

An Approach to Observer-Based Decentralized Control under Periodic Protocols

N.W. Bauer, M.C.F. Donkers, W.P.M.H. Heemels, N. van de Wouw

Abstract—This paper provides an approach to analyze and design decentralized observer-based controllers for large-scale linear plants subject to network communication constraints and varying sampling intervals. Due to communication constraints, it is impossible to transmit all input and output data simultaneously over the communication network that connects sensors, actuators and controllers. A protocol orchestrates what data is sent over the network at each transmission instant. To handle these communication constraints, it is fruitful to adopt a switched observer structure that switches based on the transmitted information. By taking a discrete-time switched linear system perspective, we are able to derive a general model that captures all these aspects and provides insight into how they influence each other. Focusing on the class of so-called ‘periodic protocols’ (of which the well-known Round Robin protocol is a special case), we provide a method to assess robust stability using a polytopic overapproximation and LMI-based stability conditions. Although the design problem is in general non-convex, we provide a procedure to find stabilizing control laws by simplifying the control problem. The design of the controller exploits the periodicity of protocols and ignores the global coupling between subsystems of the plant and variation of the sampling intervals. To assess the robust stability of the resulting closed-loop system including the ignored effects, an *a posteriori* analysis is conducted based on the derived LMIs.

I. INTRODUCTION

Recently, there has been an enormous interest in control of large-scale systems that are physically distributed over a wide area. Examples of such distributed systems are electrical power distribution networks, water transportation networks, industrial factories and energy collection networks (such as wind farms). This work considers stability analysis and controller design for this class of systems. This problem setting has a number of features that seriously challenge the controller design.

Firstly, the controller is decentralized in the sense that it consists of a number of local controllers that do not share information. Although a centralized controller could alternatively be considered, the achievable bandwidth associated with using a centralized control structure would be limited by long delays induced by the communication between the centralized controller and distant sensors and actuators over a (e.g. wireless) communication network [1].

Secondly, when considering control of a large-scale system, it would be unreasonable to assume that all states are

measured. Therefore an output-based controller is needed. In particular, we consider an observer-based control setup. Note that an observer-based controller offers the advantage of reducing the number of sensors, which alleviates the demands on the network design. However, it has been proven that, in general, it is hard to obtain decentralized observers providing state estimate converging to the ‘true’ states [11].

Finally, the observer-based controller needs to have certain robustness properties when using a communication network. Indeed, the advantages of using a wired/wireless network are inexpensive and easily modifiable communication links. However, the drawback is that the control system is susceptible to undesirable (possibly destabilizing) side-effects see e.g. [6], [15]. There are roughly five recognized Networked Control System (NCS) side-effects: time-varying delays, packet dropouts, varying sampling intervals, quantization and communication constraints (the latter meaning that not all information can be sent over the network at once). For modeling simplicity, we only consider varying sampling times and communication constraints in this work.

Resuming, we note that although this decentralized observer-based control structure is reasonable to use, its design is extremely complex due to the fact that we simultaneously face the issues of (i) a decentralized control structure (ii) limited measurement information and (iii) communication side-effects. The contribution of this paper is threefold: a model describing the controller decentralization and the communication side-effects is derived for analysis, a way to assess robust stability of the closed loop in the face of communication imperfections is given and an approach towards the design of observer-based controllers is provided.

The outline of this paper is as follows: In Section II the general problem description and the closed-loop model will be constructed. Construction of the model covers the plant decomposition needed to establish a decentralized controller structure, the network constraints and the descriptions of the observer-based decentralized control design. We will then propose LMI-based stability conditions in Section III. In Section IV we present a constructive design procedure for these decentralized observer-based controllers for the case of periodic communication protocols. Finally an example will be presented in Section V and some suggestions for future work will be discussed in Section VI.

A. Nomenclature

The following notational conventions will be used. $\text{diag}(A_1, \dots, A_N)$ denotes a block-diagonal matrix with the matrices A_1, \dots, A_N on the diagonal and $A^\top \in \mathbb{R}^{m \times n}$

This work is supported by the European Community through the FP7-ICT-2007-2 thematic programme under the WIDE-224168 project.

Nicolas Bauer, Tijs Donkers, Maurice Heemels and Nathan van de Wouw are with the Department of Mechanical Engineering, Eindhoven University of Technology, P.O.Box 513, 5600 MB Eindhoven, The Netherlands, {n.w.bauer, m.c.f.donkers, m.heemels, n.v.d.wouw}@tue.nl.

denotes the transpose of the matrix $A \in \mathbb{R}^{n \times m}$. For a vector $x \in \mathbb{R}^n$, we denote $\|x\| := \sqrt{x^\top x}$ its Euclidean norm. We denote by $\|A\| := \sqrt{\lambda_{\max}(A^\top A)}$ the spectral norm of a matrix A , which is the square-root of the maximum eigenvalue of the matrix $A^\top A$. For brevity, we sometimes write symmetric matrices of the form $\begin{bmatrix} A & B \\ B^\top & C \end{bmatrix}$ as $\begin{bmatrix} A & B \\ \star & C \end{bmatrix}$.

II. THE MODEL & PROBLEM DEFINITION

We consider a continuous-time linear plant:

$$\begin{cases} \dot{x}(t) = Ax(t) + B\hat{u}(t) \\ y(t) = Cx(t) \end{cases} \quad (1)$$

with state $x \in \mathbb{R}^n$, control input $\hat{u} \in \mathbb{R}^m$ and measured output $y \in \mathbb{R}^p$. The goal of the paper is to present an approach for the analysis and design of a stabilizing controller that has the following features:

- discrete-time;
- decentralized;
- output-based;
- robust with respect to uncertain time-varying sampling intervals $h_k \in [\underline{h}, \bar{h}]$ for all $k \in \mathbb{N}$;
- communication constrained: not all outputs and inputs can be communicated simultaneously and a protocol schedules which information is sent at a transmission instant

The decentralized controllers $C^{(i)}$, $i = 1, \dots, N$, communicate with the sensors and actuators of the plant via a shared network. The general setup is depicted in Fig. 1.

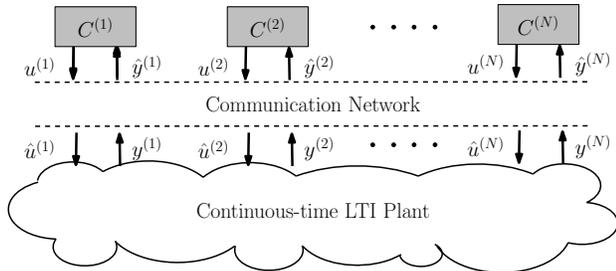


Fig. 1. Decentralized Networked Control System.

In this paper, the plant will be divided into subsystems, each of which, are controlled by a discrete-time observer-based controller whose subsystem model is based on a nominal sampling interval. In Section II-A, we determine a nominal sampling interval with a corresponding plant discretization and present a decomposition of the plant. In Section II-B a description of the network imperfections is provided. In Section II-C a switching observer-based control structure will be presented and, finally, in Section II-D a closed-loop model suitable for stability analysis is derived.

A. Plant Decomposition

Since we are aiming to design N model-based discrete-time linear observer-based controllers, the continuous-time plant needs to be divided into N discrete-time subsystems to use as sub-models. First, we will discretize the continuous-time plant, after which, the states of the continuous-time plant will be partitioned, leading to N disjoint discrete-time systems.

1) *Plant Discretization*: It is well known that a linear continuous-time system (1) with a zero-order-hold assumption on the inputs $\hat{u}(t)$ can be exactly discretized to

$$\mathcal{P} := \begin{cases} x_{k+1} = A^* x_k + B^* \hat{u}_k \\ y_k = C x_k \end{cases}, \quad k \in \mathbb{N}_{\geq 0}, \quad (2)$$

where h_* is a suitably chosen nominal constant sampling interval, $A^* := e^{A h_*}$ and $B^* := \int_0^{h_*} e^{A s} ds B$. In (2), $x_k = x(t_k)$, $y_k = y(t_k)$, with t_k the sampling instants, and \hat{u}_k is the discrete-time control action available at the plant at $t = t_k$, i.e. $\hat{u}(t) = \hat{u}_k$ for all $t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$.

2) *Plant Decomposition*: The system \mathcal{P} in (2) will be decomposed into N interconnected subsystems. Choosing the decomposition is a challenging task, as there are many aspects to be taken into account, i.e. sensor/actuator physical location, subsystem interaction, computational effort of the control law, input/output relations, etc. For *decentralized* design, reducing the interactions between subsystems is highly desired since smaller interaction will generally increase the likelihood of the decentralized controllers being successful. Considering that the goal of the decomposition is keeping the interaction between the subsystems small (while maintaining a minimal number of subsystems), we propose to use the ϵ -decomposition technique [11].

The ϵ -decomposition is an algorithm for finding a permutation matrix P such that *each element* of all subsystem coupling matrices between subsystems, has magnitude no greater than $\epsilon \geq 0$. This algorithm offers the advantage of only searching for permutations of a system, which preserves any physical meaning of the original state vector. By using this algorithm we can find a P that expresses the entire plant as a collection of interconnected (coupled) subsystems

$$\mathcal{P}^{(i)} := \begin{cases} z_{k+1}^{(i)} = \bar{A}_i z_k^{(i)} + \bar{B}_i \hat{u}_k^{(i)} \\ \quad + \sum_{j=1, j \neq i}^N (\bar{A}_{i,j} z_k^{(j)} + \bar{B}_{i,j} \hat{u}_k^{(j)}) \\ y_k^{(i)} = \bar{C}_i z_k^{(i)} + \sum_{j=1, j \neq i}^N \bar{C}_{i,j} z_k^{(j)} \end{cases}, \quad (3)$$

for $i = 1, \dots, N$, where $z = P^{-1}x$ is the state vector of the permuted system $\bar{A} = P^{-1}A^*P$, $\bar{B} = P^{-1}B^*$, $\bar{C} = CP$, $u_k^{(i)} \in \mathbb{R}^{m_i}$, and $y_k^{(i)} \in \mathbb{R}^{p_i}$. Without loss of generality, we only consider disjoint decompositions, that is, $z = (z^{(1)\top}, z^{(2)\top}, \dots, z^{(N)\top})^\top$, $u = (u^{(1)\top}, u^{(2)\top}, \dots, u^{(N)\top})^\top$, $y = (y^{(1)\top}, y^{(2)\top}, \dots, y^{(N)\top})^\top$. As such, every state, output and input are attributed to only one subsystem and the subsystem interaction matrices are denoted $\bar{A}_{i,j}, \bar{B}_{i,j}, \bar{C}_{i,j}$, $j \neq i$. Throughout this paper we use the decomposition $\bar{A} = A_d + A_c$, where $A_d := \text{diag}(\bar{A}_1, \bar{A}_2, \dots, \bar{A}_N)$. The \bar{B}, \bar{C} matrices can be expressed similarly. With this notation, we can equivalently express (3) as

$$\mathcal{P} = \begin{cases} z_{k+1} = A_d z_k + B_d \hat{u}_k + (A_c z_k + B_c \hat{u}_k) \\ y_k = C_d z_k + C_c z_k \end{cases}.$$

The control structure for a chosen decomposition is depicted in Fig. 2, where the i^{th} controller is controlling only the i^{th} subsystem.

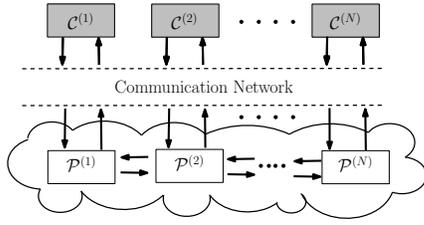


Fig. 2. Decentralized NCS After Decomposition

B. Network Description

Communication between sensors, actuators and controllers will take place via a shared network, see Fig. 2. Here, we will consider two network effects: namely, varying sampling intervals and communication constraints, where the latter imposes the need for a scheduling protocol to determine what input and output data is transmitted at each sampling time.

In Section II-A, we assumed a constant sampling interval h_* to arrive at subsystem models used by the controller. However, due to the nature of the network, the actual sampling times $t_k, k \in \mathbb{N}$, are not necessarily equidistant in time. Assuming that the sampling intervals $h_k = t_{k+1} - t_k$ are contained in $[\underline{h}, \bar{h}]$ for some $0 \leq \underline{h} \leq \bar{h}$, i.e. $h_k \in [\underline{h}, \bar{h}]$ for all $k \in \mathbb{N}$, the exact discrete-time equivalent of (1), after the permutation, is

$$\mathcal{P}_{h_k} := \begin{cases} z_{k+1} &= \bar{A}_{h_k} z_k + \bar{B}_{h_k} \hat{u}_k \\ y_k &= \bar{C} z_k, \end{cases} \quad (4)$$

where $\bar{A}_{h_k} := P^{-1} e^{A h_k} P$, $\bar{B}_{h_k} := P^{-1} \int_0^{h_k} e^{A s} ds B$ and $\bar{C} := CP$. It is important to note that the observer-based controllers will use subsystem models that are based on the constant sampling interval h_* , so variation in the sampling interval prevent the state estimation error from converging to zero.

Since the plant and controller are communicating through a network with communication constraints, the actual input of the plant $\hat{u}_k \in \mathbb{R}^m$ is not equal to the controller output u_k and the actual input of the controller $\hat{y}_k \in \mathbb{R}^p$ is not equal to the plant output $y_k \in \mathbb{R}^p$. Instead, \hat{u}_k and \hat{y}_k are networked versions of u_k and y_k , respectively.

To explain the effect of communication constraints and thus the difference between \hat{y}_k and y_k and \hat{u}_k and u_k one has to realize that the plant has n_u sensors and n_y actuators. These sensors and actuators are grouped into $n_T \leq n_u + n_y$ nodes. At each sampling time t_k , one node obtains access to the network and transmits its corresponding u and/or y values. Only the transmitted values will be updated, while all other values remain unchanged. This means the constrained data exchange can be expressed as

$$\begin{cases} \hat{u}_k &= \Gamma_{\sigma_k}^u u_k + (I - \Gamma_{\sigma_k}^u) \hat{u}_{k-1} \\ \hat{y}_k &= \Gamma_{\sigma_k}^y y_k + (I - \Gamma_{\sigma_k}^y) \hat{y}_{k-1}, \end{cases} \quad (5)$$

where $\Gamma_l^u \in \mathbb{R}^{m \times m}$ and $\Gamma_l^y \in \mathbb{R}^{p \times p}$, for $l = 1, \dots, n_T$, are diagonal matrices where the j^{th} diagonal value is 1 if the j^{th} input or output, respectively, belongs to node l and zero elsewhere. Without loss of generality, we will assume the matrices Γ_l^u and Γ_l^y can be divided in $\Gamma_{i,l}^u$ and $\Gamma_{i,l}^y$, for

$i \in \{1, \dots, N\}$ such that $\Gamma_l^u := \text{diag}(\Gamma_{1,l}^u, \Gamma_{2,l}^u, \dots, \Gamma_{N,l}^u)$ and $\Gamma_l^y := \text{diag}(\Gamma_{1,l}^y, \Gamma_{2,l}^y, \dots, \Gamma_{N,l}^y)$, where $\Gamma_{i,l}^u \in \mathbb{R}^{m_i \times m_i}$ and $\Gamma_{i,l}^y \in \mathbb{R}^{p_i \times p_i}$ are matrices corresponding to inputs and outputs, respectively, of the i^{th} subsystem.

The value of $\sigma_k \in \{1, 2, \dots, n_T\}$ indicates which node is given access to the network. The switching functions determining σ_k are known as protocols. In this paper we focus on the general class of periodic protocols [4], which are characterized by $\sigma_{k+\tilde{N}} = \sigma_k$ for some period $\tilde{N} \geq n_T$, $\tilde{N} \in \mathbb{N}$. The well-known Round Robin protocol [12] belongs to this class of periodic protocols.

Finally, we introduce the network-induced errors

$$\begin{cases} e_k^u &:= \hat{u}_{k-1} - u_k \\ e_k^y &:= \hat{y}_{k-1} - y_k, \end{cases} \quad (6)$$

where e_k^u and e_k^y will be referred to as the (network-induced) input error and output error, respectively.

C. Decentralized Networked Observer-Based Controllers

In this paper we will use decentralized observer-based controllers in the sense that for each subsystem of the plant we have one observer-based controller and the controllers do not exchange information. Therefore, the individual observers have no information about externally coupled states. As a consequence, it is desired to ensure that coupling between subsystems is minimal since ignored coupled dynamics will act as an unknown disturbance input to the decoupled observers. Furthermore, the model-based controllers will adopt switching gains to deal with the communication constraints effectively. The i^{th} networked observer-based controller is given by

$$\mathcal{C}_{\sigma_k}^{(i)} := \begin{cases} \tilde{z}_{k+1}^{(i)} &= \bar{A}_i \tilde{z}_k^{(i)} + \bar{B}_i \hat{u}_k^{(i)} \\ &+ L_{i,\sigma_k} \Gamma_{i,\sigma_k}^y (\hat{y}_k^{(i)} - \bar{C}_i \tilde{z}_k^{(i)}) \\ u_k^{(i)} &= -K_{i,\sigma_k} \tilde{z}_k^{(i)}, \end{cases} \quad (7)$$

where $\tilde{z}_{k+1}^{(i)}$ represents the state estimate of the i^{th} observer at time $k+1$ for the plant state $z_{k+1}^{(i)}$ and the output injection matrices L_{i,σ_k} , $i \in \{1, \dots, N\}$, $\sigma_k \in \{1, \dots, n_T\}$ will be designed to stabilize the dynamics of the state estimation error $\eta_k := \tilde{z}_k - z_k$. We adopt a switched-observer structure (notice the σ_k -dependence in (7)) to deal with the presence of communication constraints. Switched observers have received much attention in the past decade [2], [13], [14]. Γ_{i,σ_k}^y in (7) is used so that the standard output injection is only applied to the newly received measurements. If no measurements are received ($\Gamma_{i,\sigma_k}^y = 0$ for some σ_k) then (7) reduces to a standard model-based prediction step.

Similar to the plant, the dynamics of all the controllers (7) can be described by a single discrete-time system, which will consist of block diagonal matrices due to the decoupled nature of the controllers

$$\mathcal{C}_{\sigma_k} = \begin{cases} \tilde{z}_{k+1} &= A_d \tilde{z}_k + B_d \hat{u}_k \\ &+ L_{\sigma_k} \Gamma_{\sigma_k}^y (\hat{y}_k - C_d \tilde{z}_k) \\ u_k &= -K_{\sigma_k} \tilde{z}_k, \end{cases} \quad (8)$$

where $L_{\sigma_k} = \text{diag}(L_{1,\sigma_k}, L_{2,\sigma_k}, \dots, L_{N,\sigma_k})$ and $K_{\sigma_k} = \text{diag}(K_{1,\sigma_k}, K_{2,\sigma_k}, \dots, K_{N,\sigma_k})$.

D. Closed-Loop Model

To derive an expression for the closed loop, we will adopt the state vector $\bar{x}_k = [\eta_k^\top \ z_k^\top \ e_k^u{}^\top \ e_k^y{}^\top]^\top$. Combining (4), (5), (6), and (8) the entire closed-loop system can be represented by the following switched uncertain discrete-time system

$$\bar{x}_{k+1} = \tilde{A}_{c,h_k,\sigma_k} \bar{x}_k, \quad (9)$$

where $\tilde{A}_{c,h_k,\sigma_k}$ is given by (10) with

$$\begin{aligned} \Delta A_{c,h_k} &:= \bar{A}_{h_k} - A_d = (\bar{A}_{h_k} - \bar{A}) + A_c \\ \Delta B_{c,h_k} &:= \bar{B}_{h_k} - B_d = (\bar{B}_{h_k} - \bar{B}) + B_c. \end{aligned}$$

The $\Delta A_{c,h_k}$ term consists of two terms being $(\bar{A}_{h_k} - \bar{A})$, which is caused by a ‘clock skew’ ($h_k - h_*$) effect, added to A_c , which is caused by the subsystem coupling. The same applies to $\Delta B_{c,h_k}$. The ‘clock skew’ effect is an arbitrarily time-varying term (due to time-varying sampling intervals) while the ‘neglected coupling’ effect is a deterministic disturbance, which is the result of both the nominal sampling time and the chosen decomposition.

III. STABILITY ANALYSIS

In this section, we analyze whether the system (9), (10), with given K_{σ_k} and L_{σ_k} , is stable for some given bounds on the sampling interval, i.e. $h_k \in [\underline{h}, \bar{h}]$ for all $k \in \mathbb{N}$. The stability analysis is based on the ideas in [3], in which stability of networked control systems is discussed. As in [3], the uncertain parameter h_k , $k \in \mathbb{N}$ appears nonlinearly in (10) through \bar{A}_{h_k} and \bar{B}_{h_k} . To make the system amenable for analysis, a procedure is proposed to overapproximate system (9), (10) by a polytopic system with norm-bounded additive uncertainty, i.e.,

$$\bar{x}_{k+1} = \sum_{j=1}^M \alpha_k^j (F_{\sigma_k,j} + G_j \Delta_k H_{\sigma_k}) \bar{x}_k, \quad (11)$$

where $F_{l,j} \in \mathbb{R}^{n \times n}$, $G_j \in \mathbb{R}^{n \times q}$, $H_l \in \mathbb{R}^{q \times n}$, for $l \in \{1, \dots, n_T\}$ and $j \in \{1, \dots, M\}$, with M the number of vertices of the polytope. The vector $\alpha_k = [\alpha_k^1 \ \dots \ \alpha_k^M]^\top \in \mathcal{A}$, $k \in \mathbb{N}$, is time-varying with

$$\mathcal{A} = \left\{ \alpha \in \mathbb{R}^M \mid \sum_{j=1}^M \alpha^j = 1 \text{ and } \alpha^j \geq 0 \right. \\ \left. \text{for } j \in \{1, \dots, M\} \right\}$$

and $\Delta_k \in \mathbf{\Delta}$, where $\mathbf{\Delta}$ is a norm-bounded set of matrices in $\mathbb{R}^{q \times q}$ that describes the additive uncertainty. Equation (11)

is an overapproximation of (9) in the sense that for all $l \in \{1, \dots, n_T\}$, it holds that

$$\left\{ \tilde{A}_{c,h,l} \mid h \in [\underline{h}, \bar{h}] \right\} \\ \subseteq \left\{ \sum_{j=1}^M \alpha^j (F_{l,j} + G_j \Delta H_l) \mid \alpha \in \mathcal{A}, \Delta \in \mathbf{\Delta} \right\}. \quad (12)$$

We now provide a gridding-based procedure to overapproximate system (9), such that (12) holds, after which we can provide conditions for stability.

Procedure 1

- Select M distinct sampling intervals $\tilde{h}_1, \dots, \tilde{h}_M$ as grid points, such that $\underline{h} =: \tilde{h}_1 \leq \tilde{h}_2 < \dots < \tilde{h}_{M-1} \leq \bar{h}_M =: \bar{h}$.
- Define

$$F_{l,j} := \tilde{A}_{c,\tilde{h}_j,l}.$$

Decompose the matrix A , as given in (1), into its real Jordan form [7], i.e. $A := T \Lambda T^{-1}$, where T is an invertible matrix and

$$\Lambda = \text{diag}(\Lambda_1, \dots, \Lambda_L)$$

with $\Lambda_i \in \mathbb{R}^{n_i \times n_i}$, $i \in \{1, \dots, L\}$, the i -th real Jordan block of A .

- Compute for each line segment $S_m = [\tilde{h}_m, \tilde{h}_{m+1}]$, $m \in \{1, \dots, M-1\}$, and for each real Jordan block Λ_i , $i \in \{1, \dots, L\}$, the worst case approximation error, i.e.

$$\tilde{\delta}_{i,m}^A = \sup_{\substack{\bar{\alpha}^1 + \bar{\alpha}^2 = 1, \\ \bar{\alpha}^1, \bar{\alpha}^2 \geq 0}} \left\| e^{\Lambda_i(\bar{\alpha}^1 \tilde{h}_m + \bar{\alpha}^2 \tilde{h}_{m+1})} - \sum_{j=1}^2 \bar{\alpha}^j e^{\Lambda_i \tilde{h}_{m+j-1}} \right\|, \quad (13a)$$

$$\tilde{\delta}_{i,m}^E = \sup_{\substack{\bar{\alpha}^1 + \bar{\alpha}^2 = 1, \\ \bar{\alpha}^1, \bar{\alpha}^2 \geq 0}} \left\| \sum_{j=1}^2 \bar{\alpha}^j \int_{\tilde{h}_{m+j-1}}^{\bar{\alpha}^1 \tilde{h}_m + \bar{\alpha}^2 \tilde{h}_{m+1}} e^{\Lambda_i s} ds \right\|. \quad (13b)$$

For a detailed explanation of the origin of the approximation error bounds, see [3].

- Map the obtained bounds (13) at each line segment S_m , $m \in \{1, \dots, M-1\}$, for each Jordan block Λ_i , $i \in \{1, \dots, L\}$, to their corresponding vertices $j \in \{1, \dots, M\}$, according to

$$\delta_{j,i}^A = \begin{cases} \tilde{\delta}_{j,i}^A & i \in \{1, N\}, \\ \max\{\tilde{\delta}_{j,i-1}^A, \tilde{\delta}_{j,i}^A\} & i \in \{2, \dots, N-1\}, \end{cases}$$

$$\delta_{j,i}^E = \begin{cases} \tilde{\delta}_{j,i}^E & i \in \{1, N\}, \\ \max\{\tilde{\delta}_{j,i-1}^E, \tilde{\delta}_{j,i}^E\} & i \in \{2, \dots, N-1\}. \end{cases}$$

$$\tilde{A}_{c,h_k,\sigma_k} = \begin{bmatrix} A_d - L_{\sigma_k} \Gamma_{\sigma_k}^y C_d + \Delta B_{c,h_k} K_{\sigma_k} & L_{\sigma_k} \Gamma_{\sigma_k}^y C_c - \Delta A_{c,h_k} + \Delta B_{c,h_k} K_{\sigma_k} & -\Delta B_{c,h_k} (I - \Gamma_{\sigma_k}^u) & 0 \\ -\bar{B}_{h_k} K_{\sigma_k} & \bar{A}_{h_k} - \bar{B}_{h_k} K_{\sigma_k} & \bar{B}_{h_k} (I - \Gamma_{\sigma_k}^u) & 0 \\ K_{\sigma_k} (A_d - L_{\sigma_k} \Gamma_{\sigma_k}^y C_d - B_d K_{\sigma_k} - I) & K_{\sigma_k} (A_d + L_{\sigma_k} \Gamma_{\sigma_k}^y C_c - B_d K_{\sigma_k} - I) & (K_{\sigma_k} B_d + I) (I - \Gamma_{\sigma_k}^u) & 0 \\ \bar{C} \bar{B}_{h_k} K_{\sigma_k} & \bar{C} (I - \bar{A}_{h_k} + \bar{B}_{h_k} K_{\sigma_k}) & -\bar{C} \bar{B}_{h_k} (I - \Gamma_{\sigma_k}^u) & I - \Gamma_{\sigma_k}^y \end{bmatrix} \quad (10)$$

- Finally, with B and C given in (1), define

$$H_\sigma := \begin{bmatrix} T^{-1} & 0 & 0 & 0 \\ T^{-1}BK_l & T^{-1}BK_l & 0 & T^{-1}B(I - \Gamma_l^u) \end{bmatrix}$$

and

$$G_j := \begin{bmatrix} T & T \\ T & T \\ -CT & -CT \\ 0 & 0 \end{bmatrix} \cdot U_j$$

in which

$$U_j = \text{diag}(\delta_{1,j}^A I_1, \dots, \delta_{L,j}^A I_L, \delta_{1,j}^E I_1, \dots, \delta_{L,j}^E I_L)$$

with I_i is the identity matrix with size of the i -th real Jordan Block. The additive uncertainty set $\Delta \subseteq \mathbb{R}^{2n_x \times 2n_x}$ is now given by

$$\Delta = \{ \text{diag}(\Delta^1, \dots, \Delta^{2L}) \mid \Delta^{i+jL} \in \mathbb{R}^{n_i \times n_i}, \|\Delta^{i+jL}\| \leq 1, i \in \{1, \dots, L\}, j \in \{1, 2\} \}.$$

Theorem 1 Consider system (9), (10), where $h_k \in [\underline{h}, \bar{h}]$, $k \in \mathbb{N}$. If system (11) is obtained by following Procedure 1, (11) is an overapproximation of (9) in the sense that (12) holds.

Proof: The proof can be obtained along the lines of the proof of Theorem 1 of [3] and is omitted for the sake of brevity. ■

Using this overapproximation, stability of system (9), (10) can be analyzed using the following theorem from [3], in which

$$\mathcal{R} := \{ \text{diag}(r_1 I_1, \dots, r_L I_L, r_{L+1} I_1, \dots, r_{2L} I_L) \in \mathbb{R}^{2n_x \times 2n_x} \mid r_i > 0 \}$$

with I_i the identity matrix of size n_i , complying with the i -th real Jordan Block.

Theorem 2 Consider the closed-loop NCS (9), (10), dictated by a periodic protocol with period \tilde{N} , and an overapproximation constructed using Procedure 1. Assume that there exist positive definite matrices P_ℓ , $\ell \in \{1, \dots, \tilde{N}\}$, and

matrices $R_{\ell, \sigma_j} \in \mathcal{R}$, $\ell \in \{1, \dots, \tilde{N}\}$ and $j \in \{1, \dots, M\}$, satisfying the LMIs

$$\begin{bmatrix} F_{\sigma_\ell, j}^\top P_{\ell+1} F_{\sigma_\ell, j} - P_\ell + H_{\sigma_\ell}^\top R_{\ell, \sigma_j} H_{\sigma_\ell} & F_{\sigma_\ell, j}^\top P_{\ell+1} G_j \\ G_j^\top P_{\ell+1} F_{\sigma_\ell, j} & G_j^\top P_{\ell+1} G_j - R_{\ell, \sigma_j} \end{bmatrix} \prec 0, \quad (15)$$

where $P_{\tilde{N}+1} := P_1$, for all $\ell \in \{1, \dots, \tilde{N}\}$ and $j \in \{1, \dots, M\}$. Then, the system (11) is GAS and consequently, the system (9), (10) is globally asymptotically stable (GAS).

Proof: The proof is given in [3]. ■

Remark 1 Using a reasoning similar as in [10], it can be shown that GAS of the discrete-time model also implies stability of the sampled-data NCS including intersample behavior.

Remark 2 NCS with other protocols (e.g. TOD) can be analyzed in a similar manner using the ideas in [3].

IV. DESIGN FOR PERIODIC PROTOCOLS

In the previous sections, we derived a model describing an LTI plant interconnected with a decentralized switched observer-based controller by a communication network and presented a procedure to assess stability of the model for given K_l and L_l , $l \in \{1, 2, \dots, n_T\}$. In this section, we will present a procedure for obtaining the controller and observer gains K_l and L_l , respectively, in (8) for periodic protocols.

As the decentralized and networked constrained control problem is known to be non-convex and hard to solve in general, for the design of K_l and L_l we ignore two aspects of the problem; namely the clock skew effects and the coupling terms between the subsystems. Due to the available robust stability test (see Section III), we can verify stability including these ignored effects *a posteriori*. In other words, first we design a switched observer-based controller for the system with constant sampling interval h_* and without subsystem coupling and, second, perform a robust stability analysis including varying sampling-time effects and subsystem coupling terms using Theorem 2. In case the *a posteriori* test fails, a modification is made to the design problem and solved again.

Alternatively, the robust stability analysis can be split into two steps. The first step is to include only the coupling terms and assess stability. Since the resulting is a periodic switched linear system, stability can be assessed using a standard eigenvalue test [8]. This will provide the designer

$$\tilde{A}_{\sigma_k} = \left[\begin{array}{c|c|c|c} A_d - L_{\sigma_k} \Gamma_{\sigma_k}^y C_d & 0 & 0 & 0 \\ \hline -B_d K_{\sigma_k} & A_d - B_d K_{\sigma_k} & B_d (I - \Gamma_{\sigma_k}^u) & 0 \\ K_{\sigma_k} (A_d - L_{\sigma_k} \Gamma_{\sigma_k}^y C_d - B_d K_{\sigma_k} - I) & K_{\sigma_k} (A_d - B_d K_{\sigma_k} - I) & (K_{\sigma_k} B_d + I) (I - \Gamma_{\sigma_k}^u) & 0 \\ \hline C_d B_d K_{\sigma_k} & C_d (I - A_d + B_d K_{\sigma_k}) & -C_d B_d (I - \Gamma_{\sigma_k}^u) & I - \Gamma_{\sigma_k}^y \end{array} \right] \quad (16a)$$

$$\tilde{A}_{\sigma_k} = \left[\begin{array}{c|c|c} A_d - L_{\sigma_k} \Gamma_{\sigma_k}^y C_d & 0 & 0 \\ \hline -B_d K_{\sigma_k} & A_d - B_d K_{\sigma_k} & 0 \\ \hline C_d B_d K_{\sigma_k} & C_d (I - A_d + B_d K_{\sigma_k}) & I - \Gamma_{\sigma_k}^y \end{array} \right] \quad (16b)$$

with a check whether to re-design. Depending on the design freedom available, a re-design may consist of choosing a coarser subsystem decomposition or modifying the periodic protocol. If the coupled system is stable, then we are guaranteed to have some margin of robustness against time-varying sampling intervals, i.e. the NCS will be stable for $h_k \in [\underline{h}, \bar{h}]$, $k \in (N)$ for some $\underline{h} < h_* < \bar{h}$. Therefore the second step is to add the clock skew terms and use the stability analysis of Section III to determine the values of \underline{h} and \bar{h} that guarantee stability and verify if the range $[\underline{h}, \bar{h}]$ is sufficiently large.

Returning to the design, if we ignore clock skew effects and subsystem coupling terms, the system (9) changes into

$$\bar{x}_{k+1} = \tilde{A}_{\sigma_k} \bar{x}_k \quad (18)$$

with \tilde{A}_{σ_k} as in (16a). This simplifies the design problem to stabilizing a cascade of three smaller systems (recognize the block triangular structure in \tilde{A}_{σ_k} in (16a)). We can further reduce design complexity by assuming that the controller can access all actuators at every transmission time ($\Gamma_{\sigma_k}^u = I$). This assumption yields the system (18) with \tilde{A}_{σ_k} as in (16b). In the following, we will first design for the case when $\Gamma_{\sigma_k}^u = I$ and then briefly provide insight into the design for general $\Gamma_{\sigma_k}^u$ matrices.

For the special case, $\Gamma_{\sigma_k}^u = I$ (actuators always accessible), one can modify the model (18) with (16a) by removing the third column and row (as $e_k^u = 0$, $k \in \mathbb{N}$) and design for the model (18) with \tilde{A}_{σ_k} given by (16b). This case is certainly of practical interest since it is a common industrial configuration to hardwire actuators directly to a controller while measurement data is received through (wireless) sensor networks. The following theorem formalizes the LMI-based design of a switched observer-based controller under periodic protocols with $\Gamma_{\sigma_k}^u = I$ for all k .

Theorem 3 Consider the system (18) with \tilde{A}_{σ_k} as in (16b). Moreover, consider the protocol to be periodic, such that $\sigma_{k+\tilde{N}} = \sigma_k$ holds for all $k \in \mathbb{N}$ with $\tilde{N} \geq n_T$ and $\{\sigma_k \mid 1 \leq k \leq \tilde{N}\} = \{1, 2, \dots, n_T\}$, i.e. all nodes are addressed in one period of the protocol. Suppose that, for each i^{th} subsystem, $i = 1, \dots, N$, the following conditions are satisfied:

- 1) There exist matrices $P_{i,l} = P_{i,l}^\top \succ 0 \in \mathbb{R}^{n_i \times n_i}$ and $S_{i,l} \in \mathbb{R}^{n_i \times p_i}$ for $l = 1, 2, \dots, n_T$, such that for all $\ell = 1, 2, \dots, \tilde{N}$

$$\begin{bmatrix} P_{i,\sigma(\ell-1)} & \bar{A}_i^\top P_{i,\sigma\ell} - \bar{C}_i^\top (\Gamma_{i,\sigma\ell}^y)^\top S_{i,\sigma\ell}^\top \\ \star & P_{i,\sigma\ell} \end{bmatrix} \succ 0; \quad (19)$$

- 2) There exist matrices $Q_i = Q_i^\top \succ 0 \in \mathbb{R}^{n_i \times n_i}$ and $Z_i \in \mathbb{R}^{m_i \times n_i}$ such that

$$\begin{bmatrix} Q_i & Q_i A_i^\top - Z_i^\top \bar{B}_i^\top \\ \star & Q_i \end{bmatrix} \succ 0. \quad (20)$$

Then the controller gains $K_l = K = \text{diag}(Z_1 Q_1^{-1}, \dots, Z_N Q_N^{-1})$ and observer gains $L_l = \text{diag}(L_{1,l}, \dots, L_{N,l})$ with $L_{i,l} = P_{i,l}^{-1} S_{i,l}$ will

render the system $\bar{x}_{k+1} = \tilde{A}_{\sigma_k} \bar{x}_k$, with \tilde{A}_{σ_k} as in (16b) GAS.

Note that Theorem 3 provided convex LMI conditions (19), (20) to design switched observer-based controllers for the special case that all actuators can be updated at each sampling interval. The conditions in Theorem 3 are independent LMIs to solve for K_l and L_l , $l \in \{1, 2, \dots, n_T\}$ separately, such that the independent periodic systems corresponding to the diagonal blocks of (16b) are stable. In the general case, i.e. (18) with \tilde{A}_{σ_k} given by (16a), the convexity of the design problem is lost. Indeed, K_{σ_k} appears in a quadratic form in the second diagonal block of (16a). However, considering a time-dependent quadratic Lyapunov function candidate and using currently available software, PENBMI [9], it is possible to solve for K_l directly (using the second diagonal block of (16a)) as a polynomial matrix inequality [5].

V. EXAMPLE

In this section, we will illustrate the design procedure by using a numerical example. Let us consider the unstable continuous-time plant given by (1) with

$$\left[\begin{array}{c|c} A & B \\ \hline C & \end{array} \right] = \left[\begin{array}{cccc|ccc} 0.6 & -4.2 & 0.1 & 2.1 & 0.7 & 1.9 & -0.02 \\ 0.1 & -2.1 & 0.01 & 0 & 0 & 1 & -0.01 \\ \hline 0 & 0 & -3.2 & -0.2 & 0 & 0 & -0.8 \\ 0 & -0.03 & 5.3 & -0.2 & 0 & 0 & -0.4 \\ \hline 1 & 4 & 0 & 0.05 & & & \\ 0.2 & 1 & 0 & 0 & & & \\ \hline 0 & 0 & -2 & -0 & & & \end{array} \right], \quad (21)$$

where the decomposition into two subsystems ($N = 2$) is shown using dashed lines and the nominal sampling interval is chosen as $h_* = 1$ second. The periodic protocol, with $\tilde{N} = 3$, is given by $\sigma_1 = 1, \sigma_2 = 2, \sigma_3 = 3$ and

$$\Gamma_1^y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Gamma_2^y = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \Gamma_3^y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Gamma_l^u = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (22)$$

for all $l \in \{1, 2, 3\}$. This specific protocol indicates that the controllers have access to the actuators at each transmission time, but the sensor data available to the controller is constrained. Solving the LMI's (19), the following innovation (output injection) matrices were found

$$L_1 = \begin{bmatrix} 6.24 & -24.89 & 0 \\ -0.73 & 3.46 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.32 & 2.65 & 0 \\ 0.16 & 0.15 & 0 \\ 0 & 0 & 0.28 \\ 0 & 0 & 3.27 \end{bmatrix}, \quad L_3 = \begin{bmatrix} 0.57 & 0.44 & 0 \\ 0.04 & 0.02 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (23)$$

Solving the LMIs (20), the following state feedback matrix were found

$$K_l = K = \begin{bmatrix} 1.94 & -1.40 & 0 & 0 \\ -0.56 & -0.86 & 0 & 0 \\ 0 & 0 & 1.36 & 0.81 \end{bmatrix} \quad (24)$$

for all $l \in \{1, 2, 3\}$. The matrices shown in (23) and (24) will stabilize the decoupled version of (21) (the off-diagonal blocks of $\bar{A}, \bar{B}, \bar{C}$ equal to zero) under the protocol given in (22) and for the nominal sampling interval h_* . Including the off-diagonal blocks into the closed-loop model shows

a degradation in performance but preservation of stability. Finally, using Procedure 1 and Theorem 2 with sampling intervals $\tilde{h}_l = \{0.9, 0.96, 1, 1.04, 1.1\}$ as grid points, it was determined that this control system can withstand all possible sampling interval variations in the interval $h_k \in [0.9, 1.1]$ for all k .

Fig. 3 illustrates this design procedure by plotting the closed-loop state evolution for $\bar{x}_0 = [1, \dots, 1]^T$ after each step of the procedure.

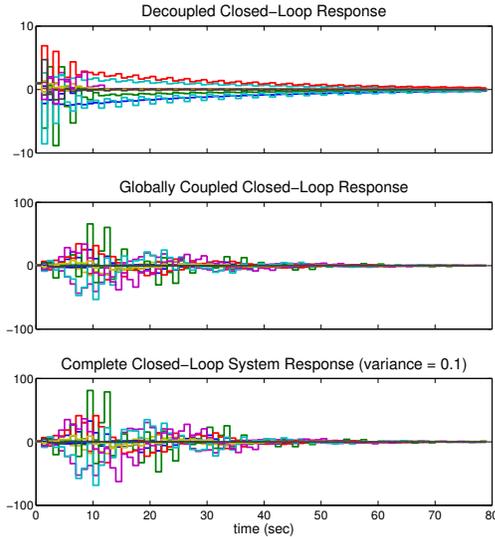


Fig. 3. Design Example: Closed-loop state evolution to non-zero initial conditions for the system without coupling and nominal sampling interval $h_* = 1$ after (top) implementing Theorem 3, (middle) including global subsystem coupling and (bottom) simulating uniformly distributed random sampling intervals $h_k \in [0.9, 1.1]$.

This example shows that the derived theory can be used to design stabilizing output-based decentralized controllers in the presence of communication constraints and network imperfections.

VI. CONCLUSIONS AND FUTURE WORK

In this paper, we developed a model for analyzing decentralized observer-based controllers in the presence of network-induced communication constraints and time-varying sampling intervals. We provided LMI-based stability conditions for verifying stability of the closed-loop NCS. A procedure was presented to design the decentralized observer-based controllers which guarantee stability in the face of communication constraints on the measurement data, but for constant sampling intervals and a decoupled plant. In the case all control inputs are transmitted at each sampling instant, LMI-based design conditions were obtained, otherwise PMI conditions can be solved. Robust stability of the designed controller was verified *a posteriori* by first assessing stability when including coupling in the plant and then testing for the range of time-varying sampling intervals that the closed loop can withstand.

The derived results show the overall structure and complexity of observer-based control design over shared networks. This confirms earlier observations that indicated the

complexity of decentralized control design (even without the presence of communication constraints and variations of sampling intervals and availability of full state information). Interestingly, by ignoring varying sampling intervals and global coupling terms, the closed-loop system matrix reveals a lower block triangular structure that can be exploited to obtain simpler LMI conditions for controller/observer synthesis and smaller polynomial matrix inequalities to perform the overall design. In the particular (but industrially relevant) case that all control inputs are communicated at each sampling interval, the design reduces to an LMI.

The presented results can serve as a platform for future developments towards more efficient design conditions for the general case under periodic and other protocols. Another topic for future work is to extend the design to include a distributed control structure, where the controllers can communicate their state information over the network.

REFERENCES

- [1] A.T. Al-Hammouri, M.S. Branicky, V. Liberatore, and S.M. Phillips. Decentralized and dynamic bandwidth allocation in networked control systems. In *Parallel and Distributed Processing Symposium, 2006. IPDPS 2006. 20th International*, pages 8 pp.–, April 2006.
- [2] A. Alessandri and P. Coletta. Switching observers for continuous-time and discrete-time linear systems. In *Proceedings of the American Control Conference, 2001*, volume 3, pages 2516–2521 vol.3, 2001.
- [3] M. C. F. Donkers, L. Hetel, W. P. M. H. Heemels, N. van de Wouw, and M. Steinbuch. Stability analysis of networked control systems using a switched linear systems approach. In *HSCC '09: Proceedings of the 12th International Conference on Hybrid Systems: Computation and Control*, pages 150–164, Berlin, Heidelberg, 2009. Springer-Verlag.
- [4] M.C.F. Donkers, W.P.M.H. Heemels, N. van de Wouw, L. Hetel, and M. Steinbuch. Stability analysis of networked control systems using a switched linear systems approach. *Submitted for journal publication*.
- [5] D. Henrion, J. Lofberg, M. Kocvara, and M. Stingl. Solving polynomial static output feedback problems with PENBMI. In *Decision and Control, 2005 and 2005 European Control Conference. CDC-ECC '05. 44th IEEE Conference on*, pages 7581–7586, Dec. 2005.
- [6] J.P. Hespanha, P. Naghshtabrizi, and Yonggang Xu. A survey of recent results in networked control systems. *Proceedings of the IEEE*, 95(1):138–162, Jan. 2007.
- [7] R. Horn and C. R. Johnson. *Matrix Analysis*. Cambridge University Press, 1985.
- [8] D. Liberzon. *Switching Systems and Control*. Birkhäuser, 2003.
- [9] M. Stingl, M. Kočvara. PENBMI Version 2.1. 2004.
- [10] D. Nešić, A. R. Teel, and E. D. Sontag. Formulas relating KL stability estimates of discrete-time and sampled-data nonlinear systems. *Systems & Control Letters*, 38(1):49 – 60, 1999.
- [11] D.D. Šiljak. *Decentralized control of complex systems*. Academic Press, Boston, 1991.
- [12] G.C. Walsh and H. Ye. Scheduling of networked control systems. *Control Systems Magazine, IEEE*, 21(1):57–65, Feb 2001.
- [13] B. Wang, G. Guo, and H. Jin. State estimation problem with communication constraints in NCSs. In *Automation and Logistics, 2008. ICAL 2008. IEEE International Conference on*, pages 2643–2648, Sept. 2008.
- [14] D. Xie, N. Xu, and X. Chen. Stabilisability and observer-based switched control design for switched linear systems. *Control Theory & Applications, IET*, 2(3):192–199, March 2008.
- [15] W. Zhang, M.S. Branicky, and S.M. Phillips. Stability of networked control systems. *IEEE Control Systems Magazine*, 21:84–99, 2001.