ASSESSMENT OF NONLINEAR PREDICTIVE CONTROL TECHNIQUES FOR DC-DC CONVERTERS

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Abstract: This article focuses on the application of two nonlinear predictive control strategies to fixed frequency switch-mode dc-dc converters. The first approach uses an averaged nonlinear model for predictions and provides an a priori robust stability guarantee for the resulting closed-loop system. The implementation of this scheme requires solving a single linear program on-line. The second approach employs a piecewise affine (PWA) approximation of the converter equations as prediction model and yields an explicit controller off-line, which allows for an a posteriori stability check. An assessment of the synthesis procedures and methodological characteristics is given and simulation results are shown to substantiate the effectiveness of the proposed approaches.

Keywords: predictive control, power circuits, robustness.

1. INTRODUCTION

Model predictive control (MPC) has established itself as a widespread and systematic control method, specifically because it can be applied for complex, multivariable systems subject to constraints by directly formulating an approximate control model of the plant. As a consequence of these characteristics and due to theoretical advancements in systems theory and increased availability of computational power it has attracted interest in a diverse range of applications, e.g. power systems operation (Larsson, 2000), vehicle dynamics steering (Borrelli et al., 2001), fuel cell control (Vahidi et al., 2006) power electronics (Geyer and Papafotiou, 2005), and many more.

We are specifically interested in the field of power electronics in which one of the most extensively used classes of circuits are dc-dc converters, employed for example in electric and hybrid vehicles, solar plants, dc motor drives and regulated dc power supplies.

Existing control techniques for dc-dc converters mainly rely on PID controllers. A number of publications have appeared in the past few years, see, for example, (Geyer et al., 2004), (Lazar and Keyser, 2004), (Papafotiou et al., 2004), where the application of predictive control synthesis techniques for dc-dc converters was investigated.

The design of predictive controllers for dc-dc converters is a challenging task, due to the very small sampling period (modern low power converters run at frequencies in the MHz range) on one hand, which imposes stringent limits on the computational complexity, and the highly nonlinear/hybrid nature of the power circuit on the other hand, which makes it difficult to guarantee robust stability and performance. In view of these issues the presented work describes two nonlinear MPC design methodologies, the first based on the averaged model of the circuit and the second based on a PWA approximation of the converter equations.

The chosen topology is a fixed-frequency synchronous buck-boost dc-dc converter. The converter is driven via semiconductor switches operated by a pulse sequence with constant switching frequency. The main control objective is to act on the semiconductor switches in order to steer the dc component of the output voltage to a specified operating point, which must be maintained despite variations in the load and/or the voltage source.

The paper is organized as follows. In Section 2, the continuous time physical equations of the buck-boost converter are derived, whereas Section 3 presents the alternative discrete time models employed for control purposes. The specific control problem formulation is given in Section 4 and the proposed MPC approaches are described in Section 5. Simulation results and a brief assessment of the control schemes are shown in Section 6, and an outlook and conclusions are outlined in Section 7.

2. PHYSICAL MODEL OF THE CONVERTER

The *schematic* topology of the ideal (i.e. neglecting parasitics) synchronous buck-boost converter is shown in Fig. 1. The output load r_o is assumed to be ohmic, x_ℓ and x_c represent the inductance and the capacitance values of the converter components and v_s denotes the source voltage. The schematic comprises two switches S_1 and S_2 that are operated dually. The switches are driven by a pulse sequence of constant switching frequency f_s (period T_s), which characterizes the operation of the converter. The dc component of the output voltage can be regulated through the duty cycle (ratio) d, which is defined as $d := \frac{t_{on}}{T_s}$, where t_{on} represents the length of the interval within the switching period during which switch S_1 is in conduction; note that $0 \leq d \leq 1$ by definition. In the following a normalized time scale will be employed, with the time unit being equal to the switching period T_s; additionally, the discrete time instant k refers to (real) time instant $t = kT_s$.

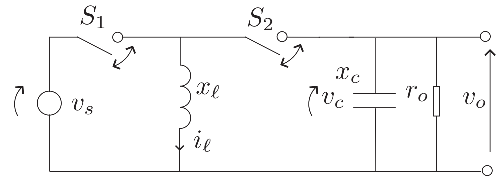

Fig. 1. Topology of the buck-boost converter

The state vector is naturally chosen as $x(t) = [i_\ell(t)\ v_c(t)]^\top$, comprising the inductor current and the capacitor voltage. The system is described by the following pair of affine continuous time state-space equations

$$\dot{x}(t) = \begin{cases} F_1 x(t) + f_1 v_s, & k \leqslant t < (k + d(k)) \\ F_2 x(t) + f_2 v_s, & (k + d(k)) \leqslant t < (k + 1) \end{cases}$$
$$v_o(t) = [0\ \ 1] x(t)$$

The matrices F_1, F_2 and the vectors f_1, f_2, are of appropriate dimensions and depend on the converter parameters through elementary circuit theory (Mohan et al., 1989). Notice that the evolution of the system is externally influenced *only* via the switches; the control input is therefore the duty cycle $d(k)$, which specifies the switching instant for period k. Furthermore, from an implementation point of view, it is preferable to measure the *output* voltage, rather than the capacitor voltage, as it is directly accessible. For the chosen ideal circuit configuration, with no parasitic components, it trivially holds that $v_c(t) = v_o(t)$, so that in this case the state vector can be redefined as $x(t) = [i_\ell(t)\ \ v_o(t)]^\top$. Notice that the presented approaches are not inherently limited by this assumption, which has been made only for the sake of simplicity.

3. MODELLING FOR CONTROL PURPOSES

3.1 Averaged Nonlinear Control Model

By taking a forward-Euler based average model (Lazar and Keyser, 2004) a discrete time approximation of the converter dynamics can be immediately formulated. Its expression amounts to

$$\begin{aligned} x(k+1) = x(k) &+ (F_1 x(k) + f_1 v_s) d(k) \\ &+ (F_2 x(k) + f_2 v_s)(1 - d(k)), \end{aligned} \tag{2}$$

which, when rearranged, yields a formulation of the type

$$x(k+1) = A x(k) + (B x(k) + D v_s) d(k) \tag{3}$$

where the matrices A, B and D depend on the converter parameters and the sampling period (Lazar and Keyser, 2004). Using the discrete time model (3), for a desired constant output voltage value $v_{o,ref}$, one can obtain the corresponding steady state duty cycle d_{ref} and inductor current

$i_{\ell,ref}$ and perform the following coordinate transformation on (3):

$$\bar{x}(k) := x(k) - x_{ref}, \quad u(k) := d(k) - d_{ref},$$

where $x_{ref} = [i_{\ell,ref} \; v_{o,ref}]^\top$. This yields the following system description

$$\bar{x}(k+1) = \bar{A}\bar{x}(k) + (\bar{B}\bar{x}(k) + \bar{D})u(k), \tag{4}$$

for some suitable matrices $\bar{A}$, $\bar{B}$ and $\bar{D}$. The first MPC algorithm uses (4) as prediction model.

3.2 Hybrid Control Model

To account for variations in the voltage source v_s directly, the (to be derived) optimal control law would need to be parameterized over v_s. To obviate this requirement and as will further be explained in Section 5.2, the voltage source v_s is removed from the model equations by defining the scaled state vector

$$x'(t) := [i'_\ell(t) \; v'_o(t)]^\top := \left[\frac{i_\ell(t)}{v_s} \; \frac{v_o(t)}{v_s}\right]^\top .$$

Next, a discrete time model is formulated by using a sampling interval equal to T_s. The employed method considers a direct least squares fitting (LSF) approximation over several regions of the control input of the exact system equations, yielding a PWA description of the associated nonlinear expressions (Beccuti et al., 2006). These can be written as

$$x'(k+1) = \Phi(d(k))x'(k) + \Gamma(d(k)) \tag{5}$$

where $\Phi(d(k))$ and $\Gamma(d(k))$ are matrices that depend nonlinearly on the duty cycle $d(k)$, calculated by integrating the converter equations from k to $k+1$. Expression (5) is approximated by determining the matrices $\bar{A}_i$, $\bar{B}_i$ and $\bar{f}_i$ that describe the system in terms of

$$x'(k+1) = \bar{A}_i x'(k) + \bar{B}_i d(k) + \bar{f}_i \tag{6a}$$

$$\text{if } d(k) \in D_i \quad i = 1, \ldots, \nu \tag{6b}$$

$$0 \le d(k) \le 1 \tag{6c}$$

and that minimize the sum of quadratic error terms

$$\big(\Phi(d(k))x'(k) + \Gamma(d(k)) - \big(\bar{A}_i x'(k) + \bar{B}_i d(k) + \bar{f}_i\big)\big)^2 \tag{7}$$

over a gridded series of points $x'(k)$ in the state-space region $[i'_{\ell,lower}, i'_{\ell,upper}] \times [v'_{o,lower}, v'_{o,upper}]$, where D_i are the ν intervals $[0, \frac{1}{\nu}], ..., [\frac{\nu-1}{\nu}, 1]$. The second MPC algorithm uses (6) as prediction model.

4. THE CONTROL PROBLEM

The main control objective for the buck-boost dc-dc converter is to regulate the dc component of the output voltage v_o to its reference $v_{o,ref}$ with an accuracy of $\pm 1\%$. This regulation has to be achieved in the presence of the hard constraints on the manipulated variable (the duty cycle) which is bounded between 0 and 1, and on the inductor current, which must not exceed its maximal value $i_{\ell,max}$ [1]. The regulation must also be maintained despite the changes in the load r_o and the source voltage v_s. Finally, the controller must render a steady state operation with constant duty cycle, thus avoiding the occurrence of fast-scale instabilities (subharmonic oscillations).

5. MPC SCHEMES

5.1 Averaged Nonlinear Model Predictive Control

Using the state transformation presented in Section 3.1, the control objective for the averaged nonlinear MPC scheme can be formulated as to robustly stabilize (4) around the equilibrium $[0 \; 0]^\top$ while fulfilling the following constraints (for any vector $x \in \mathbb{R}^n$ let x_i denote its i-th component):

$$\bar{x}_1(k) \in [\underline{b}^1, \overline{b}^1], \quad u(k) \in [\underline{b}^u, \overline{b}^u], \tag{8}$$

where $\underline{b}^1$, $\overline{b}^1$, $\underline{b}^u$ and $\overline{b}^u$ are suitable bounds. Let P, Q, R and P_V, Q_V denote known full-column rank matrices of appropriate dimensions, let $g(\bar{x}(k), u(k)) := \bar{A}\bar{x}(k) + (\bar{B}\bar{x}(k) + \bar{D})u(k)$ and let $\|\cdot\|$ denote the infinity norm. The averaged nonlinear MPC algorithm consist of the two following steps:

Step 1: At time $k \ge 0$ measure the state $x(k)$, let $\bar{x}(k) = x(k) - x_{ref}$ and minimize over u the cost

$$J(\bar{x}(k), u) := \|Pg(\bar{x}(k), u)\| + \|Q\bar{x}(k)\| + \|Ru\|,$$

subject to

$$\|P_V g(\bar{x}(k), u)\| - \|P_V \bar{x}(k)\| \le -\|Q_V \bar{x}(k)\|, \tag{9a}$$

$$g_1(\bar{x}(k), u) \in [\underline{b}^1, \; \overline{b}^1], \tag{9b}$$

$$u \in [\underline{u}, \; \overline{u}]. \tag{9c}$$

Step 2: Let $u^*(\bar{x}(k))$ be an optimal control obtained by solving the optimization problem defined at Step 1. Apply to the dc-dc converter the duty cycle $d(k) := u^*(\bar{x}(k)) + d_{ref}$, increment k by one and go to Step 1.

One of the attractive features of the proposed algorithm is that feasibility (instead of optimality) of the optimization problem in Step 1 is sufficient to guarantee input-to-state stability (ISS)

[1] Small violations are admissible in practice; rather, it is more important that i_ℓ should remain bounded.

for the closed-loop system. This can be proven by showing that $V(x) := \|P_V x\|$ is an ISS Lyapunov function (Lazar, 2006) for the MPC closed-loop system, i.e. there exist class $\mathcal{K}$ functions $\alpha_1, \alpha_2, \alpha_3$ and σ such that

$$\alpha_1(\|x\|) \leq V(x) \leq \alpha_2(\|x\|) \quad (10)$$

$$V(g(x, u^*(x))) - V(x) \leq -\alpha_3(\|x\|) + \sigma(\|w\|), \quad (11)$$

for all $x := \bar{x}(k)$ for which the optimization problem in Step 1 is feasible, and where w is an unknown additive disturbance that affects the system. Due to full-column rankness of P_V there exist $\zeta_{P_V}, \xi_{P_V} > 0$ such that $\xi_{P_V}\|x\| \leq \|P_V x\| \leq \zeta_{P_V}\|x\|$ for all x, which yields (10) for $\alpha_1(s) := \xi_{P_V} s$ and $\alpha_2(s) := \zeta_{P_V} s$. Furthermore, consider the case when a disturbance affects the system, i.e. $\bar{x}(k+1) = g(\bar{x}(k), u^*(\bar{x}(k))) + w(k)$, where $w(k)$ takes values in a bounded set containing the origin for all $k \geq 0$. Then, the "stabilization constraint" (9a) ensures (11) as follows:

$$\begin{aligned}
&V(\bar{x}(k+1)) - V(\bar{x}(k)) = \\
&\|P_V(g(\bar{x}(k), u^*(\bar{x}(k))) + w(k))\| - \|P_V\bar{x}(k)\| \leq \\
&\|P_V g(\bar{x}(k), u^*(\bar{x}(k)))\| + \|P_V w(k)\| - \|P_V\bar{x}(k)\| \\
&\leq -\|Q_V\bar{x}(k)\| + \|P_V w(k)\| \\
&\leq -\alpha_3(\|\bar{x}(k)\|) + \sigma(\|w(k)\|),
\end{aligned}$$

for $\alpha_3(s) := \xi_{Q_V} s$ (where ξ_{Q_V} satisfies $\|Q_V x\| \geq \xi_{Q_V}\|x\|$ for all x, which is possible as Q_V has full column rank) and $\sigma(s) := \zeta_{P_V} s$.

From a numerical point of view, the aforementioned algorithm has the following advantage. Since by definition $\|x\| = \max_{1\leq i\leq n}|x_i|$, where $|\cdot|$ denotes the absolute value, for a constraint $\|x\| \leq c$ with $c > 0$ to be satisfied, it is necessary and sufficient to require that $\pm x_i \leq c$ for all $1 \leq i \leq n$. Therefore, as $\bar{x}(k)$ in (9a) is just the measured state, which is known at every $k \geq 0$, for (9a) to be satisfied it is necessary and sufficient to impose $\pm\left(P_V(\bar{A}\bar{x}(k) + (\bar{B}\bar{x}(k) + \bar{D})u)\right)_i \leq \|P_V\bar{x}(k)\| - \|Q_V\bar{x}(k)\|$ for all $1 \leq i \leq n$, which yields $2n$ linear inequalities in the control variable u. Furthermore, instead of directly minimizing the cost $J(\bar{x}(k), u)$, one can introduce two auxiliary optimization variables ε_1 and ε_2 and solve the following optimization problem instead:

$$\begin{aligned}
&\min \ \varepsilon_1 + \varepsilon_2 \\
&\text{subject to} \ \ \varepsilon_1 \geq 0, \quad \varepsilon_2 \geq 0, \\
&\qquad -\varepsilon_1 \leq \|Pg(\bar{x}(k), u)\| + \|Q\bar{x}(k)\| \leq \varepsilon_1, \\
&\qquad -\varepsilon_2 \leq \|Ru\| \leq \varepsilon_2.
\end{aligned}$$

Using the technique for rewriting inequalities that include ∞-norms as linear inequalities in combination with the above optimization problem, one can formulate a simple linear program (LP) whose solution minimizes the cost $J(\bar{x}(k), u)$ and satisfies all constraints in (9). This makes the proposed algorithm viable for controller implementation for fast nonlinear systems, such as dc-dc converters. For more details and additional results on ∞-norm based Lyapunov functions the reader is referred to (Christophersen and Morari, 2007).

5.2 Hybrid Model Predictive Control

The constraint on the inductor current can be formulated by imposing

$$i'_\ell(k) + \frac{d(k)}{x_\ell} \leq i'_{\ell,max} \quad (12)$$

wherein the expression on the left hand side can be shown to yield the value of the peak inductor current during period k and where $i'_{\ell,max}$ denotes the scaled inductor current limit.

The main control objective is to regulate the output voltage to its reference as fast and with as little overshoot as possible, or equivalently, to minimize the absolute scaled output voltage error $v'_{o,err}(k) = |v'_o(k) - v'_{o,ref}|$. Additionally, let $\Delta d(k) = |d(k) - d(k-1)|$ indicate the absolute value of the difference between two consecutive duty cycles. This term is introduced in order to reduce the presence of unwanted chattering in the input when the system has almost reached stationary conditions. Define the penalty matrix $Q = \text{diag}(q_1, q_2)$ with $q_1, q_2 > 0$ and the vector $\varepsilon(k) = [v'_{o,err}(k) \ \Delta d(k)]^\top$. Consider the objective function

$$J(D(k), x'(k), d(k-1)) = \sum_{l=0}^{L-1} \|Q \ \varepsilon(k+l|k)\|_1 \quad (13)$$

penalizing the predicted evolution of $\varepsilon(k+l|k)$ from k over the horizon L using the 1-norm; move blocking is employed for the sequence of control moves $D(k) = [d(k), \ldots, d(k+L-1)]^T$ throughout the horizon to reduce the computational burden. The control input at instant k is then obtained by minimizing the objective function (13) over $D(k)$ subject to the model equations and constraints (6a), (6b), (6c), (12); the resulting optimization program is referred to as the constrained finite time optimal control (CFTOC) problem.

In (Borrelli, 2003) it is shown how to reformulate and solve a discrete-time CFTOC problem for a PWA system as a multi-parametric program by treating the state vector as a parameter. Note that the CFTOC problem corresponding to (13) is not only a parametric function of $x(k)$, but also of the last control move $d(k-1)$. Furthermore, as it is necessary to solve the CFTOC problem for all possible values of $v'_{o,ref}$ and $i'_{\ell,max}$, these values also enter the augmented state vector, which therefore results into a 5-dimensional augmented state vector. Again, it should be noticed that normalizing the system equations over v_s allows one to define a model independently of the more complex parameterization that would be required for the voltage source, and therefore to obtain a simpler explicit state-feedback law, see (Papafotiou et al., 2004) for details.

As proven in (Borrelli, 2003) the optimal state-feedback control law is a PWA function of the (augmented) state vector defined on a polyhedral partition of the feasible (augmented) state space. As a result, such a state-feedback controller facilitates controller implementation, since computing the control input does not require any on-line optimization but simply amounts to determining the polyhedron in which the measured state lies and then simply evaluating the corresponding affine control law.

Additionally, through an a posteriori analysis based on the explicit controller, a Lyapunov function can be sought (Lazar, 2006) by closing the control loop with the PWA model (6a), (6b). This leads by definition to a PWA autonomous system for which, in the case of the nominal setup presented in Section 6 a piecewise quadratic (PWQ) Lyapunov function proving the global exponential stability of the overall system can be found.

6. SIMULATION RESULTS AND ASSESSMENT OF CONTROL SCHEMES

The proposed control strategies are tested on a common buck-boost dc-dc converter with $f_s = 10$ kHz, which yields the sampling period $T_s = 0.1$ milliseconds. The output voltage reference is set to -8 V, the maximum admissible current is 2.3A and the circuit parameters are given by $x_c = 2200\mu$F and $x_\ell = 4.2$mH. The source voltage and output load are initially set to 15V and 165Ω. Two simulation scenarios are considered, namely:

(1) The first scenario concerns the start-up of the converter from zero initial conditions, i.e. with initial state $x(0) = [0 \;\; 0]^\top$ to the desired steady state operation point.
(2) The second scenario examines the response of the controller to step changes in the source voltage and output load. Starting from the previous steady state, the load decreases by 50 % at $t = 0.015$ (i.e. after 150 switching periods) and the voltage source increases by 50 % at $t = 0.022$ (i.e. after 220 switching periods).

6.1 Averaged Nonlinear Model Predictive Control

To implement the MPC scheme summarized in Section 5.1, we first obtain the model (4) in the transformed coordinates for the steady state output voltage $v_{o,ref} = -8$V. Then, we compute off-line the weight P_V of the artificial Lyapunov function $V(x) = \|P_V x\|$ for $Q_V = 0.001 I_2$, using a method presented in (Lazar, 2006), which yields the solution $P_V = \left[\begin{smallmatrix} 2.2577 & -1.1527 \\ -0.2156 & 3.8191 \end{smallmatrix}\right]$. The MPC cost matrices have been chosen as follows, to ensure a good performance: $P = Q = \left[\begin{smallmatrix} 1 & 0 \\ 0 & 4 \end{smallmatrix}\right]$, $R = 0.5$. For the prediction model, $v_s = 15$ and $r_o = 165$ at all times during the simulation.

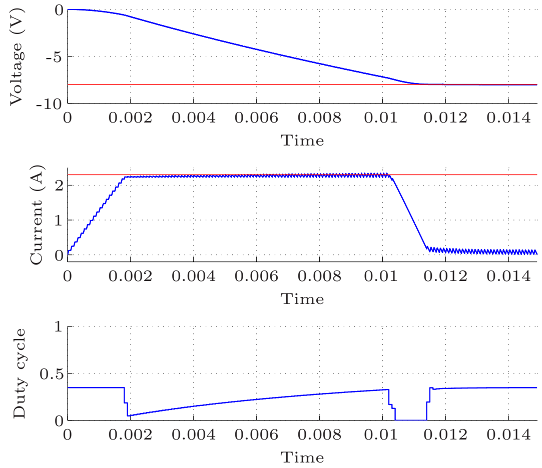

Fig. 2. Averaged Nonlinear MPC: Scenario (1)

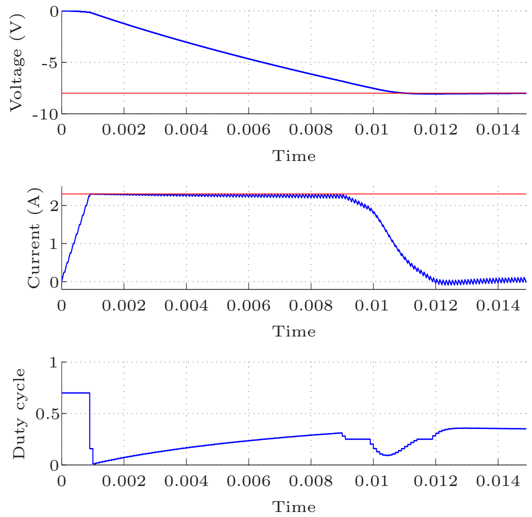

Fig. 3. Hybrid MPC: Scenario (1)

6.2 Hybrid Model Predictive Control

The model was derived for a range of values of $[-1, 6] \times [0, 3]$ for the scaled inductor current and output voltage; three affine dynamics were calculated, with the intervals D_i being $[0, 0.25]$, $[0.25, 0.7]$, and $[0.7, 1]$. For the cost function, the penalty matrix was taken as $Q = \text{diag}(10, 1)$ and the horizon set to $L = 15$. The explicit state-feedback controller has 189 regions (in the 5-dimensional space). To account for load resistance variations, an estimation scheme was incorporated in the control loop, as presented in (Beccuti et al., 2006) .

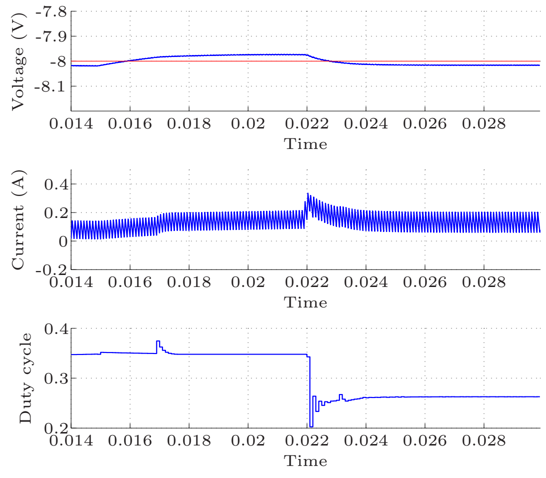

Fig. 4. Averaged Nonlinear MPC: Scenario (2)

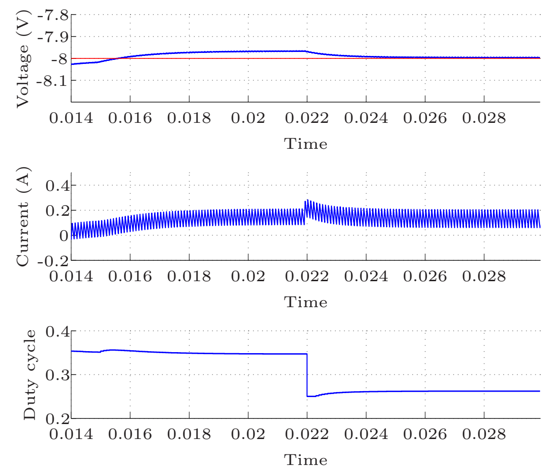

Fig. 5. Hybrid MPC: Scenario (2)

6.3 Assessment of Control Schemes and Results

The two presented control schemes differ fundamentally inasmuch as the first scheme includes robust stability constraints in its synthesis, whereas the second controller does not feature any such guarantee and relies on an a posteriori analysis to verify the resulting closed loop properties; ensuring stability a priori however intrinsically requires regulating the system to a specific operating point in the state space. These facts are mirrored in the setup chosen for the two schemes, since the first controller is based on a one step prediction (so that the averaged nonlinear model actually only yields an LP in the resulting optimal control problem), which is sufficient because of the aforementioned constraints, but is defined only for a fixed output voltage reference. The second scheme viceversa requires a considerably longer horizon ($L = 15$) to capture the non-minimum phase dynamics of the converter but features $v'_{o,ref}$ as a parameter that may take on any value in its admissible range. The resulting closed loop performance is in any case very similar: as shown in Fig. 2 and Fig. 3 both controllers steer the output voltage to its reference with virtually no undershoot whilst respecting the current constraint and within slightly more than 100 sampling periods (i.e. 0.01 seconds). For the second scenario the results are shown in Fig. 4 and Fig. 5, respectively; despite the applied disturbances the required accuracy is consistently maintained.

7. CONCLUSIONS

Two alternative MPC strategies for controlling dc-dc converters have been presented. The first approach uses an averaged nonlinear model, while the second approach employs a PWA approximation of the converter equations as prediction model; simulation results show the effectiveness of the proposed approaches. For future work it would be interesting to assess the real-time implementation feasibility of the two approaches. Additionally, the PWA converter model can be used in combination with the first MPC scheme presented. Then, an explicit solution can also be obtained for the optimization problem defined therein, leading to a robust explicit controller of low complexity.

8. ACKNOWLEDGEMENTS

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