem III.4 hold, then the dynamic CETC system (7) with (8), (28), and η_T given by (29) or (30) is ISES with decay rate ρ, and has an $\mathcal{L}_2$-gain from w to z smaller than or equal to θ.

While the static continuous event-generator only has design parameters h and Q, the state-based dynamic event-generator has design parameters h, Q, ρ, N_F, N_N, β, P_{0h}, and P_{10}^d, $d \in \mathcal{D}$, and the output-based dynamic event-generator has design parameters h, Q, ρ, N_F, N_N, β, and N_T.

However, these extra design parameters directly follow from the ISES and $\mathcal{L}_2$-gain analysis of the static CETC system in Theorem III.4. Hence, the design of these extra parameters follows directly and naturally from the design and stability analysis of the static event-generator. Of course, manual tuning of one or more of these parameters is also possible, but can be difficult given the large design space.

In contrast to the static CETC case, in the dynamic CETC case it can make sense to choose $N_F \neq O$, as η grows with (28a) during flow with $\tau \in [0, h]$, and thus, the average inter-event times might become larger when N_F is increased. This indicates the presence of a tradeoff between control performance and resource utilization, as an increase in N_F typically also leads to an increase in θ or a decrease in ρ. On the other hand, as we can give hard guarantees on the minimum inter-event time (given by h), but not on the average inter-event time, it often makes more sense to choose $N_F = O$ and to make the tradeoff between control performance and resource utilization via the parameter h.

Remark III.6: Even though we will often choose $N_F = O$, the parameter N_F can be useful in some cases. For example, in our work [17], we chose $N_F \neq O$ in order to model the dynamic continuous event-generator of [20] in our proposed new framework.

Remark III.7: In [20], we were able to guarantee GAS and $\mathcal{L}_p$-stability for all possible delays $\tau_k \in [0, \tau_{mad}]$, where $\tau_{mad} \in \mathbb{R}_{\geqslant 0}$ is the maximum allowable delay. In contrast, here we guarantee ISES and $\mathcal{L}_2$-stability for a finite set $\mathcal{D}$ of possible delays. As an engineering solution, we can approach the situation of [20] by including sufficiently many delays out of the set $[0, \tau_{mad}]$ in the set $\mathcal{D}$ (gridding). Assuming that the ETC system has a small amount of robustness against deviations of the delays from the set $\mathcal{D}$, this could also lead to a stability guarantee for all possible delays $\tau_k \in [0, \tau_{mad}]$, $k \in \mathbb{N}$, cf., [41], which uses a similar approach for the stability analysis of networked control system with varying transmission intervals.

IV. Main Results for the PETC Case

Consider again the control setup of Fig. 1 with plant $\mathcal{P}$ given by (1) and controller $\mathcal{C}$ given by (2). Instead of continuously monitoring the output y (which is sometimes difficult to realize in digital implementations), we now *periodically* sample the output y at sample times $\{s_n\}_{n \in \mathbb{N}}$ given by $s_n = nh$, where $h \in \mathbb{R}_{>0}$ is the sample period. At each sample time s_n, $n \in \mathbb{N}$, the event-generator decides whether the sampled output should be transmitted to the controller or not.

In the PETC case, the dynamic variable η will evolve according to

$$\frac{\mathrm{d}}{\mathrm{d}t}\eta(t) = \Psi(\hat{o}(t)), \qquad t \in \mathbb{R}_{\geqslant 0} \setminus \{s_n\}_{n\in\mathbb{N}} \tag{31a}$$

$$\eta(t^+) = \eta_T(\hat{o}(t)), \qquad t \in \{t_k\}_{k\in\mathbb{N}} \tag{31b}$$

$$\eta(t^+) = \eta_N(\hat{o}(t)), \qquad t \in \{s_n\}_{n\in\mathbb{N}} \setminus \{t_k\}_{k\in\mathbb{N}} \tag{31c}$$

where the functions Ψ, η_T, and η_N are to be designed and where the signal $\hat{o} : \mathbb{R}_{\geqslant 0} \to \mathbb{R}^{2n_y} \times [0, h] \times \mathbb{N} \times \{0, 1\} \times \mathbb{R}$ given by

$$\hat{o}(t) := (\zeta(s_n), \tau(t), \kappa(t), l(t), \eta(t)), \quad t \in (s_n, s_{n+1}] \tag{32}$$

is the information that is available to the periodic event-generator.

We can now describe the closed-loop system as

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\xi(t)\\ \tau(t)\\ \kappa(t)\\ l(t)\\ \eta(t)\end{bmatrix} = \begin{bmatrix}A\xi(t)+Bw(t)\\ 1\\ 0\\ 0\\ \Psi(\hat{o}(t))\end{bmatrix}, \quad \begin{array}{l} t \in \mathbb{R}_{\geqslant 0}\\ t \notin \{s_n\}_{n\in\mathbb{N}}\\ t \notin \{t_k+\tau_k\}_{k\in\mathbb{N}}\end{array} \tag{33a}$$

$$\begin{bmatrix}\xi(t^+)\\ \tau(t^+)\\ \kappa(t^+)\\ l(t^+)\\ \eta(t^+)\end{bmatrix} = \begin{bmatrix}J_0\xi(t)\\ 0\\ \kappa(t)\\ 1\\ \eta_T(\hat{o}(t))\end{bmatrix}, \quad \begin{array}{l} l = 0\\ t \in \{t_k\}_{k\in\mathbb{N}}\end{array} \tag{33b}$$

$$\begin{bmatrix}\xi(t^+)\\ \tau(t^+)\\ \kappa(t^+)\\ l(t^+)\\ \eta(t^+)\end{bmatrix} = \begin{bmatrix}J_1\xi(t)\\ \tau(t)\\ \kappa(t)+1\\ 0\\ \eta(t)\end{bmatrix}, \quad \begin{array}{l} l = 1\\ t \in \{t_k+\tau_k\}_{k\in\mathbb{N}}\end{array} \tag{33c}$$

$$\begin{bmatrix}\xi(t^+)\\ \tau(t^+)\\ \kappa(t^+)\\ l(t^+)\\ \eta(t^+)\end{bmatrix} = \begin{bmatrix}\xi(t)\\ 0\\ \kappa(t)\\ l(t)\\ \eta_N(\hat{o}(t))\end{bmatrix}, \quad \begin{array}{l} l = 0\\ t \in \{s_n\}_{n\in\mathbb{N}}\\ t \notin \{t_k\}_{k\in\mathbb{N}}\end{array} \tag{33d}$$

$$z(t) = C\xi(t) + Dw(t). \tag{33e}$$

In this model, $\tau \in [0, h]$ tracks the time that has elapsed since the last *sample time*, in contrast with the CETC case, in which τ tracked the time since the last *transmission*. All other variables have the same interpretation as in the CETC case. Furthermore, (33b) models the jumps at transmission times t_k, $k \in \mathbb{N}$, (33c) models the jumps at update times $t_k + \tau_k$, $k \in \mathbb{N}$ (when the transmitted data arrives at the controller), (33d) models the jumps at sample times $s_n \neq t_k$, $n, k \in \mathbb{N}$, at which no transmission occurs, and (7a) models the flow in between jumps.

In the PETC case, the sequence of transmission times $\{t_k\}_{k\in\mathbb{N}}$ will be generated by the dynamic periodic event-generator

$$t_0 = 0,\ t_{k+1} = \min\{t > t_k \mid \eta_N(\hat{o}(t)) \leqslant 0 \wedge \zeta^\top(t)Q\zeta(t) \geqslant 0, t \in \{s_n\}_{n\in\mathbb{N}}\}. \tag{34}$$

As in the CETC case, the model (31), (34) can also capture *static* periodic event-generators by choosing $\eta(0) = 0$ and

$$\Psi(\hat{o}) = \eta_T(\hat{o}) = \eta_N(\hat{o}) = 0 \tag{35}$$

for all $\hat{o} \in \mathbb{R}^{2n_y} \times [0, h] \times \mathbb{N} \times \{0, 1\} \times \mathbb{R}$, as then we have that $\eta(t) = 0$ for all $t \in \mathbb{R}_{\geqslant 0}$, and the dynamic periodic event-generator (34) reduces to the static periodic event-generator

$$t_0 = 0,\ t_{k+1} = \min\{t > t_k \mid \zeta(t)^\top Q\zeta(t) \geqslant 0, t \in \{s_n\}_{n\in\mathbb{N}}\}. \tag{36}$$

Definitions for ISES and $\mathcal{L}_2$-stability of the PETC system (33), (34) can be given mutatis mutandis, but are omitted for space reasons.

A. Static PETC

As in the CETC case, we will first consider *static* periodic event-generators of the form (36).

To analyze ISES and $\mathcal{L}_2$-stability of the static PETC system (33) with (35) and (36), we will again use the Lyapunov/storage function U given by (15), but now with V given by

$$V(\xi, \tau, \kappa, l) = \begin{cases}\xi^\top P_1^{\tau_\kappa}(\tau)\xi, & \tau \in [0, \tau_\kappa] \text{ and } l = 1\\ \xi^\top P_0(\tau)\xi, & \tau \in [0, h] \text{ and } l = 0\end{cases} \tag{37}$$

where again $P_0 : [0, h] \to \mathbb{R}^{n_\xi \times n_\xi}$ is a continuously differentiable function satisfying (17) and for all $d \in \mathcal{D}$, $P_1^d : [0, d] \to \mathbb{R}^{n_\xi \times n_\xi}$ is a continuously differentiable function satisfying (18).

Theorem IV.1: If Assumption II.1 holds, and there exist matrices $N_F, N_T, N_N \in \mathbb{R}^{2n_y \times 2n_y}$ with $N_F, N_T, N_N \succeq 0$, $P_{0h}, P_{1d}^d \in \mathbb{R}^{n_\xi \times n_\xi}$ with $P_{0h}, P_{1d}^d \succ 0$, and scalars $\theta, \rho, \beta, \mu^d \in \mathbb{R}_{\geqslant 0}$, $d \in \mathcal{D}$, such that Assumption III.3 holds and inequalities (26), (27), and

$$\begin{bmatrix}T^\top G_0(h)T & T^\top F_{11}(h)^{-\top}P_{0h}S(h)\\ \star & S(h)^\top P_{0h}S(h)\end{bmatrix} \prec \begin{bmatrix}T^\top\left(P_{0h} - Y^\top(N_N - \beta Q)Y\right)T & O\\ \star & I\end{bmatrix} \tag{38}$$

hold for all $d \in \mathcal{D}$, then the static PETC system (33) with (35) and (36) is ISES with decay rate ρ, and has an $\mathcal{L}_2$-gain from w to z smaller than or equal to θ.

Note that Theorem IV.1 extends [22, Th. III.2] to the case with delays. When particularized to the delay-free case, Theorem IV.1 becomes equivalent to [22, Th. III.2], see also [31].

B. Dynamic PETC

In this section, we present our design for a *dynamic* periodic event-generator, which follows from the analysis in Section IV-A. As in Section III-B, the idea is to store the "extra" decrease of V in the dynamic variable η, which is then used to reduce the number of transmissions while maintaining the same $\mathcal{L}_2$-gain θ and decay rate ρ as the static PETC system.

We select the flow dynamics (31a) of η as

$$\Psi(\hat{o}) = -2\rho\eta, \text{ for } \tau \in [0, h]. \tag{39}$$

Remark IV.2: As $\eta^+ = \eta$ at update times $t_k + \tau_k$, $k \in \mathbb{N}$, and Ψ is given by (39), it follows that $\eta(s_{n+1}) = e^{-2\rho h}\eta(s_n^+)$. Thus, since the event-generator only needs to know the value of η at sample times s_n, $n \in \mathbb{N}$, the variable η does not need to continuously evolve according to (31a) in the event-generator. Instead, we can use the discrete-time dynamics just described.

For the functions η_T and η_N, we will provide the following two different designs:

1) State-based dynamic PETC:

$$\eta_T(\hat{o}) = \eta + \min_{d\in\mathcal{D}} \xi^\top \left(P_{0h} - J_0^\top P_{10}^d J_0\right)\xi \tag{40a}$$

$$\eta_N(\hat{o}) = \eta + \xi^\top (P_{0h} - P_{00})\xi. \tag{40b}$$

2) Output-based dynamic PETC:

$$\eta_T(\hat{o}) = \eta + \zeta^\top N_T \zeta \tag{41a}$$

$$\eta_N(\hat{o}) = \eta + \zeta^\top (N_N - \beta Q)\zeta. \tag{41b}$$

Here, the scalars ρ and β, and the matrices N_T, N_N, $P_{00}, P_{0h}, P_{10}^d \in \mathbb{R}^{n_\xi \times n_\xi}$, $d \in \mathcal{D}$, follow from the stability analysis of the static PETC system in Theorem IV.1.

The first design requires that the full state $\xi(s_n)$ is known to the event-generator at sample times s_n, $n \in \mathbb{N}$. The second design is more conservative, but can also be used in case the event-generator does not have access to the full state ξ. Hence, this choice can be used for output-based dynamic PETC.

Theorem IV.3: If the conditions of Theorem IV.1 hold, then the dynamic PETC system (33) with (34), (39), and η_T and η_N given by (40) or (41) is ISES with decay rate ρ, and has an $\mathcal{L}_2$-gain from w to z smaller than or equal to θ.

Remark IV.4: Note that in contrast to the CETC case, (39) does not include the term $\zeta^\top N_F \zeta$, as in the PETC case we do not continuously measure the output y. Hence, our dynamic PETC designs do not involve the matrix N_F, and we can simply let $N_F = O$ in Theorems IV.1 and IV.3. The matrix N_N appears linearly in the LMI (38), and thus can be easily synthesized numerically via semidefinite programming.

V. Reduced Conservatism Using State-Space Partitioning

The ISES and $\mathcal{L}_2$-gain analysis in Sections III and IV are based on the "common" (timer-dependent) quadratic Lyapunov function V, in the sense that the same matrix functions P_0 and $P_1^{\tau_\kappa}$ are used for all $\xi \in \mathbb{R}^{n_\xi}$. In this section, we present even less conservative conditions to analyze ISES and $\mathcal{L}_2$-stability of the proposed CETC and PETC systems, using a *piecewise quadratic* Lyapunov/storage functional technique as proposed in [23] for static PETC systems without delays.

Define the regions

$$\mathcal{X}_i := \{\xi \in \mathbb{R}^{n_\xi} \mid X_i\xi \geqslant 0\}, i \in \{1, 2, \ldots, N\}, N \in \mathbb{N} \tag{42}$$

where the matrices $X_i \in \mathbb{R}^{n_\xi \times n_\xi}$, $i \in \{1, 2, \ldots, N\}$ are such that $\{\mathcal{X}_1, \mathcal{X}_2, \ldots, \mathcal{X}_N\}$ forms a partition of $\mathbb{R}^{n_\xi}$, i.e., the sets $\mathcal{X}_i$, $i \in \{1, 2, \ldots, N\}$ have nonempty interior, $\bigcup_{i=1}^N \mathcal{X}_i = \mathbb{R}^{n_\xi}$, and $\mathcal{X}_i \cap \mathcal{X}_j$ is of zero measure for all $i \neq j$, $i, j \in \{1, 2, \ldots, N\}$.

A. Continuous Event-Triggered Control

Consider the CETC system (7), (8), and define the functional

$$V(\xi, \tau, \kappa, l, w, t) = \begin{cases} \displaystyle\max_{\substack{i\in\{1,2,\ldots,N\} \\ s.t.\ \bar{\xi}(t_\kappa+\tau_\kappa-t,\xi,w_t)\in\mathcal{X}_i}} \xi^\top P_{i,1}^{\tau_\kappa}(\tau)\xi, & \tau \in [0, \tau_\kappa],\ l = 1 \\ \displaystyle\max_{\substack{i\in\{1,2,\ldots,N\} \\ s.t.\ \bar{\xi}(t_{\kappa+1}-t,\xi,w_t)\in\mathcal{X}_i}} \xi^\top P_{i,0}(\tau)\xi, & \tau \in [\tau_\kappa, h],\ l = 0 \\ \displaystyle\max_{\substack{i\in\{1,2,\ldots,N\} \\ s.t.\ \bar{\xi}(t_{\kappa+1}-t,\xi,w_t)\in\mathcal{X}_i}} \xi^\top P_{i,0}(h)\xi, & \tau \in [h, \infty),\ l = 0 \end{cases} \tag{43}$$

where $w_t : \mathbb{R}_{\geqslant 0} \to \mathbb{R}^{n_w}$ denotes the time-shifted signal given by $w_t(s) = w(s + t)$ for $s \geqslant 0$, and $\bar{\xi}(t, \xi, w)$ denotes the solution to $\frac{\mathrm{d}}{\mathrm{d}t}\bar{\xi} = A\bar{\xi} + Bw$ at time t with initial condition $\bar{\xi}(0) = \xi$ and disturbance signal $w : \mathbb{R}_{\geqslant 0} \to \mathbb{R}^{n_w}$. Furthermore, $P_{i,0} : [0, h] \to \mathbb{R}^{n_\xi \times n_\xi}$, $i \in \{1, 2, \ldots, N\}$ are continuously differentiable functions satisfying (17) and $P_{i,1}^d : [0, d] \to \mathbb{R}^{n_\xi \times n_\xi}$, $d \in \mathcal{D}$, $i \in \{1, 2, \ldots, N\}$, are continuously differentiable functions satisfying (18).

Note that V given by (43) depends on the value of $\xi(t_k + \tau_k)$ for $t \in [t_k, t_k + \tau_k]$ when $l = 1$ (as in this interval the index i depends on the value $\xi(t_k + \tau_k)$), and depends on the value of $\xi(t_{k+1})$ for $t \in [t_k, t_{k+1}]$ when $l = 0$, (as in this interval the index i depends on the value $\xi(t_{k+1})$). Hence, V depends not only on the delay sequence $\{\tau_k\}_{k\in\mathbb{N}}$, but also on *future* values of the disturbance w. As such, we have a trajectory/disturbance-dependent Lyapunov/storage functional, which deviates from the common literature on GES and $\mathcal{L}_2$-gain analysis, as usually the Lyapunov/storage function only depends on the current (and sometimes past) values of the state, but typically not on future values. Even though the interpretation of $U = V + \eta$ as a genuine storage function is now less natural, we will see below that we are still able to prove ISES and $\mathcal{L}_2$-stability for *any* sequence of delays $\{\tau_k\}_{k\in\mathbb{N}}$ with $\tau_k \in \mathcal{D}$ and $\mathcal{D}$ satisfying Assumption II.1 and *any* disturbance $w : \mathbb{R}_{\geqslant 0} \to \mathbb{R}^{n_w}$ with $w \in \mathcal{L}_2$.

Before defining the dynamics of η and stating our next theorem, let us introduce the notation $P_{i,00} = P_{i,0}(0)$, $P_{i,0d} = P_{i,0}(d)$, $P_{i,0h} = P_{i,0}(h)$, $P_{i,10}^d = P_{i,1}^d(0)$, $P_{i,1d}^d = P_{i,1}^d(d)$, and the functions

$$G_{i,0}(\tau) := F_{11}(\tau)^{-\top} P_{i,0h} F_{11}(\tau)^{-1} + F_{21}(\tau)F_{11}(\tau)^{-1} \tag{44}$$

for $\tau \in [0, h]$, $i \in \{1, 2, \ldots, N\}$, and

$$G_{i,1}^d(\tau) := F_{11}(\tau)^{-\top} P_{i,1d}^d F_{11}(\tau)^{-1} + F_{21}(\tau) F_{11}(\tau)^{-1} \quad (45)$$

for $\tau \in [0, d]$, $i \in \{1, 2, \ldots, N\}$, $d \in \mathcal{D}$.

For the dynamics of η, we again select Ψ as in (28). For the state-based dynamic continuous event-generator, we now select η_T as

$$\eta_T(o) = \eta + \max_{\substack{i \in \{1,2,\ldots,N\} \\ s.t.\ \xi \in \mathcal{X}_i}} \min_{\substack{d \in \mathcal{D} \\ j \in \{1,2,\ldots,N\}}} \xi^\top \left(P_{i,0h} - J_0^\top P_{j,10}^d J_0 \right) \xi \quad (46)$$

and for the output-based dynamic continuous event-generator, we again select η_T as in (30).

To understand (46), some comments are in order. At the transmission time t_k, we know which matrix function $P_{i,0}$, $i \in \{1, 2, \ldots, N\}$ we should have used in the interval $t \in [t_{k-1} + \tau_{k-1}, t_k]$. However, we do not know yet which matrix function $P_{j,1}^d$, $j \in \{1, 2, \ldots, N\}$, $d \in \mathcal{D}$, we should use in the interval $t \in [t_k, t_k + \tau_k]$. Hence, to make sure that the Lyapunov/storage function $U = V + \eta$ does not increase along transmissions (7b), we therefore take the minimum over all $j \in \{1, 2, \ldots, N\}$ and all $d \in \mathcal{D}$ in (46).

Theorem V.1: If Assumption II.1 holds, and there exist matrices $N_F, N_T, N_N \in \mathbb{R}^{2n_y \times 2n_y}$ with $N_F, N_T, N_N \succeq 0$, $P_{i,0h}, P_{i,1d}^d \in \mathbb{R}^{n_\xi \times n_\xi}$ with $P_{i,0h}, P_{i,1d}^d \succ 0$, $U_{ij}^d, W_{ij}^d \in \mathbb{R}_{\geqslant 0}^{n_\xi \times n_\xi}$ with $U_{ij}^d = {U_{ij}^d}^\top$ and $W_{ij}^d = {W_{ij}^d}^\top$, and scalars $\beta \geqslant 0$ and $\mu_{ij}^d \geqslant 0$, $d \in \mathcal{D}$, $i, j \in \{1, 2, \ldots, N\}$, such that Assumption III.3 holds and the matrix inequalities

$$\begin{bmatrix} T^\top (A^\top P_{i,0h} + P_{i,0h} A + Y^\top (N_N - \beta Q) Y) T & \star \\ B^\top P_{i,0h} T & O \end{bmatrix} \preceq \begin{bmatrix} T^\top (-2\rho P_{i,0h} - \theta^{-2} C^\top C) T & -\theta^{-2} T^\top C^\top D \\ \star & I - \theta^{-2} D^\top D \end{bmatrix} \quad (47)$$

$$\begin{bmatrix} J_0^\top G_{i,1}^d(d) J_0 & J_0^\top F_{11}(d)^{-\top} P_{i,1d}^d S(d) \\ \star & S(d)^\top P_{i,1d}^d S(d) \end{bmatrix} \prec \begin{bmatrix} P_{j,0h} - Y^\top (N_T + \mu_{ij}^d Q) Y - X_j^\top U_{ij}^d X_j & O \\ O & I \end{bmatrix} \quad (48)$$

and

$$\begin{bmatrix} J_1^\top G_{i,0}(h-d) J_1 & J_1^\top F_{11}(h-d)^{-\top} P_{i,0h} S(h-d) \\ \star & S(h-d)^\top P_{i,0h} S(h-d) \end{bmatrix} \prec \begin{bmatrix} P_{j,1d}^d - X_j^\top W_{ij}^d X_j & O \\ O & I \end{bmatrix} \quad (49)$$

hold for all $d \in \mathcal{D}$ and all $i, j \in \{1, 2, \ldots, N\}$, then the dynamic CETC system (7) with (8), (28), and (46) or (30) is ISES with decay rate ρ, and has an $\mathcal{L}_2$-gain from w to z smaller than or equal to θ.

Corollary V.2: If the conditions of Theorem V.1 hold, then the static CETC system (7) with (9) and (10) is ISES with decay rate ρ, and has an $\mathcal{L}_2$-gain from w to z smaller than or equal to θ.

Remark V.3: When increasing the number of regions N, the ISES and $\mathcal{L}_2$-gain analysis becomes less conservative at the cost of higher computational complexity. Moreover, the update (46) becomes more computationally intensive, leading to more complex event-generators. On the other hand, (30) is independent of N, and thus, do not lead to more complex event-generators when increasing N.

B. Periodic Event-Triggered Control

Consider the PETC system (33), (34), and define the functional

$$V(\xi, \tau, \kappa, l, w, t) = \begin{cases} \max\limits_{\substack{i \in \{1,2,\ldots,N\} \\ s.t.\ \bar{\xi}(\tau_\kappa - \tau, \xi, w_t) \in \mathcal{X}_i}} \xi^\top P_{i,1}^{\tau_\kappa}(\tau) \xi, & \tau \in [0, \tau_\kappa],\ l = 1 \\ \max\limits_{\substack{i \in \{1,2,\ldots,N\} \\ s.t.\ \bar{\xi}(h - \tau, \xi, w_t) \in \mathcal{X}_i}} \xi^\top P_{i,0}(\tau) \xi, & \tau \in [0, h],\ l = 0 \end{cases} \quad (50)$$

where again $P_{i,0} : [0, h] \to \mathbb{R}^{n_\xi \times n_\xi}$, $i \in \{1, 2, \ldots, N\}$ are continuously differentiable functions satisfying (17), and $P_{i,1}^d : [0, d] \to \mathbb{R}^{n_\xi \times n_\xi}$, $d \in \mathcal{D}$, $i \in \{1, 2, \ldots, N\}$ are continuously differentiable functions satisfying (18).

For the dynamics of η, we again choose Ψ as in (39). For the state-based dynamic periodic event-generator, we now select η_T and η_N as

$$\eta_T(\hat{o}) = \eta + \max_{\substack{i \in \{1,2,\ldots,N\} \\ s.t.\ \xi \in \mathcal{X}_i}} \min_{\substack{d \in \mathcal{D} \\ j \in \{1,2,\ldots,N\}}} \xi^\top \left(P_{i,0h} - J_0^\top P_{j,10}^d J_0 \right) \xi \quad (51a)$$

$$\eta_N(\hat{o}) = \eta + \max_{\substack{i \in \{1,2,\ldots,N\} \\ s.t.\ \xi \in \mathcal{X}_i}} \min_{j \in \{1,2,\ldots,N\}} \xi^\top \left(P_{i,0h} - P_{j,00} \right) \xi \quad (51b)$$

and for the output-based dynamic periodic event-generator, we again select η_T and η_N as in (41).

Theorem V.4: If Assumption II.1 holds, and there exist matrices $N_F, N_T, N_N \in \mathbb{R}^{2n_y \times 2n_y}$ with $N_F, N_T, N_N \succeq 0$, $P_{i,0h}, P_{i,1d}^d \in \mathbb{R}^{n_\xi \times n_\xi}$ with $P_{i,0h}, P_{i,1d}^d \succ 0$, $U_{ij}^d, W_{ij}^d, V_{ij} \in \mathbb{R}_{\geqslant 0}^{n_\xi \times n_\xi}$ with $U_{ij}^d = {U_{ij}^d}^\top$, $W_{ij}^d = {W_{ij}^d}^\top$, and $V_{ij} = {V_{ij}}^\top$, and scalars $\beta \geqslant 0$ and $\mu_{ij}^d \geqslant 0$, $d \in \mathcal{D}$, $i, j \in \{1, 2, \ldots, N\}$, such that Assumption III.3 holds and the matrix inequalities (48), (49), and

$$\begin{bmatrix} T^\top \left(P_{j,0h} - Y^\top (N_N - \beta Q) Y - X_j^\top V_{ij} X_j \right) T & O \\ \star & I \end{bmatrix} \succ \begin{bmatrix} T^\top G_{i,0}(h) T & T^\top F_{11}(h)^{-\top} P_{i,0h} S(h) \\ \star & S(h)^\top P_{i,0h} S(h) \end{bmatrix} \quad (52)$$

hold for all $d \in \mathcal{D}$ and all $i, j \in \{1, 2, \ldots, N\}$, then the dynamic PETC system (33) with (34), (39), and (51) or (41) is ISES with decay rate ρ, and has an $\mathcal{L}_2$-gain from w to z smaller than or equal to θ.

Corollary V.5: If the conditions of Theorem V.4 hold, then the static PETC system (33) with (35) and (36) is ISES with decay rate ρ, and has an $\mathcal{L}_2$-gain from w to z smaller than or equal to θ.

VI. Numerical Examples

In [31], we have already shown that in the delay-free case our new dynamic PETC designs provide the same control performance guarantees with less communication than the static PETC designs of [22], [25].

Here, we will demonstrate our static and dynamic CETC and PETC designs for the case with delays via two numerical examples.

A. Unstable Batch Reactor

Consider the unstable batch reactor of [32], [42], [43], with $n_{x_p} = 4$, $n_{x_c} = 2$, $n_y = n_w = n_u = n_z = 2$, and plant and controller dynamics given by (1) and (2) with

$$A_p = \begin{bmatrix} 1.3800 & -0.2077 & 6.7150 & -5.6760 \\ -0.5814 & -4.2900 & 0.0000 & 0.6750 \\ 1.0670 & 4.2730 & -6.6540 & 5.8930 \\ 0.0480 & 4.2730 & 1.3430 & -2.1040 \end{bmatrix}$$

$$B_p = \begin{bmatrix} 0.0000 & 0.0000 \\ 5.6790 & 0.0000 \\ 1.1360 & -3.1460 \\ 1.1360 & 0.0000 \end{bmatrix}, \; B_{pw} = \begin{bmatrix} 10 & 0 \\ 0 & 5 \\ 10 & 0 \\ 0 & 5 \end{bmatrix}$$

$$C_y = C_z = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \; C_u = \begin{bmatrix} -2 & 0 \\ 0 & 8 \end{bmatrix}$$

$$D_y = D_z = D_{zw} = A_c = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_c = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \text{ and } D_u = \begin{bmatrix} 0 & -2 \\ 5 & 0 \end{bmatrix}$$

and $\mathcal{D} = \{0.01,\ 0.0125,\ 0.0150,\ 0.0175,\ 0.02\}$. Note that for this system $y \neq (x_p, x_c)$, and thus we cannot use (29), (40), (46), or (51), but we have to resort to (30) for the dynamic CETC case and to (41) for the dynamic PETC case.

We choose $h = 0.1$, Q given by (11), $N_F = O$, and $\rho = 0.05$. For each choice of σ, we minimize the $\mathcal{L}_2$-gain θ, using Theorem III.4 for the CETC case and Theorem IV.1 for the PETC case, from which also the matrices N_T and N_N follow.

Fig. 2(a) shows the guaranteed $\mathcal{L}_2$-gain θ as a function of σ for both the CETC and PETC approaches. Fig. 2(b) shows the average inter-event times $\tau_{\text{avg}} =$ (total number of events)/(simulation time) for the static and (output-based) dynamic event-generators, which have been obtained by simulating the system 10 times for 40 time units with $\xi(0) = 0$ and disturbance w given by

$$w(t) = e^{-0.2t} \begin{bmatrix} 5\sin(3.5t) \\ -\cos(3t) \end{bmatrix}. \qquad (53)$$

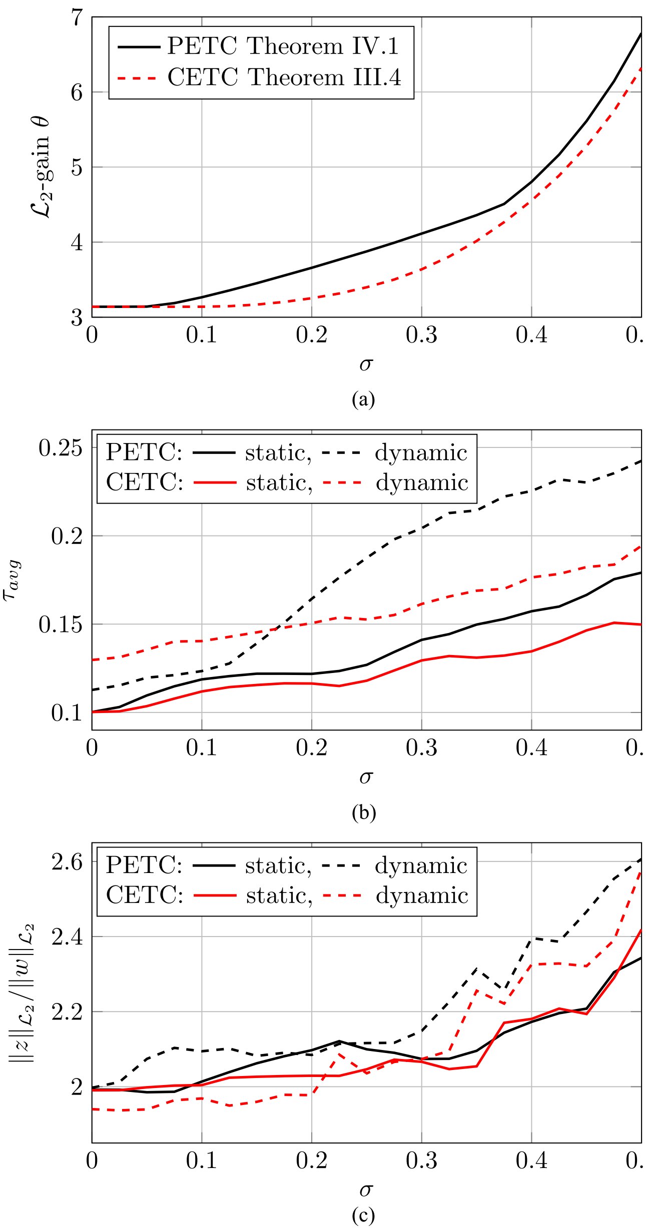

Fig. 2. Guaranteed $\mathcal{L}_2$-gain θ for varying σ (a), average inter-event times τ_{avg} for disturbance w given by (53) and different event-generators (b), and actual ratio $\|z\|_{\mathcal{L}_2}/\|w\|_{\mathcal{L}_2}$ for disturbance w given by (53) (c).

Finally, Fig. 2(c) shows the actual ratio $\|z\|_{\mathcal{L}_2}/\|w\|_{\mathcal{L}_2}$ for disturbance w given by (53), which has been obtained from the same simulations.

In Fig. 2(c) we see that, while the control performance guarantees for the dynamic CETC and PETC systems are identical to the performance guarantees for their static counterparts, the dynamic event-generators exploit (part of) the conservatism in the $\mathcal{L}_2$-gain analysis of Theorems III.4 and IV.1 to postpone the transmissions. This leads to higher ratios $\|z\|_{\mathcal{L}_2}/\|w\|_{\mathcal{L}_2}$ (but still below the guaranteed bounds in Fig. 2(b)), but also to consistently larger τ_{avg}, as can be seen in Fig. 2(b).

To compare these results with [20], note that for given ρ and θ the waiting time h (or τ_{MIET} in the terminology of [20]) of the continuous event-generator proposed in [20] cannot exceed the maximally allowable transmission interval (MATI) of [32]. Moreover, for the same example in [32, Sec. IV] we can calculate that when using the sampled-data protocol, no notion of stability can be guaranteed for MATI larger than 0.063, even without delays. In contrast, here we guarantee ISES and $\mathcal{L}_2$-stability for $h = 0.1$, in the presence of delays. Hence, our new framework tailored to linear systems is clearly much less conservative than our previous results for nonlinear systems in [20]. See also [17] for a direct comparison between the static and dynamic continuous event-generators in Section III and the event-generators proposed in [20], for the case without delays.

B. Reduced Conservatism Using Section V

In this example, we show how the conservatism of the ISES and $\mathcal{L}_2$-stability analysis can be further reduced by partitioning the state-space as in Section V.

Consider the example from [10, Sec. VI-B], with $n_{x_p} = 2$, $n_{x_c} = 0$, $n_y = n_w = n_u = n_z = 1$, and matrices

$$A_p = \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix}, \; B_p = B_{pw} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; D_u = \begin{bmatrix} -3 \end{bmatrix}$$

$$C_y = C_z = \begin{bmatrix} 1 & 0 \end{bmatrix}, \text{ and } D_y = D_z = D_{zw} = \begin{bmatrix} 0 \end{bmatrix}.$$

We will control the system using a periodic event-generator, and choose $h = 1$, $\mathcal{D} = \{0,\ 0.1,\ 0.2\}$, Q as in (11), $N_F = O$, and $\rho = 0.05$.

We partition the state-space as in (42), with $N = 10$ and matrices

$$X_i = \begin{bmatrix} -\sin(\phi_i) & \cos(\phi_i) & 0 & 0 \\ \sin(\phi_{i+1}) & -\cos(\phi_{i+1}) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{54}$$

with

$$\phi_i = (i-1)\frac{2\pi}{N}, \text{ for } i \in \{1, 2, \ldots, N+1\}. \tag{55}$$

Fig. 3 shows the guaranteed $\mathcal{L}_2$-gain θ as a function of σ for the PETC approach using Theorem IV.1, and using the less conservative conditions of Theorem V.4 with the state-space partition as defined above.

The matrices N_T and N_N and the scalar β were found numerically based on Theorem IV.1. In Theorem V.4 we then used the same values for N_N, N_T, and β, such that the resulting design for the output-based dynamic periodic event-generator is identical for both theorems, and any difference in θ can be solely attributed to the partitioning of the state-space and the use of piecewise quadratic Lyapunov functions.

Clearly, by using piecewise quadratic Lyapunov/storage functionals even tighter guarantees on the $\mathcal{L}_2$-gain can be achieved.

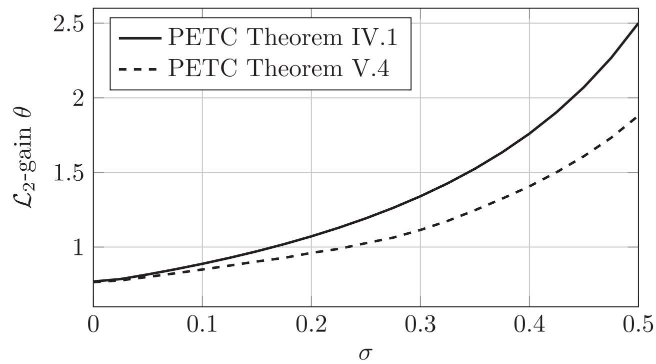

Fig. 3. Guaranteed $\mathcal{L}_2$-gain θ for varying σ.

However, while the performance guarantees become tighter when the number of regions N in the partition is increased, also the computational complexity of the required calculations becomes larger.

VII. Conclusion

We proposed a new method for the design of static and dynamic *continuous* event-generators (which require continuous measuring of the plant output) and static and dynamic *periodic* event-generators (which only require periodic sampling of the plant output) for linear control systems with communication delays. All proposed event-generators lead to GES and $\mathcal{L}_2$-stability with guaranteed decay rates and $\mathcal{L}_2$-gains, and have a guaranteed positive minimum inter-event time.

Our designs exploit Riccati-based tools tailored to linear systems, leading to a significant reduction in conservatism compared to existing results in the literature (which focus on more general nonlinear systems). In fact, we showed via a numerical example that the conservatism in the guaranteed $\mathcal{L}_2$-gain (for any of the proposed event-generators) is small.

Moreover, to the best of the authors' knowledge, the proposed dynamic periodic event-generators are the first in literature that can deal with communication delays.

Appendix

Proof of Theorem III.4: The proof is based on the storage function U given by (15) with V as defined in (16), P_0 satisfying (17), and P_1^d, $d \in \mathcal{D}$, satisfying (18). However, we only need to consider the function V, as it holds that $\eta(t) = 0$ for all $t \in \mathbb{R}_{\geqslant 0}$ (cf., (9)) and thus in this case $U = V$.

The proof consists of showing that V is a proper storage function and satisfies for all $\xi \in \mathbb{R}^{n_\xi}$, $\tau \in \mathbb{R}_{\geqslant 0}$, $\kappa \in \mathbb{N}$, and all $l \in \{0, 1\}$,

$$c_1|\xi|^2 \leqslant V(\xi, \tau, \kappa, l) \leqslant c_2|\xi|^2 \tag{56}$$

with $c_2 \geqslant c_1 > 0$, has a supply rate $\theta^{-2} z^\top z - w^\top w$ [35], [36] and decay rate 2ρ during flow (7a), and is nonincreasing along jumps (7b) and (7c).

The first property follows from Assumption III.3, as this assumption guarantees that $P_0(\tau) \succ 0$ for all $\tau \in [0, h]$ and $P_1^d(\tau) \succ 0$ for all $\tau \in [0, d]$, $d \in \mathcal{D}$, see [22], [39]. Hence, (56) holds with

$$c_1 = \min\left\{ \min_{\tau\in[0,h]} \lambda_{\min}(P_0(\tau)),\ \min_{\substack{d\in\mathcal{D}\\ \tau\in[0,d]}} \lambda_{\min}\left(P_1^d(\tau)\right)\right\} \tag{55a}$$

$$c_2 = \max\left\{ \max_{\tau\in[0,h]} \lambda_{\max}(P_0(\tau)),\ \max_{\substack{d\in\mathcal{D}\\ \tau\in[0,d]}} \lambda_{\max}\left(P_1^d(\tau)\right)\right\} \tag{55b}$$

where $c_2 \geqslant c_1 > 0$.

For brevity, we will use the notation $V(t) = V(\xi(t), \tau(t), \kappa(t), l(t))$ in the remainder of the proof.

Following the derivations in the proof of [22, Th. III.2], it can be shown that (17), (18), and (19) imply that

$$\frac{\mathrm{d}}{\mathrm{d}t}V(t) \leqslant -2\rho V(t) - \theta^{-2}z(t)^\top z(t) + w(t)^\top w(t) - \zeta(t)^\top N_F \zeta(t) \tag{58}$$

during flow (7a) with ($\tau\in[0,\tau_\kappa]$ and $l=1$) or ($\tau\in[0,h]$ and $l=0$). Additionally, during flow (7a) with $\tau\in[h,\infty)$, $l=0$, and $\zeta^\top Q\zeta \leqslant 0$, it holds that $\hat{y} = s$, which implies that

$$\begin{bmatrix} x_p(t)\\ x_c(t)\\ \hat{y}(t)\\ s(t)\end{bmatrix} = T\begin{bmatrix} x_p(t)\\ x_c(t)\\ s(t)\end{bmatrix} \text{ when } t\in(t_k+h, t_{k+1}]$$

and thus, it follows from (25) that

$$\frac{\mathrm{d}}{\mathrm{d}t}V(t) \leqslant -2\rho V(t) - \theta^{-2}z(t)^\top z(t) + w(t)^\top w(t) - \zeta(t)^\top (N_N - \beta Q)\zeta(t). \tag{59}$$

Equations (58) and (59) together with $N_F, N_N \succeq 0$, $\beta \geqslant 0$, and $\zeta(t)^\top Q\zeta(t) \leqslant 0$ for $t\in[t_k+h, t_{k+1}]$ show that

$$\frac{\mathrm{d}}{\mathrm{d}t}V(t) \leqslant -2\rho V(t) - \theta^{-2}z(t)^\top z(t) + w(t)^\top w(t) \tag{60}$$

holds during flow (7a), proving the second property.

Finally, we show that V does not increase along jumps. In [22], it is shown that (21) evaluated at $\tau = d$, $d\in\mathcal{D}$, leads to

$$P_{0d} = G_0(h-d) + F_{11}(h-d)^{-\top}\big(P_{0h} S(h-d)\left(I - S(h-d)^\top P_{0h} S(h-d)\right)^{-1} S(h-d)^\top P_{0h}\big) F_{11}(h-d)^{-1} \tag{61}$$

and for $d\in\mathcal{D}$, (22) evaluated at $\tau=0$, leads to

$$P_{10}^d = G_1^d(d) + F_{11}(d)^{-\top}\big(P_{1d}^d S(d) \left(I - S(d)^\top P_{1d}^d S(d)\right)^{-1} S(d)^\top P_{1d}^d\big) F_{11}(d)^{-1}. \tag{62}$$

Here, the existence of $\left(I - S(h-d)^\top P_{0h} S(h-d)\right)^{-1}$ and $\left(I - S(d)^\top P_{1d}^d S(d)\right)^{-1}$ is guaranteed by Assumption III.3 and $P_{0h}, P_{1d}^d \succ 0$, $d\in\mathcal{D}$, cf., [22]. By applying a Schur complement it follows from (26), $N_T \succeq 0$, and $\mu^d \geqslant 0$ for all $d\in\mathcal{D}$ that along transmissions (7b) (when $\tau\in[h,\infty)$, $l=0$, and $\zeta^\top Q\zeta \geqslant 0$) we have

$$\begin{aligned} V(t^+) &= \xi(t)^\top J_0^\top P_{10}^{\tau_{\kappa(t)}} J_0 \xi(t)\\ &\leqslant \xi(t)^\top P_{0h}\xi(t) - \zeta(t)^\top (N_T + \mu^{\tau_{\kappa(t)}} Q)\zeta(t) \quad (63a)\\ &\leqslant \xi(t)^\top P_{0h}\xi(t) = V(t) \quad (63b)\end{aligned}$$

and it follows from (27) that along updates (7c) (when $\tau = \tau_\kappa$ and $l=1$), we have

$$\begin{aligned} V(t^+) &= \xi(t)^\top J_1^\top P_{0\tau_{\kappa(t)}} J_1 \xi(t)\\ &\leqslant \xi(t)^\top P_{1\tau_{\kappa(t)}}^{\tau_{\kappa(t)}} \xi(t) = V(t). \end{aligned} \tag{64}$$

Combining (56), (60), (63b), and (64) establishes the upper bound θ on the $\mathcal{L}_2$-gain of the ETC system (7), (10) [22].

Furthermore, it follows that

$$\begin{aligned} V(t) &\leqslant e^{-2\rho t}V(0) + \int_0^t e^{-2\rho(t-s)}\|w\|_{\mathcal{L}_\infty}^2 \mathrm{d}s\\ &\leqslant e^{-2\rho t}V(0) + \frac{1}{2\rho}(1-e^{-2\rho t})\|w\|_{\mathcal{L}_\infty}^2\\ &\leqslant e^{-2\rho t}V(0) + \frac{1}{2\rho}\|w\|_{\mathcal{L}_\infty}^2\end{aligned}$$

and thus

$$|\xi(t)| \leqslant c e^{-\rho t}|\xi(0)| + (2\rho c_1)^{-1/2}\|w\|_{\mathcal{L}_\infty} \tag{65}$$

with $c = \sqrt{c_1/c_2}$, which proves that the system is ISES with decay rate ρ. ■

Proof of Theorem III.5: Consider again the Lyapunov/storage function U given by (15) with V as defined in (16).

First, we show that U is a proper storage function, by showing that $\eta(t) \geqslant 0$ for all $t\in\mathbb{R}_{\geqslant 0}$, and that U satisfies for all $\xi\in\mathbb{R}^{n_\xi}$, $\tau\in\mathbb{R}_{\geqslant 0}$, $\kappa\in\mathbb{N}$, $l\in\{0,1\}$, and all $\eta\geqslant 0$,

$$c_1|\xi|^2 + |\eta| \leqslant U(\xi,\tau,\kappa,l,\eta) \leqslant c_2|\xi|^2 + |\eta| \tag{66}$$

where c_1 and c_2 are given by (57). As $\eta(0) = 0$, it follows from (28a), $N_F \succeq 0$, and the comparison lemma [44, Lemma 3.4] that $\eta(t) \geqslant 0$ for all $t\in[0,h)$. Next, η flows according to (28b) on $[h, t_1)$, i.e., as long as $\eta \geqslant 0$ or $\zeta^\top Q\zeta \leqslant 0$ (see (8)). However, note that η can only become negative when $\eta = 0$ and $\zeta^\top Q\zeta > 0$ (see (28b) as $N_N \succeq 0$), in which case a transmission (7b) would be triggered. Hence, $\eta(t_1) = 0$. The relation $\eta_T(o(t_1)) \geqslant 0$ then follows from (63b) when η_T is given by (29), or from $N_T \succeq 0$ when η_T is given by (30). Hence, in both cases it holds that $\eta(t_1^+) \geqslant 0$. It now follows by induction that $\eta(t) \geqslant 0$ for all $t\in\mathbb{R}_{\geqslant 0}$. Property (66) then follows by combining (56) and (15).

It remains to show that U has a supply rate $\theta^{-2}z^\top z - w^\top w$ and decay rate 2ρ during flow (7a), and is nonincreasing along jumps (7b) and (7c). For brevity, we will use the notation $U(t) = U(\xi(t), \tau(t), \kappa(t), l(t), \eta(t))$ and $V(t) = V(\xi(t), \tau(t), \kappa(t), l(t))$ in the remainder of the proof.

From (19) and (28a), it follows (using (58)) that

$$\frac{\mathrm{d}}{\mathrm{d}t}U(t) \leqslant -2\rho V(t) - 2\rho\eta(t) - \theta^{-2}z(t)^\top z(t) + w(t)^\top w(t)$$
$$= -2\rho U(t) - \theta^{-2}z(t)^\top z(t) + w(t)^\top w(t) \quad (67)$$

holds during flow (7a) with $\tau \in [0, h]$, and from (19) and (28b) it follows (using (59)) that (67) holds during flow (7a) with $\tau \in [h, \infty)$.

Finally, we show that

$$U(t^+) \leqslant U(t) \quad (68)$$

holds along jumps. When using (29), we find along transmissions (7b) that (cf., (63))

$$U(t^+) = \xi(t)^\top J_0^\top P_{10}^{\tau_\kappa(t)} J_0\xi(t) + \eta(t) + \min_{d\in\mathcal{D}} \xi(t)^\top \left(P_{0h} - J_0^\top P_{10}^d J_0\right)\xi(t) \leqslant \xi(t)^\top P_{0h}\xi(t) + \eta(t) = U(t).$$

Alternatively, when using (30), we find (using (63a) and $\mu^{\tau_\kappa(t)}\zeta(t)^\top Q\zeta(t) \geqslant 0$) along transmissions (7b) that

$$U(t^+) = \xi(t)^\top J_0^\top P_{10}^{\tau_\kappa(t)} J_0\xi(t) + \eta(t) + \zeta(t)^\top N_T\zeta(t) \leqslant \xi(t)^\top P_{0h}\xi(t) + \eta(t) = U(t).$$

Along updates (7c), (68) follows from (64) (which follows from (27)) and $\eta(t^+) = \eta(t)$.

Equations (66)–(68) together prove that the system has an $\mathcal{L}_2$-gain from w to z smaller than or equal to θ [35], [36]. Furthermore, it follows that

$$U(t) = V(t) + |\eta(t)| \leqslant e^{-2\rho t}V(0) + e^{-2\rho t}|\eta(0)| + \int_0^t e^{-2\rho(t-s)}\|w\|_{\mathcal{L}_\infty}^2\mathrm{d}s \leqslant e^{-2\rho t}V(0) + e^{-2\rho t}|\eta(0)| + \frac{1}{2\rho}\|w\|_{\mathcal{L}_\infty}^2$$

and, since $\eta(0) = 0$, it follows that

$$|\xi(t)| \leqslant ce^{-\rho t}|\xi(0)| + (2\rho c_1)^{-1/2}\|w\|_{\mathcal{L}_\infty}, \text{ and} \quad (69)$$
$$|\eta(t)| \leqslant c_2e^{-2\rho t}|\xi(0)|^2 + (2\rho)^{-1}\|w\|_{\mathcal{L}_\infty}^2 \quad (70)$$

with $c = \sqrt{c_1/c_2}$, which proves that the system is ISES with decay rate ρ. ■

Proof of Theorem IV.1: Consider the Lyapunov function U given by (15), with V given by (37). As in the proof of Theorem III.4, we only need to consider the function V, as it holds that $\eta(t) = 0$ for all $t \in \mathbb{R}_{\geqslant 0}$, and thus, in this case $U = V$.

In the proof of Theorem III.4 it is shown that (56) holds with c_1 and c_2 given by (57), that (60) holds during flow (33a), that (63b) holds along transmissions (33b), and that (64) holds along updates (33c),

It remains to show that V is decreasing along nontransmission jumps (33d). From (61), it follows that (21) evaluated at $\tau = 0$ leads to

$$P_{00} = G_0(h) + F_{11}(h)^{-\top}\big(P_{0h}S(h) \big(I - S(h)^\top P_{0h}S(h)\big)^{-1} S(h)^\top P_{0h}\big)F_{11}(h)^{-1}. \quad (71)$$

By applying a Schur complement it follows from (38) and $\hat{y}(t^+) = \hat{y}(t) = s(t) = s(t^+)$ that along jumps (33d) (when $\tau = h$ and $\zeta^\top Q\zeta \leqslant 0$), we have

$$V(t^+) = \xi(t)^\top P_{00}\xi(t)$$
$$\leqslant \xi(t)^\top P_{0h}\xi(t) - \zeta(t)^\top(N_N - \beta Q)\zeta(t) \quad (72a)$$
$$\leqslant \xi(t)^\top P_{0h}\xi(t) = V(t) \quad (72b)$$

as $N_N \succeq 0$.

Similar arguments as in the proof of Theorem III.4 lead to ISES with decay rate ρ and $\mathcal{L}_2$-stability with $\mathcal{L}_2$-gain θ. ■

Proof of Theorem IV.3: Consider again the Lyapunov function U given by (15), with V given by (37).

First, we show that U is a proper storage function, by showing that $\eta(t) \geqslant 0$ for all $t \in \mathbb{R}_{\geqslant 0}$, and that U satisfies (66) for all $\xi \in \mathbb{R}^{n_\xi}$, $\tau \in [0, h]$, $\kappa \in \mathbb{N}$, $l \in \{0, 1\}$, and all $\eta \geqslant 0$. As $\eta(0) = 0$, it follows from (39) that $\eta(t) \geqslant 0$ for all $t \in [0, h]$, and hence, that $\eta(s_1) \geqslant 0$. Next, given event-generator (34), a transmission (33b) occurs in case $\zeta(s_1)^\top Q\zeta(s_1) \geqslant 0$ and $\eta_N(\hat{o}(s_1)) \leqslant 0$. In this case, $\eta_T(\hat{o}(s_1)) \geqslant 0$ follows from (63b) when η_T is given by (40a), or from $N_T \succeq 0$ when η_T is given by (41a). Otherwise, if $\zeta(s_1)^\top Q\zeta(s_1) < 0$ or $\eta_N(\hat{o}(s_1)) > 0$, no transmission occurs, and the state jumps according to (33d). Observe however that when $\zeta(s_1)^\top Q\zeta(s_1) < 0$ it holds that $\eta_N(\hat{o}(s_1)) \geqslant 0$, which follows from (72b) when η_N is given by (40b), or from $N_N \succeq 0$ and $\beta \geqslant 0$ when η_N is given by (41b). Hence, in all cases it holds that $\eta(s_1^+) \geqslant 0$. It now follows by induction that $\eta(t) \geqslant 0$ for all $t \in \mathbb{R}_{\geqslant 0}$. Property (66) then follows by combining (56) and (15), where c_1 and c_2 are given by (57).

From (19) and (39), it follows that (67) holds during flow (33a), and thus, that U has a supply rate $\theta^{-2}z^\top z - w^\top w$ and decay rate 2ρ during flow.

It remains to show that (68) holds along jumps. For transmissions (33b) and updates (33c), this has already been shown in the proof of Theorem III.5, as the functions (40a) and (29), and (41a) and (30) are identical. Along jumps (33d), we find when using (40b) that

$$U(t^+) = \xi(t)^\top P_{00}\xi(t) + \eta(t) + \xi(t)^\top\left(P_{0h} - P_{00}\right)\xi(t) = \xi(t)^\top P_{0h}\xi(t) + \eta(t) = U(t).$$

Alternatively, when using (41b), we find using (72a) (which also holds along jumps (33d) when $\tau = h$ and $\zeta^\top Q\zeta > 0$) that

$$U(t^+) = \xi(t)^\top P_{00}\xi(t) + \eta(t) + \zeta(t)^\top(N_N - \beta Q)\zeta(t) \leqslant \xi(t)^\top P_{0h}\xi(t) + \eta(t) = U(t).$$

This completes the proof. ■

Proof of Theorem V.1: The proof directly follows from the proof of Theorem III.5 by using V as defined in (43) and noting that for all $d \in \mathcal{D}$ and all $i \in \{1, 2, \ldots, N\}$ it holds that $X_j^\top U_{ij}^d X_j \geqslant 0$ and $X_j^\top W_{ij}^d X_j \geqslant 0$ when $\xi \in \mathcal{X}_j$, $j \in \{1, 2, \ldots, N\}$. ∎

Proof of Theorem V.4: The proof directly follows from the proof of Theorem IV.3 by using V as defined in (50) and noting that for all $d \in \mathcal{D}$ and all $i \in \{1, 2, \ldots, N\}$ it holds that $X_j^\top U_{ij}^d X_j \geqslant 0$, $X_j^\top W_{ij}^d X_j \geqslant 0$, and $X_j^\top V_{ij} X_j \geqslant 0$ when $\xi \in \mathcal{X}_j$, $j \in \{1, 2, \ldots, N\}$. ∎

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