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LQ-power Consistent Control: Leveraging Transmission Power Selection in Control Systems

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Abstract— **We consider a linear control system, where the sensors communicate to the controller via a wireless network in which the probability of successful data transmissions is an increasing function of the selected radio signal power. We tackle the problem of jointly selecting the transmission power at every transmission time as a function of the state, much like in event-triggered control systems, and of designing a controller in order to minimize an average quadratic cost. Our proposed power scheduler and corresponding control policy are shown to outperform the optimal control policy when the radio signal power is constant at all times, while using at most the same average transmission power. For this result to hold, the function relating the transmission power to the probability of successful transmission must be convex in the region of interest. We call such a policy LQ-power consistent.**

I INTRODUCTION

Communication is a key component of automatic control systems especially when considering complex cyber-physical systems, connecting many physical and cyber agents. In fact, the quality of communication between sensors, controllers and actuators, highly affects the control quality [1]. Fast wireless communication has opened the door for many control applications, and the recent developments in wireless communication technologies, e.g., 5G, bring even further opportunities. In some recent applications, such as platooning, wireless communication of control data is the most suitable option [2], [3]. The area of networked control systems [4]–[6] refers exactly to the study of these control loops, where data is exchanged through a (wireless) communication network.

Although communication over a wireless network can facilitate the implementation of the control scheme, it brings many design difficulties due to the underlying uncertainties and complexities [7]. The limited bandwidth of the communication channels and the restrictions on power consumption of receiving and transmitting nodes are some of these challenges, which need to be carefully accounted for. In a wireless network of sensors and controllers, which usually consists of several small battery-powered devices distributed in a large area, the power consumption management plays a crucial role in the operation time of the system [8]. In particular, the reliability of the data transmission can be increased by transmitting with a higher power. However, this results in high power consumption and consequently, reduces the power efficiency of the system. In addition, a higher transmission power increases the probability of interference with the other wireless-based communication devices and, therefore, may decrease the overall performance of the system. It is thus of importance to balance reliability and efficiency/performance. In this context, an important related challenge is to design the controller and the transmission power policy so that the desired control objectives are met by using the least possible transmission power.

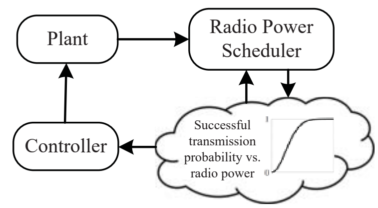

Fig. 1: Networked control system and the dependency of the successful transmissions on the radio signal power

A possible transmission power allocation strategy is to have a time-dependent policy, which consists in increasing the transmission power based on the time elapsed since the last successful transmission. In [9], the mean square stability and linear quadratic performances are guaranteed for linear wireless networked control systems (WNCS) by designing a time-dependent power scheduling policy, while the average transmission power is minimized. The transmission power can be also scheduled based on the status of the communication network at every time instant [10]. However, in control systems, we advocate that better efficiencies can be achieved by employing state-dependent power scheduling policies. In [11], [12], the power scheduling policy is state-dependent and a weighted sum of control and transmission power costs is minimized. Nevertheless, an important question remains open: are we sure that these strategies outperform a constant power scheduling policy, i.e. one that assigns the same power at any given time instant? In other words, is it worth to make the power scheduling policy state-dependent? The goal of this paper is to provide the answers to these questions.

We consider a WNCS where the transmission radio signal power can be determined by a scheduler at every time, see

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Fig. 1. The plant is linear time-invariant and has discrete-time dynamics. The control objective is to minimize an average quadratic performance of the system. Inspired by the idea of event-triggered control (ETC) e.g. [13], [14], we propose a transmission power scheduling policy, which adapts to the current state of the plant, like in [11], [12]. Then, the optimal controller associated with the proposed power scheduling policy is proved to be the linear certainty equivalent controller, where the state expectation is determined based on a Kalman filter. Finally, provided that the function relating the transmission power to the transmission probability is convex in the region of interest, as illustrated in Fig. 2, we show that our novel strategy is *LQ-power consistent*. By this we mean that the proposed strategy (i.e., transmission power policy and controller) results in a better average optimal quadratic performance than that of the constant power scheduling policy, while requiring an equal or less average transmission power.

The notion of LQ-power consistency is motivated by similar works in the literature [15]–[17], which provide ETC policies that outperform periodic control for the same average transmission rate. The main contribution of this paper is to show that a similar result can also be obtained by considering power, rather than average-transmission rate. Note that this result is of a different nature with respect to previous works in the field [9]–[12]. This novelty is expected to bridge the gap between the theory of event-triggered control and its applications. In particular, our results might help to significantly increase the battery life in applications with stand-alone battery-powered communicating devices in sensor and control networks.

Notation: $\mathsf{P}[a|b]$ denotes the conditional probability density function (pdf) of a random variable a given the information set b and $\mathcal{N}(\bar{y}, Y)$ indicates a multi-variate Gaussian pdf with mean $\bar{y}$ and covariance Y. By $\mathsf{Pr}(.)$, $\varrho(A)$ and $\mathsf{tr}(A)$, we denote the probability of an event, the spectral radius and the trace of the square matrix A, respectively. Moreover, $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$ in which $\mathbb{N}$ is the set of natural numbers.

II PROBLEM SETTING

We introduce the networked control system setting in Section II-A. Then, we discuss some required properties of the communication network characteristic curve in Section II-B. Finally, we state the considered problem in Section II-C.

II-A Networked control system setting

Consider a discrete-time linear time-invariant (LTI) system

$$x_{k+1} = Ax_k + Bu_k + w_k, \tag{1}$$

where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^m$ and w_k are the state, the control input and the disturbance, respectively, at discrete time $k \in \mathbb{N}_0$. Let $\{w_k \in \mathbb{R}^n | k \in \mathbb{N}_0\}$ be a sequence of i.i.d. Gaussian random variables with zero mean and covariance $W = \mathbb{E}[w_k w_k^\intercal]$ at every $k \in \mathbb{N}_0$. We assume W to be positive definite and the pair (A, B) to be stabilizable. The performance of the system is measured by the average quadratic cost

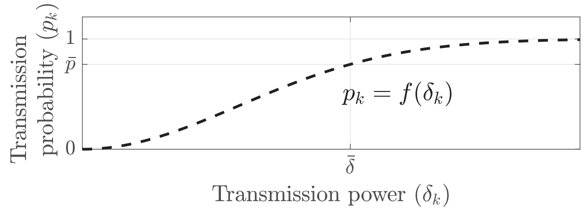

Fig. 2: The network characteristic curve describing the probability $p_k = f(\delta_k)$ of successful data transmission versus the radio signal power δ_k. We assume $f: \mathbb{R}_{\geqslant 0} \to [0,1]$ to be an increasing convex function on $[0, \bar{\delta}]$, where $\bar{\delta}$ is a limit on the transmission power. Since this limit is always present this region is called the region of interest.

$$J = \limsup_{T \to \infty} \frac{1}{T} \mathbb{E}\Big[\sum_{k=0}^{T-1} x_k^\intercal Q x_k + u_k^\intercal R u_k\Big], \tag{2}$$

where Q and R are positive semi-definite and positive definite matrices with appropriate dimensions, respectively, and $(A, Q^{\frac{1}{2}})$ is assumed to be observable.

The sensor data is transmitted to the controller through a wireless communication network (see Fig. 1), where the probability of successful data transmission depends directly on the power of the radio signal by which the data is transmitted [9]–[12]. Let $\sigma_k \in \{0,1\}$ be a binary variable indicating if there is a successful transmission at time $k \in \mathbb{N}_0$ or not. In particular, $\sigma_k = 1$ represents successful data transmission at time $k \in \mathbb{N}_0$, while $\sigma_k = 0$ indicates a packet drop. Furthermore, let us consider $\delta_k \in \mathbb{R}_{\geqslant 0}$ as the level of the radio signal power. Accordingly, we have

$$p_k := \mathsf{Pr}(\sigma_k = 1) = f(\delta_k), \tag{3}$$

where $f: \mathbb{R}_{\geqslant 0} \to [0,1]$ is a known continuous function. Hence, f describes the probability of successful data transmission with respect to the power of the radio signal. As depicted in Fig. 1, we consider that there is a power scheduler collocated with the sensors of the system, which determines the power of the radio signal at every time step. We also consider that the network sends an acknowledgment signal to the scheduler at every $k \in \mathbb{N}$ about the status of the transmission attempt at time $k-1$. We assume that the power consumption by the scheduler for the data reception is much lower than that is used for the data transmission. Therefore, the power consumption for receiving the acknowledgment signal by the scheduler can be neglected. Now given the above mentioned setup, the information available to the radio signal power scheduler and to the controller at every $k \in \mathbb{N}_0$ are

$$\mathcal{F}_k := \big\{x_t | t \in \{0, \ldots, k\}\big\} \cup \big\{\sigma_t, \delta_t | t \in \{0, \ldots, k-1\}\big\}, \tag{4}$$

and

$$\mathcal{J}_k := \big\{x_t | t \in \{0, \ldots, k\} \wedge \sigma_t = 1\big\}, \tag{5}$$

respectively. Accordingly, we can consider (joint) control and power scheduling policies as $u_k = \mu_k(\mathcal{J}_k)$

and $\delta_k = \eta_k(\mathcal{F}_k)$, respectively, for appropriate mappings $\mu_k : \mathcal{J}_k \to \mathbb{R}^m$ and $\eta_k : \mathcal{F}_k \to \mathbb{R}_{\geqslant 0}$. Moreover, let $\Delta_{(\mu,\eta)} := \limsup_{T\to\infty} \frac{1}{T}\mathbb{E}[\sum_{k=0}^{T-1} \delta_k]$ be the average radio signal power consumption of combined time-varying power scheduling $\delta_k = \eta_k(\mathcal{F}_k)$ and control $u_k = \mu_k(\mathcal{J}_k)$ policies. The average quadratic cost (2) associated to this combined power scheduling and control policies is denoted by $J_{(\mu,\eta)}$.

II-B Characteristic curve of the communication network

Let $\bar{\delta}$ be the maximum power that we can assign to the radio signal at every $k \in \mathbb{N}_0$. Then, based on the network characteristic curve, we denote by $\bar{p} = f(\bar{\delta}) \in (0,1]$ the highest achievable successful data transmission probability. In standard wireless networks, the characteristic curves usually are convex from $(\delta_k, p_k) = (0,0)$ up to a certain point and then they become concave and converge to $p_k = 1$ as $\delta_k \to \infty$ (see Fig. 2) [18]. These are often called convex-concave curves. In our work, make the following assumption on this network characteristic curve.

Assumption 1: The function $f : [0,\bar{\delta}] \to [0,\bar{p}]$ is increasing and convex on $[0,\bar{\delta}]$ and $f(0) = 0$. □

The condition $f(0) = 0$ indicates that zero transmission power results in zero successful transmission probability. The convexity of f on $[0,\bar{\delta}]$ is a crucial property in the upcoming analysis and design. We discuss the case when Assumption 1 does not hold in Section VI.

II-C Problem statement

The next definition formalizes the admissible ranges of transmission power and successful transmission probability at every $k \in \mathbb{N}_0$.

Definition 1: The admissible ranges of successful transmission probability and transmission power are $\mathcal{S} := (p_{\min}, \bar{p})$ and $\mathcal{C} := (\delta_{\min}, \bar{\delta})$, respectively, where

$$p_{\min} := \inf\{p \,|\, \varrho(\sqrt{1-p}A) < 1,\ p \in (0,\bar{p})\},$$

and $\delta_{\min} := f^{-1}(p_{\min})$.

The ranges in Definition 1 characterize the values of the transmission probability and of the transmission power ensuring mean square stability of the linear WNCS given in Section II-A. This will be discussed more after Proposition 1, where we present the optimal controller and its corresponding average quadratic cost for the networked control system when the transmission power is *constant* at all times [19].

Proposition 1: Consider the admissible regions $\mathcal{C}$ and $\mathcal{S}$ for system (1) and a control loop in which the radio signal power is constant at all times, i.e. $\delta_k = c \in \mathcal{C}$ and therefore, $p_k = p = f(c) \in \mathcal{S}$ for all $k \in \mathbb{N}_0$. Then the following statements hold.

i) The control policy $\mu^* := (\mu_0^*, \mu_1^*, \dots)$, where

$$\mu_k^*(\mathcal{J}_k) := K\hat{x}_{k|k} \tag{6}$$

minimizes the average quadratic cost (2), where

$$\begin{aligned} K &:= -(B^\intercal PB + R)^{-1}B^\intercal PA, \\ P &:= A^\intercal PA + Q - K^\intercal(B^\intercal PB + R)K, \end{aligned} \tag{7}$$

and

$$\begin{aligned} \hat{x}_{k+1|k} &= A\hat{x}_{k|k} + Bu_k, \\ \hat{x}_{k|k} &= \begin{cases} x_k, & \text{if } \sigma_k = 1, \\ \hat{x}_{k|k-1}, & \text{otherwise.} \end{cases} \end{aligned} \tag{8}$$

ii) The control loop is mean square stable, i.e., for any given initial condition x_0, there exists a $d \in [0,\infty)$ such that $\sup\{\mathbb{E}[x_k^\intercal x_k] | k \in \mathbb{N}_0\} \leqslant d$, and its corresponding average quadratic cost (2) is given by

$$J_{(\mu^*,c)} := \mathsf{tr}(PW) + \sum_{i=0}^{\infty} \big(1 - f(c)\big)^{i+1} \mathsf{tr}(A^i W A^{\intercal i} Y), \tag{9}$$

where $Y := K^\intercal(B^\intercal PB + R)K$. □

The optimal control approach toward this problem results in a model-based state estimation (8) by the controller in case of data dropout, like in [12]. It is worthwhile to mention that when A is not Schur, then the optimal average quadratic cost given in (9) is bounded if $\varrho\big((1-f(c))A^2\big) < 1$. This justifies the addmissible ranges given in Definition 1.

Our objective in this work is to construct a joint controller and transmission power policy (μ,η), which outperforms the constant power policies given in Proposition 1, in the following sense.

Definition 2: (*LQ-power consistent policy*) For a fixed $c \in \mathcal{C}$, where $\mathcal{C}$ is given in Definition 1, a combined radio signal power scheduling and control policy, i.e., $\eta = (\eta_0, \eta_1, \dots)$, $\mu = (\mu_0, \mu_1, \dots)$ such that $\delta_k = \eta_k(\mathcal{F}_k)$, $u_k = \mu_k(\mathcal{J}_k)$ and $\Delta_{(\mu,\eta)} \leqslant c$, is said to be an *LQ-power consistent policy* if it results in a lower average quadratic cost (2) than the optimal cost of the constant power scheduling policy, when $\delta_k = c$ at all $k \in \mathbb{N}_0$. In other words, if for a fixed $c \in \mathcal{C}$, $\delta_k = \eta_k(\mathcal{F}_k)$ and $u_k = \mu_k(\mathcal{J}_k)$ are such that $J_{(\mu,\eta)} < J_{(\mu^*,c)}$, while $\Delta_{(\mu,\eta)} \leqslant c$, where $J_{(\mu^*,c)}$ is given in (9), then (μ,η) is called an LQ-power consistent policy. □

In other words, the goal is to construct an LQ-power consistent policy.

III EVENT-BASED POWER SCHEDULING POLICY

Although the scheduler has access to the exact value of the state vector at every $k \in \mathbb{N}_0$, it can also implement a copy of the model-based state estimator (8) at every $k \in \mathbb{N}_0$. We introduce $e_{k|k-1} := x_k - \hat{x}_{k|k-1}$ and $\Theta_{k|k-1} := \mathbb{E}[e_{k|k-1}e_{k|k-1}^\intercal | \mathcal{J}_{k-1}]$ as the predicted state estimation error and its covariance, where they both can indeed be computed by the scheduler. Then, we propose the model-based power scheduling policy

$$\delta_k = f^{-1}\big(\mathcal{P}_k(e_{k|k-1}, \lambda_k)\big), \tag{10}$$

for any $k \in \mathbb{N}_0$, where $f^{-1} : [0,\bar{p}] \to [0,\bar{\delta}]$ indicates the inverse function of f, which exists in view of Assumption 1, and

$$\mathcal{P}_k(e_{k|k-1}, \lambda_k) := \bar{p}\Big(1 - \exp(-\frac{\lambda_k}{2} e_{k|k-1}^\intercal \Theta_{k|k-1}^{-1} e_{k|k-1})\Big), \tag{11}$$

where $\lambda_k \in \mathbb{R}_{\geqslant 0}$ is the tunning parameter of our power scheduler, which directly affects the successful data transmission probability. Based on the network characteristic curve, $\mathcal{P}_k(e_{k|k-1},\lambda_k)$ is the successful transmission probability at every $k\in\mathbb{N}_0$ and

$$\bar{\mathcal{P}}_k(\lambda_k):=\int_{\mathbb{R}^n}\mathcal{P}_k(e_{k|k-1},\lambda_k)\mathsf{P}[e_{k|k-1}|\mathcal{J}_{k-1}]de_{k|k-1} \quad (12)$$

is introduced as the *expected successful transmission probability* of (10) at every $k\in\mathbb{N}_0$. It is worthwhile to mention that the power scheduling policy (10) is such that the radio signal power is upperbounded by $\bar{\delta}=f^{-1}(\bar{p})$, which is in line with Assumption 1.

To employ the power scheduling policy (10), we have to determine $\Theta_{k|k-1}$ and then to regulate λ_k in order to have a desired expected successful transmission probability, i.e. $\bar{\mathcal{P}}_k(\lambda_k)$, at every time step. For this purpose, the next lemma states the properties of the state estimation error pdf when the power scheduler is operating based on (10).

Lemma 1: Consider that the power scheduler follows (10). Then the predicted state estimation error $e_{k|k-1}$ follows a sum of Gaussians distribution at every time step $k\in\mathbb{N}_0$. Moreover, if at a given time step $k\in\mathbb{N}_0$ we denote this sum of Gaussians distribution by

$$\mathsf{P}[e_{k|k-1}|\mathcal{J}_{k-1}]=\sum_{j=1}^{t}\alpha_k^j\mathcal{N}(0,\Theta_{k|k-1}^j) \quad (13)$$

for a given $t\in\mathbb{N}$, where $\alpha_k^j>0$, $j\in\{1,\dots,t\}$ with $\sum_{j=1}^t\alpha_k^j=1$ and $\{\Theta_{k|k-1}^1,\dots,\Theta_{k|k-1}^t\}$ is a sequence of symmetric positive definite matrices, then $\Theta_{k|k-1}=\sum_{j=1}^t\alpha_k^j\Theta_{k|k-1}^j$ and

$$\begin{aligned}&\mathsf{P}[e_{k+1|k}|\mathcal{J}_{k-1},\sigma_k=1,x_k]=\mathcal{N}(0,W),\\&\mathsf{P}[e_{k+1|k}|\mathcal{J}_{k-1},\sigma_k=0]=\frac{1}{\bar{\mathcal{Q}}_k(\lambda_k)}\sum_{j=1}^{t}\\&\quad\Big(\tilde{\alpha}_k^j\mathcal{N}(0,\tilde{\Theta}_{k+1|k}^j)+\hat{\alpha}_k^j(\lambda_k)\mathcal{N}\big(0,\hat{\Theta}_{k+1|k}^j(\lambda_k)\big)\Big),\end{aligned} \quad (14)$$

where $\bar{\mathcal{Q}}_k(\lambda_k):=\sum_{j=1}^t\big(\tilde{\alpha}_k^j+\hat{\alpha}_k^j(\lambda_k)\big)$, $\tilde{\alpha}_k^j:=(1-\bar{p})\alpha_k^j$ and

$$\hat{\alpha}_k^j(\lambda_k):=\bar{p}\alpha_k^j\det(I+\lambda_k\Theta_{k|k-1}^{-1}\Theta_{k|k-1}^j)^{-\frac{1}{2}},$$
$$\tilde{\Theta}_{k+1|k}^j:=A\Theta_{k|k-1}^jA^{\mathsf{T}}+W,$$
$$\hat{\Theta}_{k+1|k}^j\ (\lambda_k):=A\Theta_{k|k-1}^j(I+\lambda_k\Theta_{k|k-1}^{-1}\Theta_{k|k-1}^j)^{-1}A^{\mathsf{T}}+W.$$

Moreover, the expected successful transmission probability (12) at all $k\in\mathbb{N}_0$ is given by

$$\bar{\mathcal{P}}_k(\lambda_k)=1-\bar{\mathcal{Q}}_k(\lambda_k). \quad (15)$$

□

Lemma 1 not only provides the pdf of the state estimation error, but it also gives us the relationship between λ_k and the expected successful transmission probability at every $k\in\mathbb{N}_0$, see (15). The control designers usually have a better insight between stability/performance and the expected successful data transmission probability at every $k\in\mathbb{N}_0$. Therefore, (15) becomes a useful equation for determining λ_k in order to guarantee a certain expected successful transmission probability at every $k\in\mathbb{N}_0$. Hence, for a given $p\in\mathcal{S}$, the solution of the following nonlinear equation

$$\bar{\mathcal{P}}_k(\lambda_k)=p, \quad (16)$$

where $\bar{\mathcal{P}}_k(\lambda_k)$ follows (15), results in an appropriate $\lambda_k\in\mathbb{R}_{\geqslant 0}$ needed by the power scheduling policy (10). Lemma 1 also expresses an important feature of the power scheduler (10): any failure in the data transmission doubles the number of Gaussian terms of the state estimation error pdf (except the special situation in which $\bar{p}=1$, which keeps the distribution a single Gaussian at all times). On the other hand, a successful transmission resets the number of Gaussian terms to one, see (14).

The following lemma states another key feature of the power scheduling policy (10): Employing (10) results in a smaller average state estimation error covariance at every time $k\in\mathbb{N}_0$ for a given expected successful transmission probability $\bar{\mathcal{P}}_k(\lambda_k)=p\in\mathcal{S}$ in comparison with the constant power scheduler in which $\delta_k=f^{-1}(p)$.

Lemma 2: Consider that at a given time-step $k\in\mathbb{N}_0$, the pdf of the predicted state estimation error follows (13) and that the power scheduling policy (10) is employed where $\lambda_k\in\mathbb{R}_{\geqslant 0}$ is determined by solving (16) for a given $p\in\mathcal{S}\setminus\{0,\bar{p}\}$. Let the expected updated state estimation error covariance at the time-step k, i.e. $\mathbb{E}[\Theta_{k|k}]$, be denoted by Φ_{vp} and that resulting from the constant power scheduler $\delta_k=f^{-1}(p)$ be denoted by Φ_{cp}. Then $\Phi_{vp}<\Phi_{cp}$. □

IV Main Results

In the following theorem, we provide the main result of the paper. It states that the linear certainty equivalent controller (6) is optimal for the power scheduling policy (10) and together they result in an LQ-power consistent solution according to Definition 2.

Theorem 1: Suppose that Assumption 1 holds and let $\mathcal{C}$ and $\mathcal{S}$ be admissible sets according to Definition 1. Moreover, assume $c\in\mathcal{C}$ is a given radio signal power level and $p=f(c)$ is its corresponding successful transmission probability. Then the following statements hold:

i) The optimal LQG controller is linear and determined based on (6) and (8) when the power scheduling policy (10) is employed at all $k\in\mathbb{N}_0$, where $\lambda_k\in\mathbb{R}_{\geqslant 0}$ is determined by solving (16) for the given successful transmission probability $p=f(c)\in\mathcal{S}$.

ii) The combination of the controller (6), (8), and the power scheduling policy (10), where $\lambda_k\in\mathbb{R}_{\geqslant 0}$ is determined by solving (16) for fixed $p=f(c)\in\mathcal{S}\setminus\{0,\bar{p}\}$ at all $k\in\mathbb{N}_0$, results in a lower average quadratic cost than when $\delta_k=c=f^{-1}(p)$ at all $k\in\mathbb{N}_0$, while requiring an equal or less average transmission power. In other words, it is LQ-power consistent in the sense of Definition 2. □

As we can see in Theorem 1, although the proposed state-dependent power scheduling (10) is different from the one determined in [12], it results in an optimal controller with the same structure. It is worth mentioning that the power scheduling policy (10) causes an explosion in the number of Gaussian terms, which might result in long computation times for the determination of $\Theta_{k|k-1}$ and λ_k. However, when we desire to set the expected successful transmission probability $\bar{\mathcal{P}}_k(\lambda_k)$ to a constant value at all times (as this is the case in Theorem 1), then the values of λ_k just depend

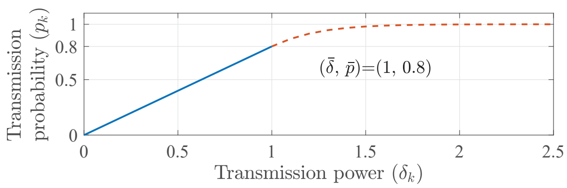

Fig. 3: The considered network characteristic curve in simulation

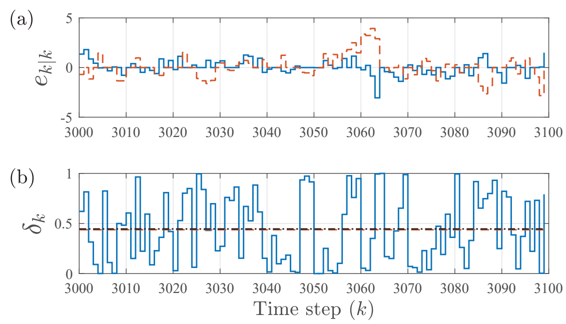

Fig. 4: Behaviour of both power scheduling policies with respect to time when $k \in Y := \{3000, \ldots, 3100\}$, dashed red lines refer to the constant power scheduling policy and solid blue lines refer to the power scheduling policy (10). (a) The trajectory of $e_{k|k}$ with respect to time. (b) The transmission power consumption of both policies. The dash-dotted line refers to the average communication power of the power scheduling policy (10) throughout Y.

on the number of time steps elapsed since the last successful transmission. Therefore, the series of the values of λ_k in between every two consequitive successful transmission times, i.e., $\{\lambda_{s_i+1}, \ldots, \lambda_{s_{i+1}}\}$, where $\sigma_{s_i} = 1$, for all $i \in \mathbb{N}_0$, is time-invariant and the scheduler can compute it offline, keep it in memory and set the values of λ_k in every time step based on that without the need for resolving (16).

V Simulation Results

Consider a scalar system where $A = 1.05$, $B = 1$, $W = 1$. Moreover, $Q = 1$, $R = 0.1$ are the parameters of the cost function (2). The LQG control gain for this system is determined as $K = -0.9626$. Furthermore, as shown in Fig. 3, the network characteristic curve is considered to be linear for $\delta_k \in [0, \bar{\delta}]$, where $\bar{\delta} = 1$ is the largest power we can assign to the radio signal, which results in $\bar{p} = 0.8$ as the highest achievable successful transmission probability at every time step. In order to guarantee the mean-square stability of the system, the minimum transmission probability at every time step is $p_{\min} = 1 - 1/A^2 = 0.0930$ as in Definition 1, which is associated with $\delta_{\min} = 0.1162$ based on the network characteristic curve.

As the first step, we consider $c = 0.4375$, which results in $p = 0.35$ as the expected successful transmission probability at every time step and implement both the power scheduling policy (10) and the constant power scheduling with the optimal controller (6), where $x_0 = 0$. The trajectory of $e_{k|k}$

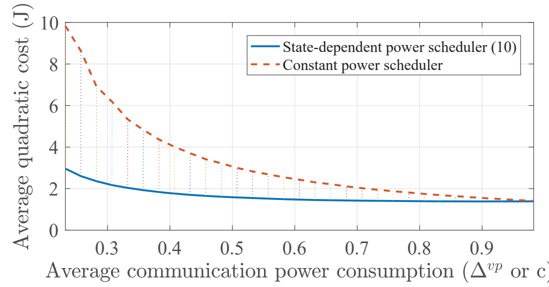

Fig. 5: Trade-off curves between the average quadratic performance and the average transmission power consumption for a control loop operating based on the power scheduling policy (10) and the constant power scheduling policy.

when $k \in Y := \{3000, \ldots, 3100\}$ is shown in Fig. 4(a). As we know, the performance of the system directly depends on $e_{k|k}$ and, as we see in Fig. 4(a), the power scheduling policy (10) has a better performance in the attenuation of $e_{k|k}$. Moreover, the communication power consumption of both policies with respect to time is shown in Fig. 4(b), which indicates that they both require the same average transmission power.

As a next step, for every $c \in [2\delta_{k,\min}, \bar{\delta})$, we implement the base policy introduced in Proposition 1 for 100 Monte-Carlo runs, each for 20000 time steps and with zero initial condition. The average quadratic performance is shown with respect to the constant communication power by the red dashed line in Fig. 5. Then for every $c \in [2\delta_{k,\min}, \bar{\delta})$, which is associated with a $p \in [2p_{k,\min}, \bar{p})$, we implement the same Monte-Carlo runs, however, assuming that the power scheduler is operating based on (10), where λ_k at every time step is regulated to guarantee the average transmission probability of p by solving (16). The trade-off curve associated to this condition is shown by the blue solid line in Fig. 5. Every point on this curve is connected by a dotted line to a point on the curve related to the constant power scheduling policy. In principle, these connected points are associated with the condition in which the expected successful transmission probability of both scheduling polices are the same at every $k \in \mathbb{N}_0$. Based on these plots in Fig. 5, we can easily conclude the LQ-power consistency of the power scheduling policy (10) together with the linear controller (6) based on Definition 2.

VI Performance-power Trade-off Curve for different Network Characteristic Curves

Theorem 1 is valid as far as Assumption 1 holds, i.e., the characteristic curve of the communication network is convex on $[0, \bar{\delta}]$. However, this condition may not be valid in all wireless communication networks and therefore, we cannot argue the LQ-power consistency of (6) and (10) for all networked control systems defined in Section II-A. In this section, we discuss the LQ-power consistency of (6) and (10) based on the trade-off curve between the average quadratic cost and the average transmission power consumption for different shapes of the network characteristic curve.

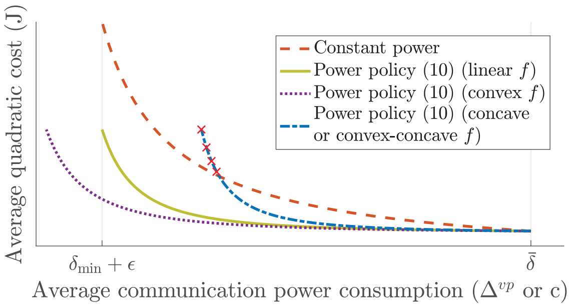

Fig. 6: Comparison of the trade-off curves between the average quadratic cost and the average communication power consumption for different shapes of f. Consider $\epsilon \in (0, \bar{\delta} - \delta_{\min})$ as a small positive scalar. Red crosses indicate the none LQ-power consistent region when f is non-convex on $[0, \bar{\delta}]$.

Suppose that the dashed red line in Fig. 6 demonstrates the trade-off curve for a linear system when the transmission power is constant at all times, consistently with what we observed in the example of Section V in Fig. 5. If we employ (6) and $\hat{\eta}_p = (\delta_0, \delta_1, \dots)$, where all δ_k, $k \in \mathbb{N}_0$, follow (10) for a constant expected successful transmission probability $p \in \mathcal{S}$, then $J_{(\mu^*, \hat{\eta}_p)} < J_{(\mu^*, c)}$, where $c = f^{-1}(p)$ for any arbitrary f as the network characteristic curve. However, based on the shape of f on $[0, \bar{\delta}]$, several conditions may occur for the average transmission power consumption, i.e. $\Delta_{(\mu^*, \hat{\eta}_p)}$.

By using the Jensen's inequality, we can show that for a linear f, $\Delta_{(\mu^*, \hat{\eta}_p)} = c$ and for a convex f, $\Delta_{(\mu^*, \hat{\eta}_p)} < c$. Therefore, in these situations, the trade-off curves are always below the curve related to the constant power scheduling policy for every linear system (1), as illustrated in Fig. 6. This indicates the LQ-power consistency of (6) and (10) for all linear and convex network characteristic curves. Now, suppose that f be a concave function, then $\Delta_{(\mu^*, \hat{\eta}_p)} > c$ (again by resorting to the Jensen's inequality) and we cannot argue the LQ-power consistency of (6) and (10) for all linear system based on Definition 2. Furthermore, for a convex-concave or any general f, we cannot even compare the values of $\Delta_{(\mu^*, \hat{\eta}_p)}$ and c. In these situations, by running Monte-Carlo simulations, we can find the trade-off curves related to a given linear system and a network characteristic curve for both constant power scheduler and power scheduling policy (10). Then, for the values of $c \in \mathcal{C}$ in which the trade-off curve associated with (10) is below that of the constant power scheduling policy (the region of the blue line with no crosses in Fig. 6), (6) and (10) is LQ-power consistent based on Definition 2.

VII Conclusions

We investigated the problem of communication power scheduling in a networked control framework, where the data successful transmission probability directly depends on the radio signal power. We introduced the notion of LQ-power consistency, which refers to combined time-varying power scheduling and control policies that result in a lower average quadratic cost than the optimal cost of any constant power scheduler, while it consumes less or at most equal average transmission power. We proposed a controller and a power scheduler which depends on the state estimation error percieved by the controller and proved that they are LQ-power consistent given the condition that the network characteristic curve is convex in the region of interest.

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