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Robust Self-Triggered Model Predictive Control for Constrained Discrete-Time LTI Systems based on Homothetic Tubes

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Abstract— **In this paper we present a robust self-triggered model predictive control (MPC) scheme for discrete-time linear time-invariant systems subject to input and state constraints and additive disturbances. In self-triggered model predictive control, at every sampling instant an optimization problem based on the current state of the system is solved in order to determine the input applied to the system until the next sampling instant, as well as the next sampling instant itself. This leads to inter-sampling times that depend on the trajectory of the system. By maximizing the inter-sampling time, the amount of communication in the control system is reduced. In order to guarantee robust constraint satisfaction, Tube MPC methods are employed. Specifically, in order to account for the uncertainty in the system, homothetic sets are used in the prediction of the future evolution of the system. The proposed controller is shown to stabilize a closed and bounded set including the origin in its interior.**

I. INTRODUCTION

In a control system where the cost of communication between the controller and the plant cannot be neglected, control schemes that achieve large inter-sampling times become relevant. In particular, it has been found that making the inter-sampling time state-dependent and, hence, in general aperiodic, may lead to a great reduction of overall communication when compared to schemes with periodic sampling, see for example [1] and the references therein.

In this work, we employ the so-called "self-triggered" control paradigm, where at each sampling instant the next sampling instant is calculated based on the current state, see [2]. In self-triggered control, the control input to the plant is strictly open-loop between sampling instants and hence may be computed at the same time when the next sampling instant is decided. For an overview of self-triggered control, we refer to [3].

If satisfaction of hard constraints on the inputs and the states are part of the control objective, model predictive control (MPC) is a viable choice for the controller design. In MPC, an optimal control problem is solved at every sampling instant based on the current state of the system. The optimal input trajectory is applied until the next sampling instant, where the optimization is repeated based on updated state information, thus inducing feedback in the control system. For an overview of MPC, we refer to [4], [5], and [6].

In [7] a self-triggered model predictive controller for constrained linear systems was presented. In [8] a self-triggered LQR subject to stochastic disturbances was proposed, which could be perceived as the unconstrained version of [7]. In [9] an approach similar to [7] was pursued, the main difference being that in [7] the length of the inter-sampling time is maximized (subject to constraints on the cost function), while in [9] the inter-sampling time is directly included in the cost function. An alternative (unconstrained) receding horizon control strategy was presented in [10], in which a cost function was minimized subject to constraints on the average communication rate. Reducing the frequency of control updates may also be used to achieve "sparse control", that is, input signals that change only at infrequent points in time (or are non-zero only at infrequent points in time). In [11] a sparse control scheme was presented, where a ℓ_1-norm based cost is minimized.

Based on [7], a self-triggered model predictive controller for constrained linear systems subject to bounded additive disturbances was recently proposed in [12], where Tube MPC methods from [13] were employed in order to guarantee robust constraint satisfaction. In Tube MPC, the possible future evolution of the system under the (unknown) disturbances is predicted using a sequence of sets (a so-called "tube") in the state-space. The main idea in Tube MPC is to assume that feedback will occur at later points in time, ensuring that the size of the predicted sets does not grow unboundedly.

In [12], the predicted sets were of fixed shape and size and were parametrized by a translation in the state-space only, thereby following the principles of [13]. As in [7], the control algorithm in [12] attempted to maximize at every sampling instant the time until the next sampling instant. The control input, computed at the same time, was then applied in an open-loop fashion until the next sampling instant. Hence, the crucial assumption in [13], the presence of feedback at every point in time, was not met in this setup, leading to a rather large size of the predicted sets and, hence, possibly conservative constraint tightening. Note that feedback was assumed in the predictions employed in [12], however occurring only after an assumed maximum inter-sampling time.

In this paper, which is partly based on the first author's

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diploma thesis [14], we propose a novel self-triggered model predictive controller that ensures robust constraint satisfaction in the presence of additive disturbances. In order to alleviate the conservatism in [12], we employ a more advanced Tube MPC method recently proposed in [15]. In [15], the predicted sets are not only parametrized by a translation, but also by an additional scaling factor, leading to more flexible tubes when compared to [13]. The main benefit of the inclusion of additional parameters in the predictions is the reduction of conservativeness, hence leading in general to larger sets in the state space where the algorithm is feasible. Furthermore, in [12] a maximal inter-sampling time had to be fixed, which determined the shape and size of the sets in the predicted tube. In the scheme presented in this paper, the maximal inter-sampling time does not influence the shape of the sets in the prediction such that no re-design is necessary if the maximal inter-sampling time is increased. Applied to self-triggered MPC, this approach has the advantage of allowing the predicted sets to grow faster during the assumed open-loop phase at the beginning of the prediction horizon, and to grow slower (or even shrink) in the later phase of the prediction horizon, were feedback is assumed. Hence, the size of the tube depends naturally on the length of the assumed open-loop phase, which was not the case in [12]. As in [12], the controller in this paper is shown to stabilize a closed and bounded set in the state-space whose size is traded off with the average sampling time in the control system. A major benefit of the present paper, from a theoretical point of view, is that the set, which is stabilized, can be easily determined a priori, while in [12] the stabilized set is defined as a sublevel set of the optimal cost which is in general not easily obtainable. A numerical example illustrates the reduction of conservatism of the present approach when compared to [12], that is, the present approach leads to a larger feasible region of the MPC scheme.

The remainder of the paper is organized as follows. The introduction section concludes with a few notes on notations. The problem setup is presented in Section II. In Section III, a Tube MPC problem is defined where the input is assumed to be strictly open-loop for the first M prediction steps. The novel self-triggered Tube MPC scheme is proposed in Section IV. Results concerning robust constraint satisfaction and stability are given in Section V. Section VI contains a numerical example illustrating the approach and Section VII concludes the paper.

Notation: The notation is adapted from [12] and [15] with only minor changes. Let $\mathbb{N}$ and $\mathbb{R}$ denote the set of natural numbers including 0 and the set of real numbers, respectively. Moreover, let $\mathbb{N}_{\geq q}$ denote $\{r \in \mathbb{N} \mid r \geq q\}$. The sets $\mathbb{N}_{>q}$, $\mathbb{N}_{[q,\,s]}$, $\mathbb{R}_{\geq 0}$, and $\mathbb{R}_{>0}$ are defined analogously. For given sets $\mathbb{A}, \mathbb{B} \subset \mathbb{R}^n$, scalar $c \in \mathbb{R}$, and matrix $A \in \mathbb{R}^{m\times n}$, define the sets $c\,\mathbb{A} := \{c\,a \mid a \in \mathbb{A}\}$, $A\,\mathbb{A} := \{A\,a \mid a \in \mathbb{A}\}$, and the Minkowski set addition $\mathbb{A} \oplus \mathbb{B} := \{a+b \mid a \in \mathbb{A},\ b \in \mathbb{B}\}$. Furthermore, define for a given vector $v \in \mathbb{R}^n$ $v \oplus \mathbb{A} := \{v\} \oplus \mathbb{A}$. Let $\mathrm{convh}(\mathbb{A})$ denote the convex hull of a set $\mathbb{A}$. The distance of a vector $v \in \mathbb{R}^n$ to a set $\mathbb{S} \subseteq \mathbb{R}^n$ is given by $|v|_{\mathbb{S}} := \inf\{|v-s| \mid s \in \mathbb{S}\}$ with the Euclidean norm $|\cdot| := \|\cdot\|_2$. A closed, bounded, and convex set containing the origin is denoted as a C-set. A C-set containing the origin in its (non-empty) interior is called a proper C-set or a PC-set. A function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ belongs to class $\mathscr{K}$ if it is continuous, strictly increasing and $\alpha(0) = 0$. A function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ belongs to class $\mathscr{K}_\infty$ if it is a $\mathscr{K}$-function and $\lim_{s\to\infty} \alpha(s) = \infty$.

II. PROBLEM SETUP AND PRELIMINARIES

We consider a discrete-time linear time-invariant system with additive bounded disturbances of the form

$$x_{t+1} = A\,x_t + B\,u_t + w_t \tag{1a}$$

$$x_0 \in \mathbb{R}^n \tag{1b}$$

with states $x_t \in \mathbb{R}^n$, inputs $u_t \in \mathbb{R}^m$, and unknown bounded disturbances $w_t \in \mathbb{W} \subset \mathbb{R}^n$ at time $t \in \mathbb{N}$. The goal is to asymptotically stabilize an *a priori known* closed and bounded set containing the origin of system (1) while satisfying the constraints

$$\forall t \in \mathbb{N},\ \ x_t \in \mathbb{X},\ \ u_t \in \mathbb{U} \tag{2}$$

and reducing the communication between the system and the controller to a minimum. The following assumption holds throughout the paper.

Assumption 1 (cf. [15]): The sets $\mathbb{X}$ and $\mathbb{U}$ are PC-sets. The set $\mathbb{W}$ is a C-set.

Definition 1 (Positively invariant set; cf. [16]): A set $\mathbb{A} \subset \mathbb{R}^n$ is a positively invariant set of the system $x^+ = f(x)$ with $f : \mathbb{R}^n \to \mathbb{R}^n$, if for all $x \in \mathbb{A}$ it holds that $f(x) \in \mathbb{A}$.

Definition 2 (Maximal positively invariant set; cf. [16]): The maximal positively invariant set Ω_∞ inside $\mathbb{X}$ of a system $x^+ = f(x)$ with $f : \mathbb{R}^n \to \mathbb{R}^n$ is the positively invariant set contained in $\mathbb{X} \subset \mathbb{R}^n$, that contains every positively invariant set of $x^+ = f(x)$ included in $\mathbb{X}$.

III. HOMOTHETIC TUBE MPC WITH MULTIPLE-STEP OPEN-LOOP CONTROL

In order to minimize the communication between the system and the controller, at every sampling instant the time until the next sampling instant is computed as part of the solution to an optimization problem. The input to the system until the next sampling instant is obtained as part of the solution of the same finite horizon optimal control problem parametrized by the state of the system at the current sampling instant. Hence, between two sampling instants, the control is open-loop. This control architecture is taken from [7]. In order to guarantee satisfaction of the state and input constraints under all possible disturbances, Homothetic Tube MPC as proposed in [15] is employed. The idea in [15] is to predict a sequence of sets and control laws which contain every possible future trajectory of the system under a given bound on the disturbances (i. e. specified by $\mathbb{W}$). The sets are parametrized by their translation and scaling in the state space, i.e. they are homothetic to a predefined, robustly positive invariant set. This parameterization was chosen in [15] in order to make the predicted sequence of sets more flexible when compared to earlier Tube MPC approaches where the sets were only parametrized by their translation, but had a fixed shape and a fixed size, as e.g. in [13]. However, as usual in Tube MPC, in [15] it is assumed that feedback is possible at every sampling instant, which is an assumption that is explicitly not satisfied in the self-triggered framework considered here. In the following, we propose a Tube MPC framework based on [15], where the first M steps of the predicted input sequence are applied in a strictly open-loop fashion.

Define the decision variable for the MPC problem at time point $t \in \mathbb{N}$ as

$$\mathbf{d}_N^M := [(z_{t|0}, \dots, z_{t|N}), (a_{t|0}, \dots, a_{t|N}), (y_{t|0}, \dots, y_{t|M-1}), (v_{t|0}, \dots, v_{t|N-1}), (u_{t+0}, \dots, u_{t+M-1})] \in \mathbb{D}_N^M \quad (3)$$

with $\mathbb{D}_N^M := \mathbb{R}^{(N+1)n} \times \mathbb{R}^{N+1} \times \mathbb{R}^{Mn} \times \mathbb{R}^{Nm} \times \mathbb{R}^{Mm}$, the prediction horizon $N \in \mathbb{N}_{\geq 2}$, and the inter-sampling time $M \in \mathbb{N}_{[1,N-1]}$, which is the time between two control updates.

For given states $x_t \in \mathbb{R}^n$ of system (1) define the constraints on the decision variable $\mathbf{d}_N^M$ as

$$x_t \in z_{t|0} \oplus a_{t|0} S \quad (4a)$$
$$\forall k \in \mathbb{N}_{[0,N-1]}, \quad a_{t|k} \geq 0 \quad (4b)$$
$$\forall k \in \mathbb{N}_{[0,N-1]}, \quad z_{t|k} \oplus a_{t|k} S \subseteq \mathbb{X} \quad (4c)$$
$$\forall k \in \mathbb{N}_{[0,N-1]}, \quad v_{t|k} \oplus a_{t|k} R \subseteq \mathbb{U} \quad (4d)$$
$$(z_{t|N}, a_{t|N}) \in \mathbb{G}_\mathrm{f} \quad (4e)$$
$$y_{t|0} = x_t \quad (4f)$$
$$\forall k \in \mathbb{N}_{[0,M-2]}, \quad y_{t|k+1} = A y_{t|k} + B u_{t+k} \quad (4g)$$
$$\forall k \in \mathbb{N}_{[1,M-1]}, \quad \forall \xi \in z_{t|k} \oplus a_{t|k} S, \quad A\xi + B u_{t+k} \oplus \mathbb{W} \subseteq z_{t|k+1} \oplus a_{t|k+1} S \quad (4h)$$
$$\forall k \in \{0\} \cup \mathbb{N}_{[M,N-1]}, \quad \forall \xi \in z_{t|k} \oplus a_{t|k} S, \quad A\xi + B(v_{t|k} + \sigma(\xi - z_{t|k})) \oplus \mathbb{W} \subseteq z_{t|k+1} \oplus a_{t|k+1} S \quad (4i)$$
$$\forall k \in \mathbb{N}_{[0,M-1]}, \quad u_{t+k} = v_{t|k} + \sigma(y_{t|k} - z_{t|k}) \quad (4j)$$
$$\forall k \in \mathbb{N}_{[1,M-1]}, \quad u_{t+k} = u_t, \quad (4k)$$

where the sets $S \subset \mathbb{R}^n$, $R \subset \mathbb{R}^m$, $\mathbb{G}_\mathrm{f} \subset \mathbb{R}^n \times \mathbb{R}_{\geq 0}$, and the function $\sigma : \mathbb{R}^n \to \mathbb{R}^m$ are fixed parameters of the MPC scheme and will be defined later.

Remark 1: In [12], two versions of self-triggered Tube MPC are considered: Packet-based control and sparse control. In the first case, constraint (4k) is excluded, in the second case it is included. In packet-based control, a sequence of M control inputs is transmitted at once, while in sparse control only one control input is transmitted and held constant by the actuator for M time steps. In the sparse case, overall less information has to be transmitted, while additionally the strain on the actuators might be reduced, as less changes in the input values occur. In this paper, we consider only the sparse control case, i. e. the control inputs are constant in the open-loop phase. It is possible without any restriction to apply the scheme without constraint (4k). Note that in the literature "sparse control" often refers to input signals that contain few non-zero entries, either at each point in time, if the input is a vector, or are non-zero only at infrequent points in time, cf. [11]. The scheme in this paper may easily be modified to this definition of "sparse" by changing constraint (4k) to $\forall k \in \mathbb{N}_{[1,M-1]}, u_{t+k} = 0$.

The set of all decision variables that satisfy the constraints (4) is given by

$$\mathscr{D}_N^M(x_t) = \{\mathbf{d}_N^M(x_t) \in \mathbb{D}_N^M \mid (4) \text{ holds}\} \ . \quad (5)$$

Define the cost function for self-triggered Homothetic Tube MPC as

$$J_N^M(\mathbf{d}_N^M) := \frac{1}{\beta} \sum_{k=0}^{M-1} \ell(z_{t|k}, a_{t|k}, v_{t|0}) + \sum_{k=M}^{N-1} \ell(z_{t|k}, a_{t|k}, v_{t|k}) + V_\mathrm{f}(z_{t|N}, a_{t|N}) \quad (6)$$

with the parameter $\beta \in \mathbb{R}_{\geq 1}$, the stage cost function $\ell : \mathbb{R}^n \times \mathbb{R} \times \mathbb{R}^m \to \mathbb{R}$, and the terminal cost function $V_\mathrm{f} : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$.

Remark 2: Note that for the first M time steps, $v_{t|0}$ instead of $v_{t|k}$ appears in the cost function. This choice was made in order to adapt the cost function to the fact that the input u_t is constant for the first M steps.

For a given state $x_t \in \mathbb{R}^n$ and inter-sampling time $M \in \mathbb{N}_{[1,N-1]}$, the problem formulation for the multiple-step Homothetic Tube MPC is given by

$$\mathscr{P}_N^M(x_t): \quad V_N^M(x_t) := \min_{\mathbf{d}_N^M \in \mathscr{D}_N^M(x_t)} J_N^M(\mathbf{d}_N^M) \quad (7a)$$
$$\hat{\mathbf{d}}_N^M(x_t) := \operatorname*{arg\,min}_{\mathbf{d}_N^M \in \mathscr{D}_N^M(x_t)} J_N^M(\mathbf{d}_N^M) \quad (7b)$$

with the value function V_N^M and the optimal decision variable

$$\hat{\mathbf{d}}_N^M(x_t) = [(\hat{z}_{t|0}(x_t), \dots, \hat{z}_{t|N}(x_t)), (\hat{a}_{t|0}(x_t), \dots, \hat{a}_{t|N}(x_t)), (\hat{y}_{t|0}(x_t), \dots, \hat{y}_{t|M-1}(x_t)), (\hat{v}_{t|0}(x_t), \dots, \hat{v}_{t|N-1}(x_t)), (\hat{u}_{t+0}(x_t), \dots, \hat{u}_{t+M-1}(x_t))] \ . \quad (8)$$

Remark 3: Note that the optimizer might not be unique. Assume in the remainder of the work, that $\hat{\mathbf{d}}_N^M$ is selected to be any minimizer of the cost function.

Assumption 2 (cf. [15]): The local controller σ is a continuous, positively homogeneous function of the first degree, that is, for all $\gamma \in \mathbb{R}_{\geq 0}$, $s \in \mathbb{R}^n$ it holds that $\sigma(\gamma s) = \gamma \sigma(s)$.

The sets that form the different tubes, are homothetic to the basic sets S, R and S^+, cf. [15].

Assumption 3 (cf. [15]): The set $S \subset \mathbb{R}^n$ is a C-set. Furthermore, for all $s \in S$ it holds that $As + B\sigma(s) \oplus \mathbb{W} \subseteq S$.

Define the set $R \subset \mathbb{R}^m$ as $R := \operatorname{convh}(\{\sigma(s) \mid s \in S\})$ and the set $S^+ \subset \mathbb{R}^n$ as $S^+ := \operatorname{convh}(\{As + B\sigma(s) \mid s \in S\})$, cf. [15].

Remark 4: By Assumption 2 and the definition of R, the following constraint is trivially satisfied

$$\forall k \in \mathbb{N}_{[0,N-1]}, \forall \xi \in z_{t|k} \oplus a_{t|k} S, \ \sigma(\xi - z_{t|k}) \in a_{t|k} R \ , \quad (9)$$

cf. [15].

For the construction of the terminal set and a terminal cost function, a linear terminal feedback controller of the form $u = Kx$ with $K \in \mathbb{R}^{m \times n}$ [13] and terminal tube size dynamics [15] are used. These will be introduced through the following definitions and assumptions.

Assumption 4 (cf. [15]): It holds that the absolute value of each eigenvalue of the matrix $(A + BK)$ is less than 1.

Define $\mu := \min\{\eta \in \mathbb{R}_{\geq 0} \mid \mathbb{W} \subseteq \eta S\}$ and $\lambda := \min\{\eta \in \mathbb{R}_{\geq 0} \mid S^+ \subseteq \eta S\}$, cf. [15]. Define further the parameter $\tilde{\mu} \in \mathbb{R}$.

Assumption 5: It holds that $\lambda \in [0,1)$ and that $\tilde{\mu} \geq \mu \geq 1$.

Define the constraint set for the terminal dynamics as

$$\mathbb{G} := \{(z,a) \mid z \oplus aS \subseteq \mathbb{X}, Kz \oplus aR \subseteq \mathbb{U}\} \ . \tag{10}$$

The terminal tube size dynamics is given by $a^+ = \lambda a + \tilde{\mu}$ with the equilibrium point $\bar{a} = (1-\lambda)^{-1}\tilde{\mu}$, cf. [15].

To ensure recursive feasibility and asymptotic stability of the closed-loop system, a terminal constraint is used. This terminal constraint (4e) is defined by the terminal set $\mathbb{G}_f \subset \mathbb{R}^n \times \mathbb{R}$, which we define as the maximal positively invariant set inside $\mathbb{G}$ for the terminal dynamics [15]

$$z^+ = (A+BK)z \tag{11a}$$

$$a^+ = \lambda a + \tilde{\mu} \ . \tag{11b}$$

Remark 5: The maximal positively invariant set can be computed as the *maximal output admissible set* as in [17].

Define the subset of $\mathbb{G}_f$ for fixed $a = \bar{a}$ in the state space as $\mathbb{Z}_f^{\bar{a}} := \{z \in \mathbb{R}^n \mid (z,\bar{a}) \in \mathbb{G}_f\}$. For the stability proof, we will need the following assumption.

Assumption 6: The set $\mathbb{Z}_f^{\bar{a}}$ is a PC-set.

The stage costs ℓ and the terminal cost V_f have to satisfy the following requirements.

Assumption 7 (Cost functions; cf. [12] and [15]): The stage cost function ℓ and the terminal cost function V_f are convex and continuous.

Assumption 8 (cf. [12]): There exist $\mathscr{K}_\infty$-functions α_1 and α_2 as well as $\mathscr{K}$-functions α_3, α_4, α_5, α_6, and α_7, such that for all $z \in \mathbb{R}^n$, $a \in \mathbb{R}$, $v \in \mathbb{R}^m$ it holds that

$$\ell(z,a,v) \geq \alpha_1(|z|) + \alpha_2(\max\{0, a-\bar{a}\}) + \alpha_3(|v|) \ , \tag{12}$$

and for all $(z,a) \in \mathbb{G}_f$

$$\ell(z,a,Kz) \leq \alpha_4(|z|) + \alpha_5(|a-\bar{a}|) \ , \tag{13}$$

$$V_f(z,a) \leq \alpha_6(|z|) + \alpha_7(|a-\bar{a}|) \ . \tag{14}$$

Assumption 9: There exists a scalar $\vartheta \in \mathbb{R}_{>0}$, such that for all $s \in \mathbb{R}_{>0}$ it holds that $\alpha_2(s) = \alpha_1(\vartheta s)$.

Assumption 10 (cf. [4]): For all $(z,a) \in \mathbb{G}_f$ it holds that

$$V_f((A+BK)z, \lambda a + \tilde{\mu}) \leq V_f(z,a) - \ell(z,a,Kz) \ . \tag{15}$$

IV. SELF-TRIGGERED HOMOTHETIC TUBE MPC

In this section, we present a predictive control algorithm, where the time until the next sampling instant is computed whenever the inputs are updated. This is achieved by selecting, at each sampling instant, the maximal possible length of the time span until the next sampling instant subject to constraints on the optimal cost of the associated MPC problem in (7). In particular, we require that the optimal cost of the multiple step open-loop MPC scheme is bounded by the optimal cost of a standard (that is, one-step open-loop) MPC scheme.

The self-triggered problem formulation is similar to [7] and [12]. The main difference lies in the specific choice of the cost function V_N^M and the constraint set $\mathscr{D}_N^M$ of the underlying MPC problem. Furthermore, there were no disturbances considered in [7].

Define for a given prediction horizon $N \in \mathbb{N}_{\geq 2}$ and a given maximal inter-sampling time $M_{\max} \in \mathbb{N}_{[1,N-1]}$ the self-triggered problem formulation for Homothetic Tube MPC

$$\mathscr{P}_N^{st}(x_t): \ \hat{M}(x_t) := \max_M\{M \in \mathbb{N}_{[1,M_{\max}]} \mid \mathscr{D}_N^1(x_t) \neq \emptyset, \mathscr{D}_N^M(x_t) \neq \emptyset, V_N^M(x_t) \leq V_N^1(x_t)\} \ , \tag{16a}$$

$$\hat{\mathbf{d}}_N^{\hat{M}(x_t)}(x_t) = \underset{\mathbf{d}_N^{\hat{M}(x_t)} \in \mathscr{D}_N^M(x_t)}{\arg\min} \ J_N^{\hat{M}(x_t)}\left(\mathbf{d}_N^{\hat{M}(x_t)}\right) \ . \tag{16b}$$

The resulting algorithm is given in Algorithm 1.

Algorithm 1 Self-triggered Homothetic Tube MPC

1: initialize $t = 0$
2: at time point t, measure the state of the system x_t
3: solve the self-triggered Homothetic Tube MPC problem $\mathscr{P}_N^{st}(x_t)$, obtain $\hat{M}(x_t)$ and $\hat{\mathbf{d}}_N^{\hat{M}(x_t)}(x_t)$
4: for all $k \in \mathbb{N}_{[0,\hat{M}(x_t)-1]}$ and time points $t+k$ apply the input $\hat{u}_{t+k}(x_t)$ to the system
5: at time point $t + \hat{M}(x_t)$, set $t = t + \hat{M}(x_t)$
6: go to step 2

The set of states where Algorithm 1 is feasible is given by

$$\hat{\mathbb{X}}_N := \left\{x \in \mathbb{X} \mid \mathscr{D}_N^1(x) \neq \emptyset\right\} \subseteq \mathbb{X} \subset \mathbb{R}^n \ . \tag{17}$$

The closed-loop system obtained by application of Algorithm 1 to the system (1a) is for all $i \in \mathbb{N}$ and for all $t \in \mathbb{N}_{[t_i, t_{i+1}-1]}$ given by

$$x_{t+1} = Ax_t + B\hat{u}_t(x_{t_i}) + w_t \ , \tag{18a}$$

$$t_{i+1} = t_i + \hat{M}(x_{t_i}) \ , \tag{18b}$$

$$t_0 = 0, \quad x_{t_0} = x_0 \ . \tag{18c}$$

V. CONSTRAINT SATISFACTION AND STABILITY

In this section we show that the proposed MPC scheme guarantees recursive feasibility of the optimization problem, constraint satisfaction, and that a positively invariant set containing the origin is asymptotically stabilized.

The following lemma ensures that the closed-loop system satisfies the state and input constraints.

Lemma 1: Given any $x_t \in \mathbb{X}$ with any $t \in \mathbb{N}$, such that $\mathscr{D}_N^M(x_t) \neq \emptyset$ for some $M \in \mathbb{N}_{[1,M_{\max}]}$. Let further

$$\mathbf{d}_N^M(x_t) = [(z_{t|0},\ldots,z_{t|N}),(a_{t|0},\ldots,a_{t|N}),(y_{t|0},\ldots,y_{t|M-1}), (v_{t|0},\ldots,v_{t|N-1}),(u_t,\ldots,u_{t+M-1})] \in \mathscr{D}_N^M(x_t) \tag{19}$$

and let for all $k \in \mathbb{N}_{[0,M-1]}$

$$x_{t+k+1} = Ax_{t+k} + Bu_{t+k} + w_{t+k} \ , \ w_{t+k} \in \mathbb{W} \ . \tag{20}$$

Then it holds that

$$\forall k \in \mathbb{N}_{[0,M]}, \quad x_{t+k} \in \mathbb{X} \ , \tag{21a}$$

$$\forall k \in \mathbb{N}_{[0,M-1]}, \quad u_{t+k} \in \mathbb{U} \ . \tag{21b}$$

Proof: By constraints (4a) and (4h) it follows that for all $k \in \mathbb{N}_{[0,M]}$ it holds that $x_{t+k} \in z_{t|k} \oplus a_{t|k}S$. Hence, it follows by (4c), (4d), and (9), that constraints (21a) and (21b) hold, thereby completing the proof. ■

The application of Algorithm 1 requires that the constrained optimization problem in (16) is feasible at every sampling instant $i \in \mathbb{N}$, $t_i \in \mathbb{N}$, as defined in (18). We prove this with the concept of recursive feasibility: If optimization problem (16a) is feasible at initialization, then it remains feasible at every point in the state space and any sampling time generated by the closed-loop system (18), cf. [5]. In particular, we show that if optimization problem (16a) is feasible at initialization, then optimization problem (7) is feasible for $M = 1$ at every future point in time for the closed-loop system (18), i. e. $\mathscr{D}_N^1(x_{t+1}) \neq \emptyset$. This is established in the following lemma.

Lemma 2: Let $t \in \mathbb{N}$. Assume that $\hat{M} = \hat{M}(x_t)$ and

$$\hat{\mathbf{d}}_N^{\hat{M}}(x_t) = [(\hat{z}_{t|0}, \ldots, \hat{z}_{t|N}), (\hat{a}_{t|0}, \ldots, \hat{a}_{t|N}), (\hat{y}_{t|0}, \ldots, \hat{y}_{t|\hat{M}-1}), (\hat{v}_{t|0}, \ldots, \hat{v}_{t|N-1}), (\hat{u}_{t+0}, \ldots, \hat{u}_{t+\hat{M}-1})] \tag{22}$$

is a solution of $\mathscr{P}_N^{st}(x_t)$ and that it holds that

$$\forall j \in \mathbb{N}_{[0,\hat{M}-1]}, \quad x_{t+j+1} = A x_{t+j} + B \hat{u}_{t+j} + w_{t+j} \ , \tag{23a}$$

$$w_{t+j} \in \mathbb{W} \ . \tag{23b}$$

Then it holds that $\mathscr{D}_N^1(x_{t+\hat{M}}) \neq \emptyset$, that is $x_{t+\hat{M}} \in \hat{\mathbb{X}}_N \subseteq \mathbb{X}$.

Proof: Define $\tilde{u}_{t+\hat{M}} := \hat{v}_{t|\hat{M}} + \sigma(x_{t+\hat{M}} - \hat{z}_{t|\hat{M}})$. Consider the candidate solution to $\mathscr{P}_N^{\rm st}(x_{t+\hat{M}})$

$$\begin{aligned}\tilde{\mathbf{d}}_N^{1,\hat{M}} := [&(\hat{z}_{t|\hat{M}}, \ldots, \hat{z}_{t|N}, (A+BK)\hat{z}_{t|N}, \ldots, (A+BK)^{\hat{M}}\hat{z}_{t|N}),\\ &(\hat{a}_{t|\hat{M}}, \ldots, \hat{a}_{t|N}, \lambda\hat{a}_{t|N} + \tilde{\mu}, \ldots, \lambda^{\hat{M}}\hat{a}_{t|N} + \textstyle\sum_{l=0}^{\hat{M}-1} \lambda^l \tilde{\mu}),\\ &(x_{t+\hat{M}}), (\hat{v}_{t|\hat{M}}, \ldots, \hat{v}_{t|N-1}, K\hat{z}_{t|N}, K(A+BK)\hat{z}_{t|N}, \ldots\\ &\ldots, K(A+BK)^{\hat{M}-1}\hat{z}_{t|N}), (\tilde{u}_{t+\hat{M}})] \ .\end{aligned} \tag{24}$$

To prove recursive feasibility, it is necessary to prove that (24) satisfies all the constraints in (4).

The constraints (4f), (4g), (4h), (4j) and (4k) are trivially satisfied. The satisfaction of (4a), i.e. $x_{t+\hat{M}} \in \hat{z}_{t|\hat{M}} \oplus \hat{a}_{t|\hat{M}} S$, is shown in the proof of Lemma 1.

By $\hat{\mathbf{d}}_N^{\hat{M}}(x_t) \in \mathscr{D}_N^{\hat{M}}(x_t)$ it holds that $\hat{a}_{t|\hat{M}}, \ldots, \hat{a}_{t|N} \geq 0$. As $\lambda, \tilde{\mu} \geq 0$ it holds for all $i \in \mathbb{N}_{\geq 0}$ that $\lambda^i \hat{a}_{t|N} + \sum_{l=0}^{i-1} \lambda^l \tilde{\mu} \geq 0$, which proves the satisfaction of (4b).

By $\hat{\mathbf{d}}_N^{\hat{M}}(x_t) \in \mathscr{D}_N^{\hat{M}}(x_t)$ it holds that $(\hat{z}_{t|N}, \hat{a}_{t|N}) \in \mathbb{G}_{\rm f}$. Additionally, by definition, the terminal set $\mathbb{G}_{\rm f}$ is positively invariant for the terminal dynamics in (11). Hence, by the definition of $\mathbb{G}$ and $\mathbb{G}_{\rm f}$, it holds for all $i \in \mathbb{N}_{[0,N-1]}$ that

$$(A+BK)^i \hat{z}_{t|N} \oplus \left(\lambda^i \hat{a}_{t|N} + \textstyle\sum_{l=0}^{i-1} \lambda^l \tilde{\mu}\right) S \ \subseteq \ \mathbb{X} \ , \tag{25a}$$

$$K(A+BK)^i \hat{z}_{t|N} \oplus \left(\lambda^i \hat{a}_{t|N} + \textstyle\sum_{l=0}^{i-1} \lambda^l \tilde{\mu}\right) R \ \subseteq \ \mathbb{U} \ , \tag{25b}$$

$$\left((A+BK)^i \hat{z}_{t|N}, \lambda^i \hat{a}_{t|N} + \textstyle\sum_{l=0}^{i-1} \lambda^l \tilde{\mu}\right) \in \mathbb{G}_{\rm f} \ , \tag{25c}$$

proving the satisfaction of (4c), (4d), and (4e), respectively, cf. Remark 1 in [15] .

Constraint (4i) is trivially satisfied for $k = 0, \ldots, N - \hat{M} - 1$. For $k = N - \hat{M}, \ldots, N$, the satisfaction of (4i) is ensured by the terminal constraint (4e) and the assumption on the terminal dynamics in (11), see Remark 1 in [15].

By the results above it holds that $\mathscr{D}_N^1(x_{t+\hat{M}}) \neq \emptyset$ and $x_{t+\hat{M}} \in \hat{\mathbb{X}}_N \subseteq \mathbb{X}$, which completes the proof. ■

The satisfaction of the state and input constraints are summarized in the following theorem.

Theorem 1: Given any initial value $x_0 \in \hat{\mathbb{X}}_N$. Then for the closed-loop system (18), it holds for all $t \in \mathbb{N}$ that $u_t \in \mathbb{U}$ and $x_t \in \mathbb{X}$.

Proof: The statement follows directly from Lemma 1 and Lemma 2. ■

Next, two lemmas are now introduced as auxiliary results to prove the existence of a Lyapunov function, which will be used for the main stability theorem.

Lemma 3: There exist a $\mathscr{K}_\infty$-function α_{11} and a $\mathscr{K}$-function α_{12}, such that for the closed-loop system (18), for any $i \in \mathbb{N}$, $t_i \in \mathbb{N}$, $x_0 \in \hat{\mathbb{X}}_N$, and $j \in \mathbb{N}_{[0,\hat{M}(x_{t_i})-1]}$, it holds that

$$\alpha_{11}(|x_{t_i+j}|_{\bar{a}S}) \leq V_N^1(x_{t_i}) \leq \alpha_{12}(|x_{t_i}|_{\bar{a}S}) \ . \tag{26}$$

Due to space limitations, the proof is omitted here. The proof follows along similar lines as the proof of Lemma 4 in [14], where the interested reader is referred to.

Lemma 4: For the closed-loop system (18) with $x_0 \in \hat{\mathbb{X}}_N$ and control update time points t_i with $i \in \mathbb{N}$, it holds that

$$V_N^1(x_{t_{i+1}}) \leq V_N^1(x_{t_i}) - \frac{1}{\beta}\alpha_1(|x_{t_i}|_{\bar{a}S}) \ . \tag{27}$$

Proof: By the definition of $\mathbb{G}_{\rm f}$ and Assumption 10, it holds for all $k \in \mathbb{N}$ and for all $(z,a) \in \mathbb{G}_{\rm f}$ that

$$\begin{aligned}&\ell\left((A+BK)^k z,\ \lambda^k a + \textstyle\sum_{l=0}^{k-1} \lambda^l \tilde{\mu},\ K(A+BK)^k z\right)\\ &+ V_{\rm f}\left((A+BK)^{k+1} z,\ \lambda^{k+1} a + \textstyle\sum_{l=0}^{k} \lambda^l \tilde{\mu}\right)\\ &\leq V_{\rm f}\left((A+BK)^k z,\ \lambda^k a + \textstyle\sum_{l=0}^{k-1} \lambda^l \tilde{\mu}\right) \ .\end{aligned} \tag{28}$$

Consider again the candidate solution $\tilde{\mathbf{d}}_N^{1,\hat{M}}$ given in (24) in the proof of Lemma 2 for $t = t_i$ and $t_{i+1} = t_i + \hat{M}$. With the substitution $A_{\rm cl} := A + BK$ it holds that

$$\begin{aligned}J_N^1(\tilde{\mathbf{d}}_N^{1,\hat{M}}) = &\frac{1}{\beta}\ell(\hat{z}_{t_i|\hat{M}}, \hat{a}_{t_i|\hat{M}}, \hat{v}_{t_i|\hat{M}}) + \textstyle\sum_{k=\hat{M}+1}^{N-1} \ell(\hat{z}_{t_i|k}, \hat{a}_{t_i|k}, \hat{v}_{t_i|k})\\ &+ \textstyle\sum_{k=0}^{\hat{M}-1} \ell\left(A_{\rm cl}^k \hat{z}_{t_i|N}, \lambda^k \hat{a}_{t_i|N} + \sum_{l=0}^{k-1} \lambda^l \tilde{\mu}, K A_{\rm cl}^k \hat{z}_{t_i|N}\right)\\ &+ V_{\rm f}\left(A_{\rm cl}^{\hat{M}} \hat{z}_{t_i|N}, \lambda^{\hat{M}} \hat{a}_{t_i|N} + \textstyle\sum_{l=0}^{\hat{M}-1} \lambda^l \tilde{\mu}\right) \ .\end{aligned}$$

Considering that $\beta \geq 1$, it follows that

$$\begin{aligned}J_N^1(\tilde{\mathbf{d}}_N^{1,\hat{M}}) \leq &V_N^{\hat{M}}(x_t) - \frac{1}{\beta}\textstyle\sum_{k=0}^{\hat{M}-1} \ell(\hat{z}_{t_i|k}, \hat{a}_{t_i|k}, \hat{v}_{t_i|0})\\ &+ \textstyle\sum_{k=0}^{\hat{M}-1} \ell\left(A_{\rm cl}^k \hat{z}_{t_i|N}, \lambda^k \hat{a}_{t_i|N} + \sum_{l=0}^{k-1} \lambda^l \tilde{\mu}, K A_{\rm cl}^k \hat{z}_{t_i|N}\right)\\ &+ V_{\rm f}\left(A_{\rm cl}^{\hat{M}} \hat{z}_{t_i|N}, \lambda^{\hat{M}} \hat{a}_{t_i|N} + \textstyle\sum_{l=0}^{\hat{M}-1} \lambda^l \tilde{\mu}\right) - V_{\rm f}(\hat{z}_{t_i|N}, \hat{a}_{t_i|N})\\ \leq &V_N^{\hat{M}}(x_{t_i}) - \frac{1}{\beta}\textstyle\sum_{k=1}^{\hat{M}-1} \ell(\hat{z}_{t_i|k}, \hat{a}_{t_i|k}, \hat{v}_{t_i|0}) \ ,\end{aligned} \tag{29}$$

where the last line of (29) follows from the repeated application of (28). By (16a) it holds that $V_N^{\hat{M}}(x_{t_i}) \leq V_N^1(x_{t_i})$, such that finally

$$
\begin{aligned}
V_N^1(x_{t_{i+1}}) \le J_N^1(\tilde{\mathbf{d}}_N^{1,\hat{M}}) &\le V_N^1(x_{t_i}) - \frac{1}{\beta}\sum_{k=0}^{\hat{M}-1} \ell(\hat{z}_{t_i|k},\, \hat{a}_{t_i|k},\, \hat{v}_{t_i|0}) \\
&\le V_N^1(x_{t_i}) - \frac{1}{\beta}\ell(\hat{z}_{t_i|0},\, \hat{a}_{t_i|0},\, \hat{v}_{t_i|0}) \\
&\le V_N^1(x_{t_i}) - \frac{1}{\beta}\alpha_1(|x_{t_i}|_{\bar{a}S}) \ , \qquad (30)
\end{aligned}
$$

where the last inequality follows from reasoning similar to the proof of Lemma 4 in [14]. This completes the proof. ■

The following theorem delivers the main stability results.

Theorem 2: The set $\bar{a}S$ is asymptotically stable for the closed-loop system under the control of Algorithm 1 with a region of attraction $\hat{\mathbb{X}}_N$.

Proof: By the Assumption 6, the set $\bar{a}S$ is contained in the interior of the set $\hat{\mathbb{X}}_N$. By Lemma 3 and 4, if $x_0 \in \hat{\mathbb{X}}_N$, it holds for the closed-loop system that

$$\forall i \in \mathbb{N}, \forall t \in \mathbb{N}_{[t_i, t_{i+1}-1]}, \ \ \alpha_{11}(|x_t|_{\bar{a}S}) \le V_N^1(x_{t_i}) \qquad (31)$$

and, hence,

$$\forall t \in \mathbb{N}, \ |x_t|_{\bar{a}S} \le \alpha_{11}^{-1}(V_N^1(x_0)) \le \alpha_{11}^{-1}(\alpha_{12}(|x_0|_{\bar{a}S})) \ , \qquad (32)$$

proving that the set $\bar{a}S$ is Lyapunov stable for the closed-loop system. As additionally by Lemma 4 $\lim_{i\to\infty} V_N^1(x_{t_i}) = 0$, it follows that

$$\lim_{t\to\infty} |x_t|_{\bar{a}S} = 0 \ , \qquad (33)$$

proving that the set $\bar{a}S$ is attractive for any initial condition in $\hat{\mathbb{X}}_N$, thereby completing the proof. ■

Remark 6: For larger inter-sampling times M, the number of constraints in the optimization problem increases, leading, in general, to a larger optimal cost function. Increasing the parameter β acts against this effect, enlarging the set in the state space where $V_N^M(x) \le V_N^1(x)$ holds, and in turn leading to a longer average inter-sampling time during the transient phase of the closed-loop system. The effect of the parameter β decreases for states close to the set $\bar{a}S$, and vanishes for $x_t \in \bar{a}S$, as V_N^1 is zero on this set. However, on some subset of $\bar{a}S$ it also holds that V_N^M is zero. This subset increases with increasing $\bar{a}$, making it possible to increase the average inter-sampling time for the limit-behavior of the closed-loop system by increasing the parameter $\bar{a}$.

Summarizing, the parameters β and $\bar{a}$ are tuning parameters that determine the triggering behavior of the system. Increasing the parameter β increases the average inter-sampling time during the transient phase while possibly sacrificing performance, see also [7]. Increasing the parameter $\bar{a}$ increases the infinite horizon average inter-sampling time while also increasing the asymptotic bound on the system state.

VI. NUMERICAL EXAMPLE

Example 1: In order to compare the MPC scheme in this paper with that in [12], the same numerical example as in [12] is chosen with parameters exactly the same whenever applicable. Some parameters are adapted from [15]. Consider the system

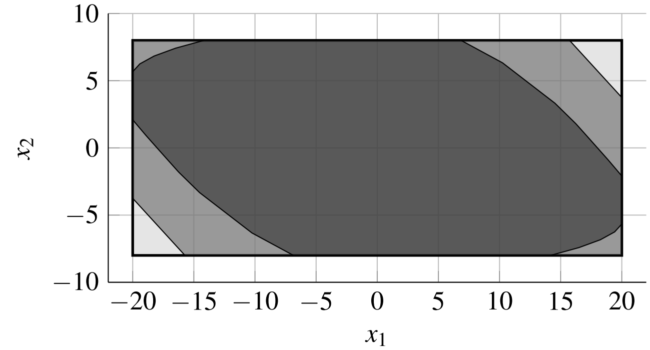

Fig. 1. Example 1: The light gray set is the state constraint set $\mathbb{X}$. The middle gray set is the region of attraction $\hat{\mathbb{X}}_{20}$. The dark gray set is the region of attraction of the scheme in [12].

$$x_{t+1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x_t + \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} u_t + w_t \qquad (34)$$

subject to the constraints $x_t \in [-20, 20] \times [-8, 8]$, $u_t \in [-8, 8]$ and $w_t \in [-0.25, 0.25] \times [-0.25, 0.25]$. Consider the matrices $Q_z = \left[\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right]$ and $Q_v = 0.1$ and the factor $q_a = 0.1$. The matrix P_z is chosen as the solution of the Discrete Algebraic Riccati Equation $P_z = Q_z + A^\top P_z A - A^\top PB(Q_v + B^\top PB)^{-1} B^\top P_z A$ and will be used as a terminal weighting matrix. The matrix K is chosen as the corresponding optimal feedback matrix $K = -(Q_v + B^\top P_z B)^{-1} B^\top P_z A$. The prediction horizon is set to $N = 20$ and the maximal inter-sampling time to $M_{\max} = 5$. The set S is equal to the $M_{\max}$-step $(A, B, K, \mathbb{W})$-invariant set $\mathbb{E}$ from [12]. Define the factor $p_a = (1-\lambda^2)^{-1} q_a$. For the local controller function it holds that $\sigma(s) = Ks$. For the sets R and S^+ it holds that $R = KS$ and $S^+ = (A + BK)S$, respectively. The regions of attraction for the different schemes are compared in Figure 1. The region of attraction for the scheme presented in this paper covers nearly the whole set $\mathbb{X}$ for this example. The enlargement of the region of attraction when compared to [12] is due to the additional degrees of freedom $a_{t|k}$ in the definition of the tube in (4).

Example 2: Consider system (34) subject to the constraints $x_t \in [-20, 20] \times [-10, 10]$, $u_t \in [-15, 15]$ and $w_t \in [-0.25, 0.25] \times [-0.25, 0.25]$. Define the cost functions

$$\ell(z, a, v) := z^\top Q_z z + q_a \min_{b \ge a - \bar{a}} b^2 + v^\top Q_v v \qquad (35a)$$

$$V_{\mathrm{f}}(z, a) := z^\top P_z z + p_a \min_{b \ge a - \bar{a}} b^2 \ . \qquad (35b)$$

The tuning parameters $\tilde{\mu}$ and β are set to $\tilde{\mu} = 4\mu$ and $\beta = 2$, respectively. The set S is chosen as a robust positively invariant outer ε-approximation of the minimal robust positively invariant set with $\varepsilon = 2$, cf. [16]. The numerical tolerance for checking the constraint $V_N^M(x_t) \le V_N^1(x_t)$ in (16a) is set to $\varepsilon_{\text{num}} = 10^{-12}$. The initial conditions are set to $t_0 = 0$ and $x_0 = \begin{bmatrix} 10 & 6 \end{bmatrix}^\top$. The simulation end time is $t_{\mathrm{f}} = 100$. The disturbances are chosen to be constant, i. e., $w_t = \begin{bmatrix} -0.25 & -0.25 \end{bmatrix}^\top$, $i \in \mathbb{N}$. The remaining parameters are the same as in Example 1. Note that the cost functions in (35) satisfy Assumptions 7 and 8.

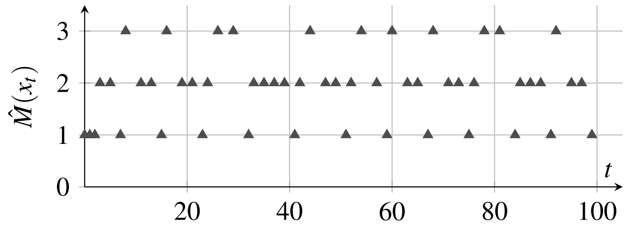

Fig. 2. Example 2: Inter-sampling times $\hat{M}(x_{t_i})$ calculated at sampling time points t_{t_i} for self-triggered Homothetic Tube MPC.

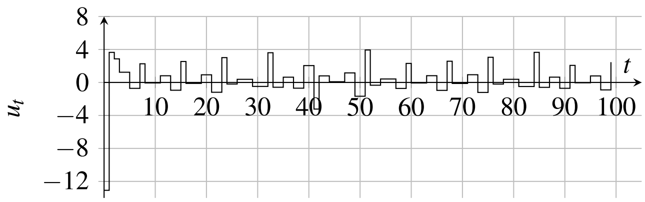

Fig. 3. Example 2: Inputs u_t by self-triggered Homothetic Tube MPC.

In the Figures 2, 3, and 4 we plotted the inputs, the inter-sampling times and the trajectory, respectively. The average inter-sampling time is $\bar{M} = 1.9231$, which leads to 48.00% less communication when compared to an implementation with sampling at every point in time.

VII. CONCLUSIONS AND OUTLOOK

In this paper, a self-triggered model predictive controller was presented, which was shown to robustly stabilize a closed and bounded set in the state space including the origin its interior while ensuring the satisfaction of hard constraints on the inputs and states of the system. It was shown that, at least for one example, the controller leads to a larger feasible set when compared to the earlier approach presented in [12]. The enlargement of the feasible set is due to the higher flexibility of the tubes employed in the scheme proposed in this paper, which is based on the robust MPC scheme presented in [15]. The extension of robust self-triggered MPC to linear system with multiplicative uncertainty and nonlinear systems is a topic of future research.

ACKNOWLEDGMENT

MATLAB R2012a together with YALMIP [18], Multi-Parametric Toolbox 3.0 [19] and IBM ILOG CPLEX Optimization Studio were used for the simulations.

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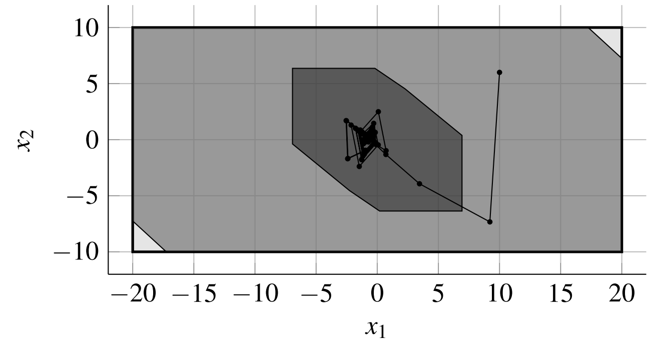

Fig. 4. Example 2: Trajectory of the closed-loop system for self-triggered Homothetic Tube MPC. The light gray set is the state constraint set $\mathbb{X}$. The middle gray set is the region of attraction $\hat{\mathbb{X}}_{20}$. The dark gray set is the stabilized set $\bar{a}S$.

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