A Multi Target Track Before Detect Application

Y. Boers and J.N. Driessen
JRS-PE-Innovation
THALES Nederland
7554 RR Hengelo
The Netherlands

F. Verschure, W.P.M.H. Heemels and A. Juloski
Electrical Engineering
Eindhoven University of Technology
5600 MB Eindhoven
The Netherlands

Abstract

This paper deals with a radar track before detect application in a multi target setting. Track before detect is a method to track weak objects (targets) on the basis raw radar measurements, e.g. the reflected target power. In classical target tracking, the tracking process is performed on the basis of pre-processed measurements, that are constructed from the original measurement data every time step. In this way no integration over time takes place and information is lost.

In this paper we will give a modelling setup and a particle filter based algorithm to deal with a multiple target track before detect situation. In simulations we show that, using this method, it is possible to track multiple, closely spaced, (weak) targets.

1. Introduction

Classical tracking methods take as an input so called plots that typically consist of range measurements, bearing measurements, elevation measurements and range rate (doppler) measurements, see [1] and [2]. In this classical tracking setting tracking consists of estimating kinematic state properties, e.g. position, velocity and acceleration on the basis of these measurements.

In the classical setup the measurements are the output of the extraction, see figure 1. In this setup there is a processing chain before the tracking, this processing chain can consist (e.g. in case of radar) of a detection stage, a clustering stage and an extraction stage, see figure 1.

In the method, that we propose here, we will use as measurements the raw measurement data, e.g. reflected power, see figure 1. The method is called Track Before Detect (TBD), see also [6], [9] and [10].

If we look at figure 1 we see that in classical tracking (i.e. the separate blocks) a detection decision is made directly on the basis of the raw measurement from one single scan. This means that already at the beginning of the processing chain a hard decision is made w.r.t. the presence of a possible target. Note that this decision is made instantaneously, i.e without using information from the (near) past.

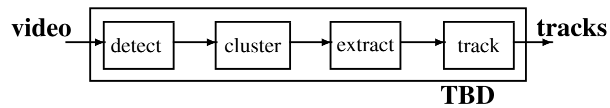

Figure 1: Classical data and signal processing (seperate boxes) and TBD (large box)

In TBD the decision is made at the end of the processing chain, i.e. when all information has been used and integrated over time. Note that information in a track has been obtained by integrating over time). This method is especially suitable for tracking weak targets, i.e. targets that in the classical setting often will not lead to a detection.

A particle filter will be used to perform the TBD. In [8, 9, 10] it has been shown that for a single target the detection can be based on the output of this filter. Somewhat related is the work in [7], where a multiple target approach for TBD on the basis of camera observations for well separated targets has been presented.

In this paper we will present an algorithm that deals with a multiple, closely spaced target, setup in which targets can popup and disappear (birth/death).

2 System setup

Consider the general system

$$s_{k+1} = f(t_k, s_k, d_k, w_k), \quad k \in \mathbb{N} \tag{1}$$

$$\text{Prob}\{d_{k+1} = i \mid d_k = j\} = [\Pi(t_k)]_{ij} \tag{2}$$

$$z_k = h(t_k, s_k, d_k, v_k), \quad k \in \mathbb{N} \tag{3}$$

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Where

- $s_k \in \mathcal{S} \subset \mathbb{R}^{n_{s(d_k)}}$ is the base state of the system.
- $d_k \in \mathcal{D} \subset \mathbb{N}$ is the modal state of the system.
- $z_k \in \mathbb{R}^p$ is the measurement.
- $t_k \in \mathbb{R}$ is time.
- w_k is the process noise and $p_{w(k,d_k)}(w)$ is the probability distribution of the process noise.
- v_k is the measurement noise and $p_{v(k,d_k)}(v)$ is the probability distribution of the measurement noise.
- f is the system dynamics function.
- h is the measurement function.
- $\Pi(t_k)$ is the Markov transition matrix.

define

$$Z_k = \{z_1, \dots, z_k\} \tag{4}$$

The optimal hybrid filtering problem can be formulated as follows.

Problem 2.1 (Optimal filtering problem)
Consider the system represented by the equations (1), (2) and (3). Assume that the initial pdf $p(s(0), d(0))$ is available. The hybrid filtering problem is the problem of constructing the a posteriori pdf

$$p(s_k, d_k \mid Z_k) \tag{5}$$

Note that given the solution to problem 2.1, the mean of the state (s_k, d_k) is obtained as

$$E_{p(s_k,d_k|Z_k))}(s_k, d_k) \tag{6}$$

Actually for any function of the state, $\phi(s_k, d_k)$, the mean

$$E_{p(s_k,d_k|Z_k))}\phi(s_k, d_k) \tag{7}$$

can be calculated on the basis of the filtering solution.

3 Particle filter solution

A solution to the above hybrid filtering problem is to extend the state in a straightforward manner with a modal state and then apply a particle filter to the extended state, see e.g. [4, 12]

The following algorithm will result in an approximation of the a posteriori filtering distribution

$$p(s_k, d_k \mid Z_k)$$

Algorithm 3.1
Consider the system represented by the equations (1), (2) and (3).

Assume that an initial pdf

$$p(s_0, d_0)$$

is given.

Choose an integer N, the sample size.

1. *Draw N samples according $p(s_0, d_0)$, to obtain $\{(\tilde{s}_0^i, \tilde{d}_0^i)\}_{i=1,\dots,N}$*
2. *Generate $\{d_k^i\}_{i=1,\dots,N}$ on the basis of $\{\tilde{d}_{k-1}^i\}_{i=1,\dots,N}$ and $\Pi(t_{k-1})$.*
3. *Draw $\{w_{k-1}^i\}_{i=1,\dots,N}$ according to $p_w(w)$ and obtain $\{(s_k^i, d_k^i)\}_{i=1,\dots,N}$ using*
$$s_k^i = f(t_{k-1}, \tilde{s}_{k-1}^i, \tilde{d}_{k-1}^i, w_{k-1}^i)$$
4. *Given z_k, define*
$$\tilde{q}_k^i = p(z_k \mid s_k^i, d_k^i, t_k), \;\; i = 1, \dots, N$$
5. *Normalize*
$$q_k^i := \frac{\tilde{q}_k^i}{\sum_{i=1}^N \tilde{q}_k^i}, \;\; i = 1, \dots, N$$
6. *Resample N times from*
$$\hat{p}(s, d) := \sum_{j=1}^N q_k^i \delta((s, d) - (s_k^i, d_k^i))$$
and obtain $\{(\tilde{s}_k^i, \tilde{d}_k^i)\}_{i=1,\dots,N}$ to construct
$$\hat{p}(s_k \mid Z_k) := \sum_{j=1}^N \frac{1}{N} \delta((s, d) - (\tilde{s}_k^i, \tilde{d}_k^i))$$
goto 2.

Remark 3.2
The above algorithm is a standard particle filter implementation. Different and better algorithms for multiple model particle filters exist, see e.g. [4].

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4 TBD system setup

In this section we will describe the models that will be used in the TBD application. We will describe the system dynamics models and the measurement models.

4.1 System dynamics

A widely used model is the constant velocity model, see e.g. [1, 2]. This model is used to describe the position and velocity using Cartesian coordinates. Furthermore the model has an additive process noise term. The discrete-time system dynamics of this model is of the form:

$$s_{k+1} = f(t_k, s_k, d_k) + g(t_k, s_k, d_k) w_k \qquad (8)$$

where

$$f(t_k, s_k, d_k) = \begin{pmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} s_k \qquad (9)$$

with the state vector $s_k = [x_k, y_k, \dot{x}_k, \dot{y}_k]^T$ where x_k and y_k are the positions and $\dot{x}_k$ and $\dot{y}_k$ are the velocities. The process noise w_k is assumed to be standard white Gaussian noise. T is the revisit time, which has been assumed to be constant here, this assumption can be relaxed, however.

The process noise input model is given by

$$g(t_k, s_k, d_k) = \begin{pmatrix} \frac{1}{2}(\frac{1}{3}a_{x,max})T^2 & 0 \\ 0 & \frac{1}{2}(\frac{1}{3}a_{y,max})T^2 \\ \frac{1}{3}a_{x,max}T & 0 \\ 0 & \frac{1}{3}a_{y,max}T \end{pmatrix} \qquad (10)$$

with maximum accelerations $a_{x,max}$ and $a_{y,max}$.

4.2 Measurement model

The measurements are measurements of reflected power. One measurement z_k consists of $N_r \times N_d \times N_b$ power measurements z_k^{ijl}, where N_r, N_d and N_b are the number of range, doppler and bearing cells.

In figure 2 we have plotted power measurements for a fixed bearing angle, but as a function of different range and doppler. The power measurements in this figure correspond to a target that has a power of 10 and a noise level such that the signal to noise ratio is 13dB.

The power measurements per range-doppler-bearing cell are defined by

$$z_k^{ijl} = |z_{A,k}^{ijl}|^2 \qquad k \in \mathbb{N} \qquad (11)$$

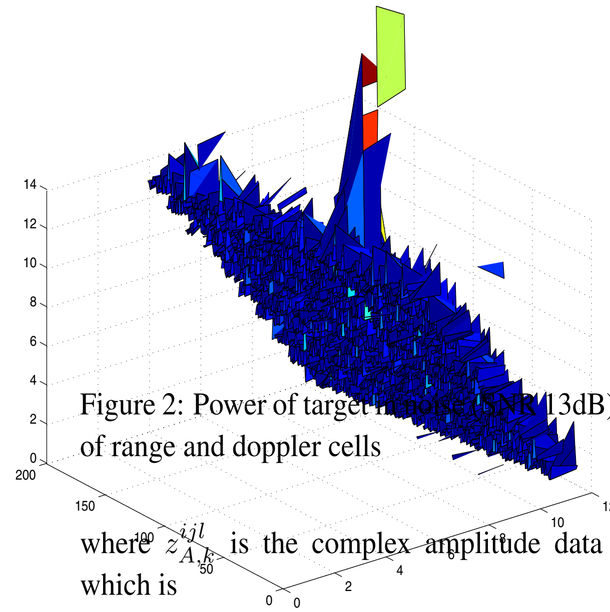

Figure 2: Power of target in noise (SNR 13dB) as a function of range and doppler cells

where $z_{A,k}^{ijl}$ is the complex amplitude data of the target which is

$$z_{A,k} = A_k h_A(s_k, t_k) + n(t_k), \qquad k \in \mathbb{N} \qquad (12)$$

where

$$A_k = \tilde{A}_k e^{\imath \phi_k}, \qquad \phi_k \in (0, 2\pi) \qquad (13)$$

is the complex amplitude of the target and $h_A(s_k, t_k)$ is the reflection form that is defined for every range-doppler-bearing cell by

$$h_A^{ijl}(s_k, t_k) = e^{-\frac{(r_i - r_k)^2}{2R} L_r - \frac{(d_j - d_k)^2}{2D} L_d - \frac{(b_l - b_k)^2}{2B} L_b}, \qquad (14)$$

$i = 1, \ldots, N_r,\ j = 1, \ldots, N_d,\ l = 1, \ldots, N_b$ and $k \in \mathbb{N}$

with

$$r_k = \sqrt{x_k^2 + y_k^2} \qquad (15)$$

$$d_k = \dot{r}_k = \frac{1}{\sqrt{x_k^2 + y_k^2}} (x_k \dot{x}_k + y_k \dot{y}_k) \qquad (16)$$

$$b_k = \arctan\left(\frac{y_k}{x_k}\right) \qquad (17)$$

which are the range, doppler and bearing respectively of the target. R, D and B are related to the size of a range, a doppler and a bearing cell. L_r, L_d and L_b represent constants of losses.

The noise is defined by

$$n(t_k) = n_I(t_k) + \imath n_Q(t_k) \qquad (18)$$

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which is complex Gaussian, where $n_I(t_k)$ and $n_Q(t_k)$ are independent, zero-mean white Gaussian with variance σ_n^2.

In this way the power measurements per range-doppler-bearing cell are defined by

$$z_k^{ijl} = |z_{A,k}^{ijl}|^2 = \tag{19}$$

$$= |A_k h_A^{ijl}(s_k, t_k) + n_I(t_k) + \imath n_Q(t_k)|^2, \qquad k \in \mathbb{N}$$

These measurements, conditioned on s_k, are now exponentially distributed

$$p(z_k^{ijl}|s_k) = \frac{1}{\mu_0^{ijl}} e^{-\frac{1}{\mu_0^{ijl}} z_k^{ijl}} \tag{20}$$

where

$$\begin{aligned}
\mu_0^{ijl} &= E[z_k^{ijl}] \\
&= E[|\tilde{A}_k e^{i\phi_k} h_A^{ijl}(s_k, t_k) + n_I(t_k) + \imath n_Q(t_k)|^2] \\
&= E[(\tilde{A}_k h_A^{ijl}(s_k, t_k)\cos(\phi_k) + n_I(t_k))^2 \\
&\quad +(\tilde{A} h_A^{ijl}\sin(\phi_k) + n_Q(t_k))^2] \\
&= \tilde{A}^2 (h_A^{ijl}(s_k, t_k))^2 + 2\sigma_n^2 \\
&= P h_P^{ijl}(s_k, t_k) + 2\sigma_n^2
\end{aligned} \tag{21}$$

with

$$h_P^{ijl}(s_k, t_k) = (h_A^{ijl}(s_k, t_k))^2 = \tag{22}$$

$$= e^{-\frac{(r_i - r_k)^2}{R} L_r - \frac{(d_j - d_k)^2}{D} L_d - \frac{(b_l - b_k)^2}{B} L_b}$$

which describes the power contribution of a target in every range-doppler-bearing cell.

5 Multiple target setting

In this section we consider the (possible) presence of two targets, where one target can originate (spawn) from the other. Think, e.g. of a missile being fired from a fighter airplane.

We can consider the general system introduced in section 2, i.e.

$$s_{k+1} = f(t_k, s_k, d_k, w_k), \;\; k \in \mathbb{N} \tag{23}$$

$$\text{Prob}\{d_{k+1} = i \mid d_k = j\} = [\Pi(t_k)]_{ij} \tag{24}$$

$$z_k = h(t_k, s_k, d_k, v_k), \;\; k \in \mathbb{N} \tag{25}$$

Furthermore, for this problem it is convenient to define a base state vector that consists of the base states of both targets. Thus,

$$S_k = \begin{pmatrix} s_k^{(1)} \\ s_k^{(2)} \end{pmatrix} \tag{26}$$

The discrete mode d_k represents one of three hypotheses

- $d_k = 0$: There is no target present.
- $d_k = 1$: There is one target present.
- $d_k = 2$: There are two targets present.

5.1 Measurements models

For the multi target setup the measurement models need to be extended.

The complex amplitude data that is received from two targets can be modelled by

$$z_{A,k} = A_k^{(1)} h_A^{(1)}(s_k, t_k) + A_k^{(2)} h_A^{(2)}(s_k, t_k) + n(t_k) \tag{27}$$

where $A_k^{(1)}$, $h_A^{(1)}$ and $A_k^{(2)}$, $h_A^{(2)}$ are the amplitude and reflection form of the first and second target respectively. Thus, the power measurements are

$$\begin{aligned}
z_k^{ijl} &= |z_{A,k}^{ijl}|^2 \\
&= |A^{(1)} h_A^{(1)ijl}(s_k, t_k) + A^{(2)} h_A^{(2)ijl}(s_k, t_k) \\
&\quad + n_I(t_k) + \imath n_Q(t_k)|^2
\end{aligned} \tag{28}$$

These will, again be exponentially distributed

$$p(z_k^{ijl}|s_k) = \frac{1}{\mu_0^{ijl}} e^{-\frac{1}{\mu_0^{ijl}} z_k^{ijl}} \tag{29}$$

where

$$\begin{aligned}
\mu_0^{ijl} &= E[z_k^{ijl}] \\
&= E[|\tilde{A}_k^{(1)} e^{\imath\phi_k} h_A^{(1)ijl}(s_k, t_k) \\
&\quad + \tilde{A}_k^{(2)} e^{\imath\phi_k} h_A^{(2)ijl}(s_k, t_k) + n_I(t_k) + \imath n_Q(t_k)|^2] \\
&= E[(\tilde{A}_k^{(1)} \cos(\phi_k^{(1)}) h_A^{(1)ijl}(s_k, t_k) \\
&\quad + \tilde{A}_k^{(2)} \cos(\phi_k^{(2)}) h_A^{(2)ijl}(s_k, t_k) + n_I(t_k))^2 \\
&\quad + (\tilde{A}_k^{(1)} \sin(\phi_k^{(1)}) h_A^{(1)ijl}(s_k, t_k) \\
&\quad + \tilde{A}_k^{(2)} \sin(\phi_k^{(2)}) h_A^{(2)ijl}(s_k, t_k) + n_Q(t_k))^2] \\
&= (\tilde{A}_k^{(1)} h_A^{(1)ijl}(s_k, t_k))^2 \\
&\quad + (\tilde{A}_k^{(2)} h_A^{(2)ijl}(s_k, t_k))^2 + 2\sigma_n^2 \\
&= P^{(1)} h_P^{(1)ijl}(s_k, t_k) \\
&\quad + P^{(2)} h_P^{(2)ijl}(s_k, t_k) + 2\sigma_n^2
\end{aligned} \tag{30}$$

using this we obtain for the mode dependent likelihood

$$p(z_k|S_k, d_k) = \begin{cases} \prod_{ijl} p_v(z_k^{ijl}) & \text{for } d_k = 0 \\ \prod_{ijl} p(z_k^{ijl}|S_k) & \text{for } d_k = 1, \; s_k^{(2)} = s_k^{(1)} \\ \prod_{ijl} p(z_k^{ijl}|S_k) & \text{for } d_k = 2 \end{cases} \tag{31}$$

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6 Multiple target tracking simulations

In this section we give a demonstration of a particle filter that is capable of tracking multiple targets.

In the scenario, initially there is no target present, the first target appears after 5 seconds at a position of $88.6km$ from the sensor and flies at a constant velocity of $200ms^{-1}$ directly to the sensor. At $t = 15$ a second target spawns from the first, accelerating to a velocity of $300ms^{-1}$ over three scans, see figure 3 for an illustration of the scenario.

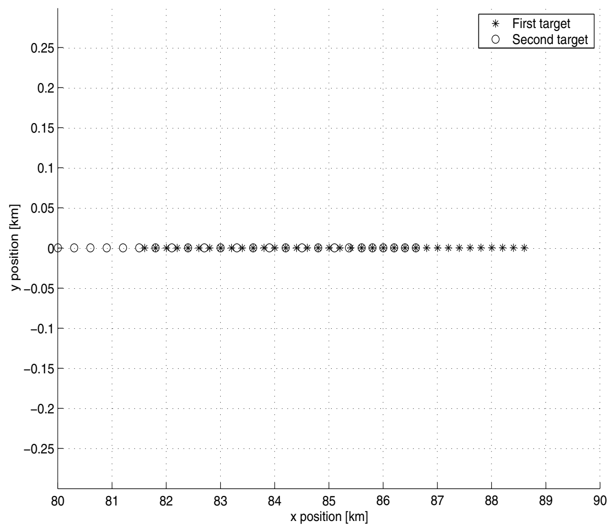

Figure 3: Target trajectories

We consider range cells in the interval $[80, 90]km$, doppler cells in the interval$[-0.34, -0.10]kms^{-1}$. We only consider one bearing cell in this example. We therefore have $N_r \times N_d \times N_b$ cells, where $N_r = 50$, $N_d = 16$ and $N_b = 1$.

Initially, 9600 particles are uniformly distributed in the state space, in an area between $[85, 90]km$ and $[-0.22, -0.10]kms^{-1}$ in the x-direction and $[-0.1, 0.1]km$ $[-0.10, 0.10]kms^{-1}$ and in the y-direction. Furthermore, initially we uniformly distribute the particles over all modes.

The transition probability matrix is assumed to be

$$\Pi(t_k) = \begin{pmatrix} 0.90 & 0.10 & 0.00 \\ 0.10 & 0.80 & 0.10 \\ 0.00 & 0.10 & 0.90 \end{pmatrix}$$

The update time, T, is set to one second.

The dynamics of both targets are captured by a constant velocity model, but with different values for their maximal acceleration. For the first target we set $a_{max,x} = a_{max,y} = 5ms^{-2}$. For the second target we choose $a_{max,x} = 35ms^{-2}$ and $a_{max,y} = 5ms^{-2}$.

We assume that the power of both targets is known, $P^{(1)} = 10$ and $P^{(2)} = 1$. The noise level is to be assumed such that a specified SNR for the first (strongest) target is realized according to

$$SNR = 10 \log \left(\frac{P^{(1)}}{2\sigma_n^2} \right) [dB]$$

Simulations are performed for two different SNR values, $SNR = 13dB$ and $SNR = 7dB$. This implies that the SNR of the second target becomes $3dB$ and $-3dB$ respectively.

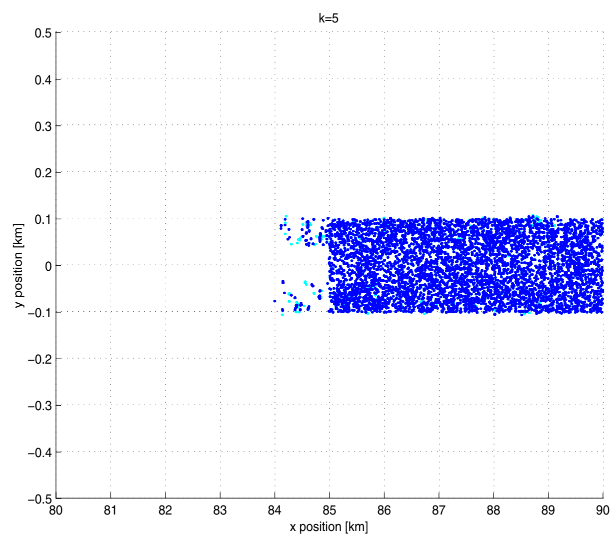

Figure 4: Particles at k=5, SNR=13dB

In the figures 4, 5 and 6, we see, for the case of SNR=13dB, the particle clouds at different time steps. In figure 7, we see the corresponding mode probabilities. Until time step k=5, the cloud remains more or less uniformly distributed. When the first target appears the cloud quickly concentrates on the target and remains there, see figure 5. The filter however, is somewhat indecisive whether one or two targets are present, see figure 7. After the birth of the second target the filter is very well able to distinguish the two targets.

In the figures 8, 9 and 10, we see for the case of SNR=7dB, the particle clouds at different time steps. In figure 11, we see the corresponding mode probabilities. Again until time step k=5, the cloud remains more or less uniformly distributed. When the first target appears the filter picks it up quite good, but is again quite indecisive on the

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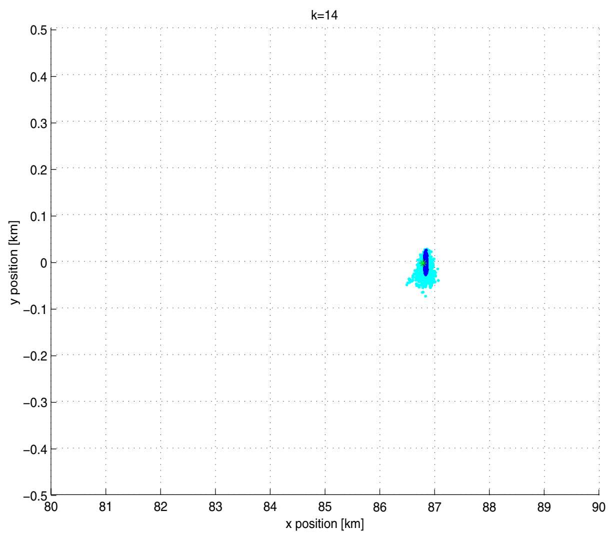

Figure 5: Particles at k=14, SNR=13dB

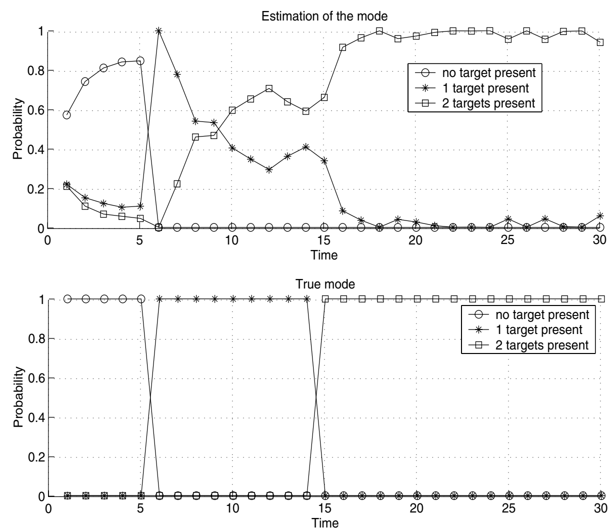

Figure 7: Mode probabilities, SNR=13dB

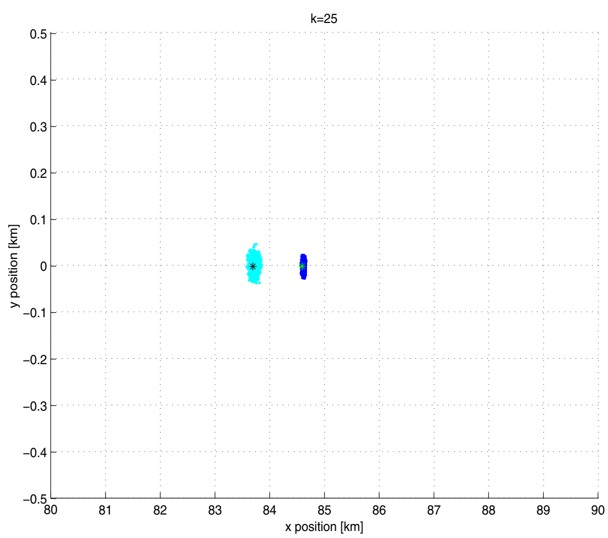

Figure 6: Particles at k=25, SNR=13dB

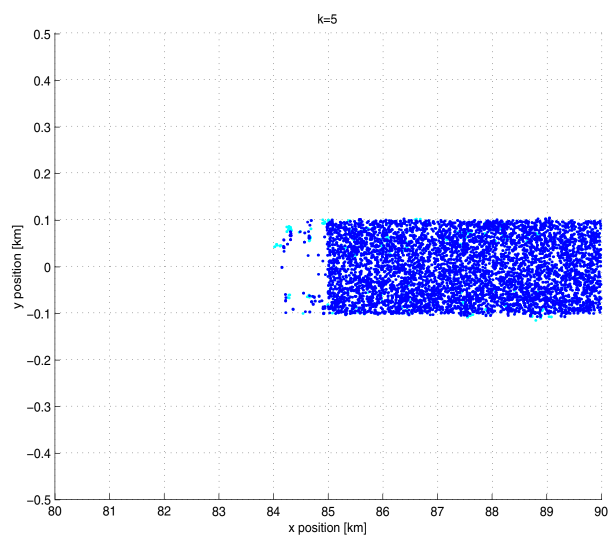

Figure 8: Particles at k=5, SNR=7dB

number of targets, see figure 11. After the birth of the second target, the filter is able to track both the first and second target, but is still quite indecisive on the number of targets. The explanation for this that the second target is a really weak one, i.e. SNR=-3dB.

7 Conclusions

In this paper we presented a modelling setup and an algorithm for a multiple target TBD application. The problem has been solved by using a multiple model particle filter. Simulations show that the method can deal with fairly weak targets, that in a classical target tracking setup never would have been detected.

In the future, the method should be extended to the case where the SNR of the targets is unknown and where fluctuation of the strength of the targets, so called radar cross section fluctuation, is taken into account. Furthermore, it should be studied to what extent other (better?) multiple model particle filter algorithms can improve the results.

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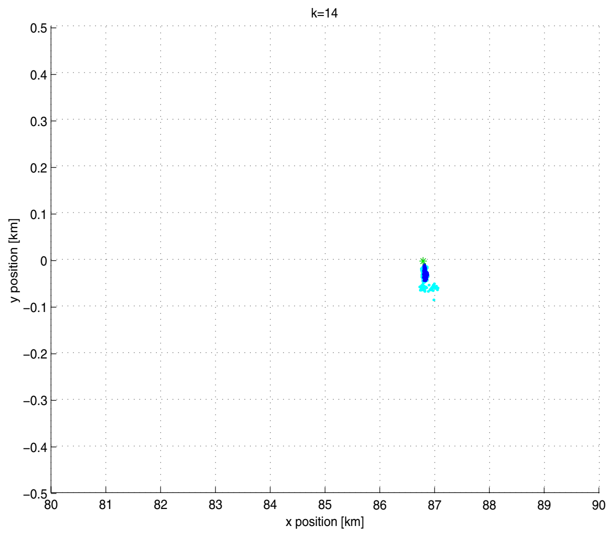

Figure 9: Particles at k=14, SNR=7dB

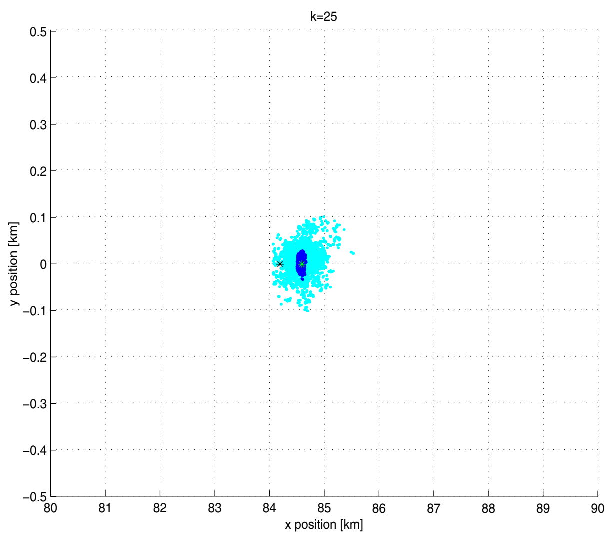

Figure 10: Particles at k=25, SNR=7dB

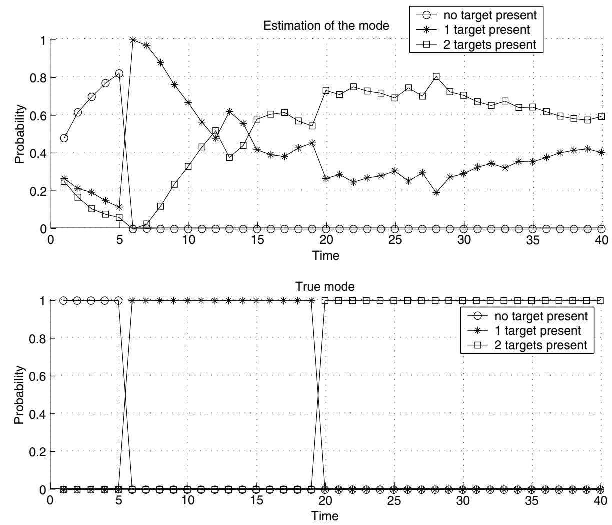

Figure 11: Mode probabilities, SNR=7dB

References

[1] S. Blackman and R. Popoli, *Design and Analysis of Modern Tracking Systems*, Artech House, Norwood, MA, 1999.

[2] Y. Bar-Shalom and X.R. Li. *Estimation and Tracking Principles, Techniques and Software*, Artech House, Norwood, MA, 1993.

[3] D. Crisan and A. Doucet. "A Survey of Convergence Results on Particle Filtering Methods", *IEEE Transactions on Signal Processing,* vol. 50, no. 3, pp. 736-746, 2002.

[4] A. Doucet, J.F.G. De Freitas and N.J. Gordon, Eds., *Sequential Monte Carlo Methods in Practice* Springer-Verlag, New York NY, 2001.

[5] N.J. Gordon, D.J. Salmond and A.F.M. Smith, "Novel approach to nonlinear/non-Gaussian Bayesian state estimation", *Proceedings IEE. F,* vol. 1400, no. 2, pp. 107-113, 1993.

[6] D.J. Salmond and H. Birch, A Particle Filter for Track-Before-Detect, *In Proceedings of the American Control Conference*, Arlington, VA, June 25-27, 2001.

[7] D.J. Ballantyne, H.Y. Chan and M.A. Kouritzin. A Branching Particle-Based Nonlinear Filter for Multi-Target tracking. *In Proceedings of the FUSION 2001 Conference*, Montreal, Canada, August 7-10, 2001.

[8] Y. Boers. On the number of samples to be drawn in particle filtering. *In proceedings of the IEE colloquium on Target Tracking*, London, UK, 1999.

[9] Y. Boers and J.N. Driessen, Particle Filter Based Detection For Tracking, *In Proceedings of the American Control Conference*, Arlington, VA, June 25-27, 2001.

[10] Y. Boers, J.N. Driessen and K. Grimmerink. Particle filter based detection schemes. *In Proceedings of the Small Targets Conference at SPIE AEROSENSE 2002*, Orlando, Fl., April 2-5, 2002.

[11] Y. Boers and J.N. Driessen. A Particle Filter Based Detection Scheme. Accepted for publication in *IEEE Signal Processing Letters*, 2003.

[12] Y. Boers and J.N. Driessen. Hybrid state estimation: a target tracking application. *AUTOMATICA*, vol. 38, no. 12, pp. 2153-2158, 2002.

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